Public Investments in Disaster Risk Reduction – A Social Cost Benefit Analysis

Dissertation

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*) Either the German or the Italian form of the title may be used.

Abstract

The cautious treatment of natural disaster risk constitutes a major challenge for governments of affected countries. In the evaluation of public risk reduction interventions it should always be warranted that the scarce invested means are allocated efficiently and thus possess the potential to increase social welfare. In the present thesis public risk reduction projects are considered as investments and are evaluated on basis of a social cost benefit analysis. In the first part of the thesis an innovative risk management framework is developed, that allows to systematically identify the social exposure to natural hazards and to quantify disaster risk probabilistically both before and after implementation of the risk reduction measure. The second focus of the thesis constitutes the economic evaluation of human safety. In particular, general cost benefit rules for pricing enhanced human safety on basis of the willingness to pay (WTP) concept are derived. Special emphasis is given on the Life Quality Index (LQI) concept, which has recently gained increasing attention as a risk management tool in civil engineering. For the latter a new time consistent derivation method is developed in a general equilibrium model that clearly reveals the economic reasoning behind the index. Based on this the conventional LQI based safety pricing rule is extended and improved in its calibration. In the last part of the thesis the concept of real options is firstly applied to the evaluation of public disaster risk reduction interventions and the traditional net present value criterion is replaced by more extensive decision rules in the presence of uncertainty. Eventually, a detailed case study for the seismic risk management of San Francisco based on the disaster loss estimation program HAZUS demonstrates the practical applicability of the presented methodology.

Zusammenfassung

Der planvolle Umgang mit Naturkatastrophen stellt eine große Herausforderung für Regierungen betroffener Länder dar. Bei der Beurteilung öffentlicher Risikoreduktionsmaßnahmen sollte stets gewährleistet sein, dass die aufgewendeten begrenzten Ressourcen effizient eingesetzt werden und potentiell zu einer Erhöhung der gesellschaftlichen Wohlfahrt beitragen. In der vorliegenden Arbeit werden gesellschaftliche Risikoreduktionsmaßnahmen als Investitionen betrachtet und anhand einer sozialen Kosten-Nutzen Analyse bewertet. Im ersten Teil der Arbeit wird hierzu zunächst ein innovatives Risikomanagementkonzept entwickelt, das es ermöglicht, die Gefährdungssituation einer Gesellschaft systematisch zu erfassen und das damit einhergehende Risiko vor und nach Durchführung der Risikoreduktionsmaßnahme probabilistisch abzuschätzen. Der zweite Schwerpunkt der Arbeit liegt in der volkswirtschaftlichen Bewertung menschlicher Sicherheit. Hierbei wird speziell die wichtige Fragestellung behandelt, wie die Zahlungsbereitschaft einer Gesellschaft zur Rettung von Menschenleben ökonomisch erfasst werden kann und welche Position der ingenieurwissenschaftliche Life Quality Index (LQI) in diesem Kontext einnimmt. Für Letzteren wird im Rahmen eines gesamtwirtschaftlichen Gleichgewichtsmodells eine neue konsistente Herleitungsmethode bereitgestellt und darauf aufbauend das bestehende LQI basierte Akzeptanzkriterium erweitert und in seiner Kalibrierung verbessert. Im letzten Teil der Arbeit wird das in der Betriebswirtschaft zunehmend an Bedeutung gewinnende Konzept sogenannter Realoptionen auf Risikoreduktionsmaßnahmen von Naturkatastrophen übertragen und schließlich das traditionelle Barwertkriterium durch innovative Entscheidungsregeln in der Gegenwart von Unsicherheit ersetzt. Eine detaillierte Fallstudie für das Erdbebenrisikomanagement von San Francisco unter Verwendung des Naturkatastrophen-Simulationsprogramms HAZUS demonstriert die Anwendbarkeit der Methoden und rundet die Arbeit ab.

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Eventually, I would like to express my gratitude to Prof. Dr. Claudio Borri for his effort as the Italian initiator and coordinator of the PhD Course, who contributed to enable living this great experience. During the previous years in this program, I enjoyed many enriching conversations and discussions with people from distinct countries around the world, I learned that there is not only one view of the world but that there are numerous possibilities to interpret certain situations and overcome difficulties. Not only the extension of my horizon but equally important the great new friendships that i made contribute to the fact that this overwhelming period of my life will remain unforgettable to me.

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Timm Pliefke

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Abbreviations and Nomenclature

α	Share of rents on capital in total GDP
β	Share of wages in total GDP
ϵ	Elasticity of marginal utility of consumption
γ	GDP growth rate per capita
Λ	Lagrange function of a constraint optimization problem
λ	Lagrange multiplier
μ	Crude mortality rate
$\mu(x)$	Age specific mortality rate
$\overline{\sigma}$	Volatility
Π	Profits
Ψ	Annual disaster occurrence probability
ρ	Rate of pure time preferences
σ	Standard deviation
σ^2	Variance
τ	Taxes
$\widehat{l}$	Annual per capita leisure time
$\widetilde{e}_d$	Age averaged discounted life expectancy in society
A	Technological knowledge factor
a	Life working fraction
a_d	Annual work demand fraction
a_s	Annual work supply fraction
AC	Annuity cost of a risk reduction project
B	Risk free bond
B^T	Total benefits of a risk reduction project
B^T/C^T	Benefit to cost ratio of a risk reduction project
B^{CSH}	Cultural-social-histrical benefit of a risk reduction project
B^{econ}	Economic benefit of a risk reduction project
B^{env}	Environmental benefit of a risk reduction project
B^{hum}	Human benefit of a risk reduction project
C	Number of consumers in society
C^I	Investment cost of a risk reduction project

C^O	Operation cost of a risk reduction project
C^R	Replacement cost of a risk reduction project
C^T	Total cost of a risk reduction project
C^{MR}	Maintenance & repair cost of a risk reduction project
C^{RV}	Residual value of a risk reduction project
C_t	Option price at time t
CSH	Cultural social historical
CV	Compensating variation
D	Dividend payments
d	Downturn in the stock price
D_e	Distance to epicenter
DA	Damage
DC	Damage control (FEMA 156 rehabilitation level)
$e(x)$	Remaining life expectancy at age x
$e = e(0)$	Life expectancy at birth
$e_d(x)$	Discounted remaining life expectancy at age x
$EaNR$	Element at non risk
EaR	Element at risk
ES	Expected shortfall
EV	Equivalent variation
EX	Exports
F	Number of producers in society
$F(.)$	Production function
$f(.)$	Per capita production function
$F_D(x)$	Cumulative probability function of not having survived until age x
$f_D(x)$	PDF of living exactly for x years
G	Government spending
g	Gravity
GDP	Gross domestic product
HDI	Human Development Index
HI	Hazard intensity
HL	Hazard load
I	Aggregate investment
IM	Imports
IO	Immediate occupancy (FEMA 156 rehabilitation level)
IRR	Internal rate of return of a risk reduction project
K	Aggregate capital
k	Per capita capital
L	Aggregate labor

l	Annual per capita labor time
LO	Loss
LQI	Life Quality Index
LS	Life safety (FEMA 156 rehabilitation level)
M	Number of deaths per year
m	Number of saved lives per year through risk reduction
M_w	Moment magnitude
MPK	Marginal product of capital
MPL	Marginal product of labor
MRS	Marginal rate of substitution
MRT	Marginal rate of transformation
$MRTS$	Marginal rate of technical substitution
N	Total population size
$NCHS$	Net cost into human safety
NPV	Net present value
P	Exercise price of a call option
p	Price of consumption
PDF	Probability density function
PGA	Peak ground acceleration
PPF	Production possibilities frontier
q	Elasticity of LQI
R	Aggregate rents on capital
r	Rent on capital, interest rate
R_D	Structural risk
R_L	Total risk
s	Safety
$S(x)$	Survival probability of living at least until age x
S^*	Threshold value from which on immediate investment becomes optimal
S_t	Price of underlying of a call option at time t
S_t	Total value of a risk reduction project at time t
SPP	Social planner's problem
SWF	Social welfare function
$SWTP$	Social willingness to pay
T	Time to maturity
t_{max}	Maximum attainable age in society
Tr	Return period of a natural hazard
U	Direct utility function
u	Upturn in the stock price
V	Indirect utility function

V_t	Portfolio value at time t
VaR	Value at risk
VSL	Value of statistical life
$VSLY$	Value of a statistical life year
W	Aggregate wages
w	Wage rate
WTA	Willingness to accept
WTP	Willingness to pay
X	Aggregate consumption
x	(Vector of) consumption good(s)
y	(Non labor) income

Chapter 1

Introduction

Natural disasters appear in many shapes and sizes. Most of them are related to the weather. Some are largely predictable - like a hurricane. Others, like an earthquake, come surprisingly. Generally speaking, a natural event is a sudden and violent change of the environment due to extreme atmospherical, hydrological or geological natural forces. A natural event per se might not have an impact on human activities and turns into a natural hazard only if it encounters exposed human assets. A natural hazard on the contrary converts into a natural disaster, if the extreme natural forces strike vulnerable elements and cause considerable harm to the affected region. A natural disaster can result in serious damage to buildings and infrastructure, in injuries and loss of life, in the devastation of the environment and in the loss of cultural, social and historical assets.

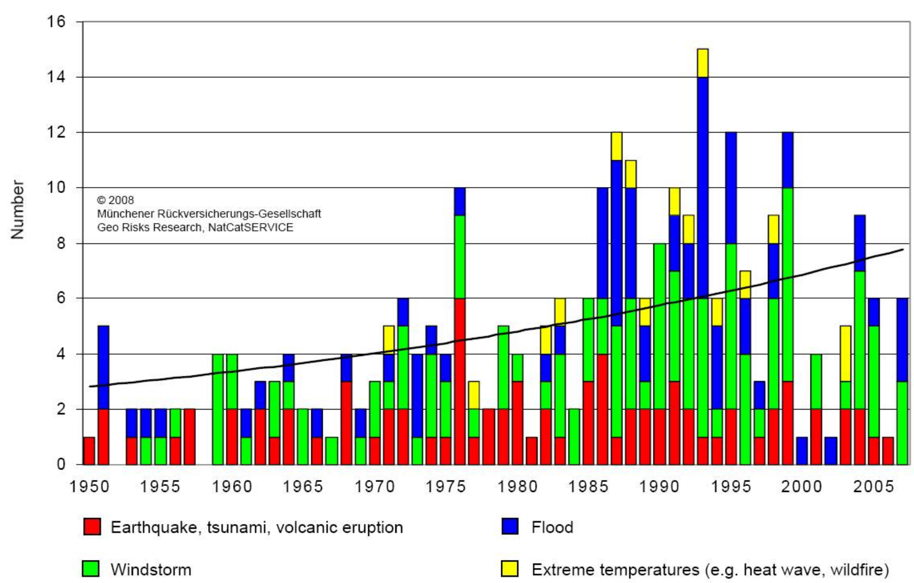


Figure 1.1: Great natural disasters 1950-2007 [Munich Re Group 2007 [99]]

An exponential population growth coinciding with the new and often fast colonization of hazard prone areas such as riversides and coastlines as well as the change of environmental conditions has led to a permanent increase in natural disaster occurrences throughout recorded history, as depicted in Figure 1.1. It is easily verified that windstorms, floods and earthquakes take the lead in the great disaster occurrences of the last 60 years, roughly accounting for a little less than one third each.

According to the understanding of the Munich Re Group [98], an event is declared as being a great disaster if it causes either substantial overall losses in relation to the country's economy or results in thousands of fatalities or hundreds of thousands of people made homeless, making interregional or even international assistance inevitable.

Major natural catastrophes caused economic losses estimated at an average of US$ 66 billion per year throughout the 1990s and resulted in an average annual number of 79,000 fatalities [26], accompanied by innumerable injuries and people that were made homeless. Whereas the major part of absolute economic losses occurred in developed countries, the number of fatalities and economic losses in relative terms, as a fraction of domestic GDP, were substantially larger in developing countries [165]. This shows evidence that despite a high concentration of values in the developed world, people and property enjoy a higher level of protection due to risk reduction measures that have been implemented in the past. Hence, there is the urgent need for national governments to engage in risk reduction activities in order to prevent the impacts of natural hazards from becoming devastating and thus, to guarantee an adequate safety level for their citizens.

Several studies have demonstrated that social risk reduction measures can bring about large benefits in terms of reduced disaster consequences and thus represent a sound investment. The World Bank and the U.S. Geological Survey estimated that an investment of US$ 40 billion into preventative disaster risk mitigation strategies could have led to a reduction of US$ 280 billion economic losses worldwide in the 1990s [25]. Independently, the U.S. National Institute of Building Sciences [190] claims that on average, a dollar spend by the FEMA[1] on hazard mitigation provides the United States around US$ 4 in future benefits.

1.1 Problem Specification

A rational response to these extreme natural events requires a collaboration of researchers from many different disciplines. Natural scientists are needed to estimate the hazard characteristic parameters such as probability of occurrence and intensity for a special location. Engineers are required to analyze, how endangered buildings and infrastructure elements are likely to respond due to natural disaster loads. Economic expert judgments are necessary to estimate the monetary consequences of the damage and harm to the affected region both short and long term. Politicians then have to process the accumulated information by deciding on how to handle the peril to protect people and property in the best possible way.

The necessity to consider disaster management throughout several disciplines and for distinct natural hazards has led to the development of a great diversity of risk management definitions and methods within scientific community. So far no consistency in the risk management terminology has been achieved, making problems in communication inevitable. Different definitions as well as ways to estimate and evaluate risk frequently lead to results which are not comparable as the underlying range of consequences that is included in the calculation is quite uneven. Contributing to this problem is the fact that numerous risk management methodologies presented in literature lack a clear mathematical formulation of the risk defining terms and of steps to be performed within the respective approach. Therefore, costly risk studies often do not provide sufficient assistance to

[1]FEMA = Federal Emergency Management Agency (U.S. government agency tasked with disaster mitigation, preparedness, response and recovery planning)

decision makers and accordingly, huge mistakes can be made. A unified and structured methodology to define and to calculate risk throughout various disciplines is indispensable for a rational quantification, comparison, and treatment of risks. Only if the frequency and the impact of potentially threatening hazards can be reliably estimated and communicated in the same language, will it finally be possible to identify and implement efficient risk reduction strategies.

The economic evaluation of public risk reduction projects is an important task within disaster management. From an economic point of view, a public risk reduction measure is characterized by a flow of benefits and a flow of cost that occur uncertainly at different points in time over the effective period. Whereas the cost arise mainly due to the material and labor needed to implement and maintain the measure and are assessable with sufficient accuracy, the benefits are represented by the reduced expected disaster consequences, that occur over a wide range of categories and are highly uncertain. They appear in terms of reduced damages to buildings and infrastructure elements, reduced economic losses, prevented fatalities and environmental damages and the preservation of cultural social and historical assets. In order to judge on project desirability, the various intangible benefits have to be converted to monetary value to compare them to the project cost. The estimation of a social price for intangible goods is further complicated by the fact that society consists of numerous heterogeneous individuals, trying to cultivate their own subjective understanding of valuation. This often results in a conflict of interest.

In particular, the evaluation of human safety constitutes a controversial and highly debated field of research. While each individual supplements her safety level herself to a certain extent by the daily choices that are made in favor of certain consumption goods and labor provision, the state, federal, and local governments often take the lead in providing a baseline safety level that applies to all [145]. Consequently, it is largely the government that decides what level of safety its limited public resources can afford. There is an increasing cost to a fixed increment of safety. Absolute safety cannot be obtained and the level provided for a reasonable cost requires a compromise. Allocating resources in disaster risk reduction means taking resources away elsewhere, such as health care, education and social services that also have the potential to enhance safety [195]. Therefore, trade-offs are inevitable in the decision processes of competing needs in order to finally achieve an efficient and affordable safety provision for society.

This ambition can be reached only if a reasonable way has been found to derive an economic value of human life. Instead of placing a value on the life of any particularly identifiable individual, which would obviously contradict with our ethical understanding that human life is priceless and unpayable, economic valuation focuses on the value of preventing statistical deaths ex ante. Seen from this perspective, prevented fatalities or "statistical lives" saved result in small reductions in the probability of death that are in turn enjoyed by any member of society. Accordingly, safety becomes an economic good that increases the wellbeing of individuals and can therefore be valued by economic theory.

The concept of willingness to pay (WTP) is one important approach to the valuation of safety. The technique is based on the principle that the maximum amount of money an individual is willing to pay for an incremental increase in her safety level is an indicator of the value of safety to her. In order to determine a social value of safety, the individual WTPs then have to be aggregated over all individuals that comprise society. It becomes clear intuitively, that the WTP concept is based on individual preferences for safety, in accordance with the basic postulate of welfare economics. As individual preferences are highly heterogeneous and context specific, they are difficult to assess reliably and consequently, a great variation of statistical life values is observable throughout literature. The range varies from about US$ 400,000 to US$ 30 million in industrialized countries as, among others, Blaeij et. al. [29] demonstrated in their meta analysis.

A conceptually distinct approach to the valuation of safety is the use of compound social indicators, that capture the quality of life the average individual of society enjoys from living under particular circumstances. The value of safety is then determined by the exchange rate between two indicators, so that total life quality is maintained. Here, the individualistic concept is abandoned and quick and stable estimates of statistical life values are enabled. This however comes at the cost of a violation of the individualistic principle of welfare economics in general, as the economic reasoning behind this approach is justifiable only under restrictive conditions. A prominent example of such a social indicator is the Life Quality Index (LQI), that recently gained increasing attention as a safety pricing tool in civil engineering.

1.2 Objectives of the Thesis

This thesis deals with the evaluation of public risk reduction interventions by means of a social cost benefit analysis, with the ambition to provide a rational decision basis for project approval in social sense. Addressing the above illustrated problem, the objectives of this thesis are threefold:

1. The first goal constitutes the development of an innovative disaster management model that provides a clear and well structured approach to managing disaster risk. Starting from delineating the model domain and addressing the intensity and frequency parameters of the hazard, the model enables a consistent assessment, evaluation and treatment of risk. By clearly illustrating the steps to be performed in mathematical terms and providing statistical characterizations of the risk defining parameters, the presented risk model is expected to contribute to a more homogeneous understanding of disaster risk and lead to greater transparency in performing risk based calculations.

2. The second objective of this thesis concerns the derivation of a social price for safety improvements. In particular, the influence of safety in a market economy is modeled and cost benefit rules for incremental safety increases are presented. The first developed approach follows the individualistic (i.e. bottom up) approach to derive a social WTP (SWTP) for safety and introduces the representative consumer model for safety pricing in general equilibrium. The other approach discusses the estimation of a SWTP in a macroeconomic (i.e. top down) context with special emphasis on the LQI. Here, a new LQI derivation approach is presented that allows far reaching interpretations and enables an improvement of the conventional LQI based safety pricing rule.

3. The third aim is to explicitly account for the uncertainties inherent in the benefit estimation process, by developing a real option approach for disaster risk reduction investments. The methodology develops a pricing rule for the flexibility to postpone the risk reduction investment to a future point in time. The possibility to delay the investment may pay because it allows to collect further information about possible project developments and thus enables more informed decisions.

1.3 Structure of the Thesis

To provide a comprehensive discussion on the problem of evaluating social risk reduction projects, this thesis is subdivided into eight chapters each of which contains valuable information. Figure 1.2 displays the logical order of the thesis; parts that contain own contributions are emphasized in red.

Chapter 2 introduces the innovative general risk management framework, provides clear definitions of the basal terms and gives an interpretation of other commonly used risk definitions. Then, risk reduction projects are mathematically characterized. **Chapter 3** presents the essentials of welfare economics and cost benefit analysis and serves as the theoretical basis of this work.

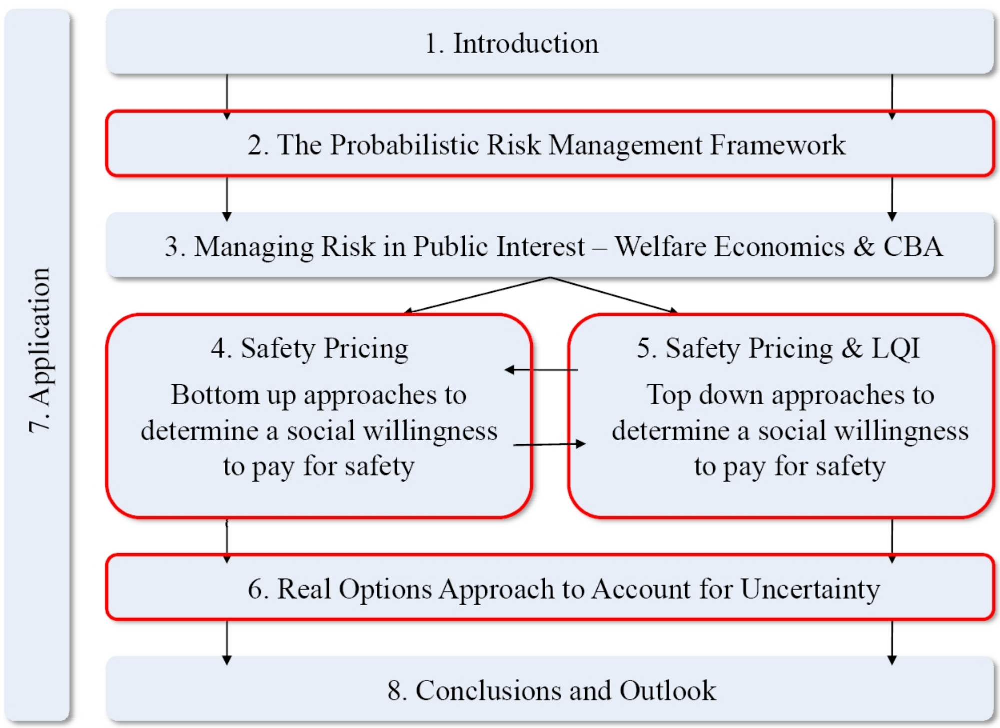


Figure 1.2: Organizational overview

Chapter 4 discusses the influence of safety in a market economy and introduces first social WTP rules for safety pricing in a bottom up fashion. Furthermore, commonly applied strategies to estimate the WTP for safety in practice are presented. Based on these results, **Chapter 5** discusses the LQI in depth and provides a new derivation of the index in a general equilibrium setting that clearly reveals the economic reasoning behind the approach and its relative standing towards alternative safety pricing methodologies. In addition, an innovative LQI based safety pricing rule is developed. **Chapter 6** presents a novel real option approach that enables the proper treatment of uncertainties involved in the benefit estimation process by evaluating the possibility to delay the risk reduction investment to a future point in time. With the option value on hand, sound recommendations whether to invest immediately, to postpone or to reject the risk reduction project are derived.

Chapter 7 presents two applications of the proposed methodology. In particular, earthquake risk for the city of San Francisco is assessed and treated based on the computer loss-estimation program HAZUS. **Chapter 8** concludes and gives an outlook where future research could be directed to.

Chapter 2

Management of Disaster Risk

This chapter introduces an innovative general risk management framework that provides reasonable definitions and a standardized language for communicating and managing disaster risk among stakeholders. To do this in a justifiable manner, firstly risk definitions and concepts existing in literature are reviewed and out of these, classes of risk calculation schemes are extracted. Subsequently, an exhaustive risk management concept is presented that covers the whole risk management chain, starting from risk identification over risk assessment up to risk treatment. The discussion of the risk management's workflow is accompanied by consistently defining the basal risk terms mathematically and schematically illustrating their interrelations. Then, the extracted risk definitions are integrated in the concept to demonstrate its generality and their advantageousness with respect to different application fields are discussed. Eventually, the essentials about public risk reduction projects are presented and their effects on disaster risk analyzed.

2.1 Statistics on Natural Disasters

The occurrence of a natural disaster rests upon the convergence of three central factors. The first is the hazard factor, which is the risk of an earthquake, hurricane, flood or an other natural event and is based on the geological, meteorological or ecological characteristics of a region. The second constitutes the exposure factor, which is best described as the number of people or property at risk of being potentially harmed by a hazard's occurrence. The third is the vulnerability factor, which describes the susceptibility of the exposed elements towards the impact of a hazard, leading to loss of lives or property, injuries or the disruption of livelihoods and economic activity.

Since 1900, more than 11,000 natural disasters have been recorded in the EM-DAT database [74]. Out of these registered events, more than 9,000 or around 80% have occurred over the last thirty years. The reason for this strongly increasing trend can be seen on the one hand in the fast colonization and the accumulation of human assets in hazard prone areas, exponential population growth and changing climate conditions but is also attributed to an improved data collection and hazard observation. In particular, advancements in surveillance technology and the launch of active data collection by the Office of U.S. Foreign Disaster Assistance (OFDA) and the Centre for Research on the Epidemiology of Disasters (CRED) in the 1970ies have led to an improved gathering of small to medium size events in the databases. Out of all recorded disasters, geophysical events take a share of around 12%, hydrological disasters of 36%, meteorological events of 29% and disasters of other categories such as droughts, wildfires and extreme temperatures are represented by a fraction of 23%.

Affected country	**Event type**	**Occurrence date**	**Number of fatalities**
China	Flood	01.07.1931	3,700,000
China	Flood	01.07.1959	2,000,000
China	Flood	01.07.1939	500,000
Bangladesh	Storm	12.11.1970	300,000
China	Earthquake	27.07.1976	242,000
China	Earthquake	22.05.1927	200,000
China	Earthquake	16.12.1920	180,000
Indonesia	Earthquake	26.12.2004	165,708
Japan	Earthquake	01.09.1923	143,000
China	Flood	18.04.1905	142,000

Table 2.1: The ten worst natural disasters in terms of fatalities 1900-2009 [EM-DAT [74]]

In Table 2.1 the ten worst disasters with respect to the number of fatalities are listed that have occurred after 1900. It might seem self-evident that disasters have a greater human impact on poorer countries with less developed economies. This relationship has been well documented in literature [100] and is due the fact that poor populations often end up living in high risk or environmentally degraded areas, have the least access to social disaster risk reduction initiatives or save infrastructure and have few savings or available credit. These factors collectively create conditions that increase a population's vulnerability to hazards, and hence lead to the more frequent occurrence and devastating impacts of disasters.

South and East Asia, particularly Bangladesh and China, are in the highest vulnerability category with a high proportion of its population being affected by natural disasters. These countries have areas of high population density, especially in river basins, and are home to populations whose livelihoods are often based on agriculture. When floods occur, the number of affected people quickly reach into the hundred thousands, and in some cases, millions as tragically evidenced by the three deadliest disasters that have occurred in China.

Although the absolute number of affected people has increased over the last decades, the number of fatalities has declined and there remains a decreasing trend in mortality through recent years [100]. Most likely, this reduction can be partially attributed to the effects of increasing risk awareness and substantial advancements in disaster prevention, preparedness, response and recovery.

In Table 2.2 the ten worst disasters in terms of economic damage costs are registered that have occurred since 1900. It becomes obvious that richer countries tend to rank frequently in these listing of the most expensive disasters, while they are clearly underrepresented in the fatality listings. This shows evidence that despite a high concentration of exposed valuable assets, the safety standards are on a comparatively high level, mainly attributed to effective risk mitigation measures that have been implemented in the past. Japan, Italy and the United States, for example, often appear on top of the list for earthquakes, which are the least predictable of all natural disasters. In addition, the time span between an earthquake threat and occurrence is the briefest among all major disasters.

Partly due to this, earthquakes top the scale of immediate mortality and structural destruction in more recent disaster events.

Affected country	**Event type**	**Occurrence date**	**Damage cost (US$ 2007 million)**
Japan (Kobe Earthquake)	Earthquake	17.01.1995	136,000
U.S. (Hurricane Katrina)	Storm	29.08.2005	132,500
China (Sichuan Earthquake)	Earthquake	12.05.2008	81,731
Italy (Irpinia Earthquake)	Earthquake	23.11.1980	50,400
U.S. (Northridge Earthquake)	Earthquake	17.01.1994	42,000
U.S. (Hurricane Andrew)	Storm	24.08.1992	39,220
China (Yangtze River Flood)	Flood	01.07.1998	38,100
Japan (Chuetsu Earthquake)	Earthquake	23.10.2004	30,800
U.S. (Hurricane Ike)	Storm	12.09.2008	28,846
Turkey (Izmit Earthquake)	Earthquake	17.08.1999	24,800

Table 2.2: The ten worst natural disasters in terms of damage costs 1900-2009 [EM-DAT [74]]

With the exception of floods and wave surges, developed nations rank on top in every other disaster category. Also because of higher property values linked to higher labor costs for reconstruction, richer countries constitute those with highest losses. The two disaster events associated with the highest losses ever recorded are the 1995 Kobe Earthquake and Hurricane Katrina, which stroke in particular the city of New Orleans in 2005.

Examining disaster data worldwide by the number of occurrences in each country provides valuable information about which countries suffer more from disasters. The U.S., together with South and Southeast Asia and Australia, are particularly prone to natural disasters, followed by Latin America, Russia and some European countries, such as Italy. However, looking at the total number of disasters relative to a country's surface or the total number of people affected relative to a country's total population might provide a different perspective on natural disaster occurrence and impact. Such an analysis accounts for variability among countries with very large and very small populations, although some countries covering large areas still come out on top.

As an indicator of severity also economic losses in absolute terms can clearly be misleading and should not be used except to compare countries with similar economic conditions. For international comparison, it is rather recommendable to apply a system of standardization by looking at the cost of a disaster relative to a country's total market value of goods and services produced, namely its gross domestic product (GDP). Thus, a clear picture emerges of the intensity of economic consequences of a disaster on a particular country. This makes comparisons between countries easier. The Kobe disaster listed above, which constitutes the most expensive disaster in absolute terms (US$ 136 billion), in fact represented less that 3% of Japan's GDP, while the devastating earthquake that occured in Guatemala in 1976 accounted for almost 25% of its GDP. Similarly, Hurricane Katrina which hit New Orleans in 2005 and is the second largest disaster in terms of cost in the EM-DAT database, represented only about 0.93% of the total U.S. GDP.

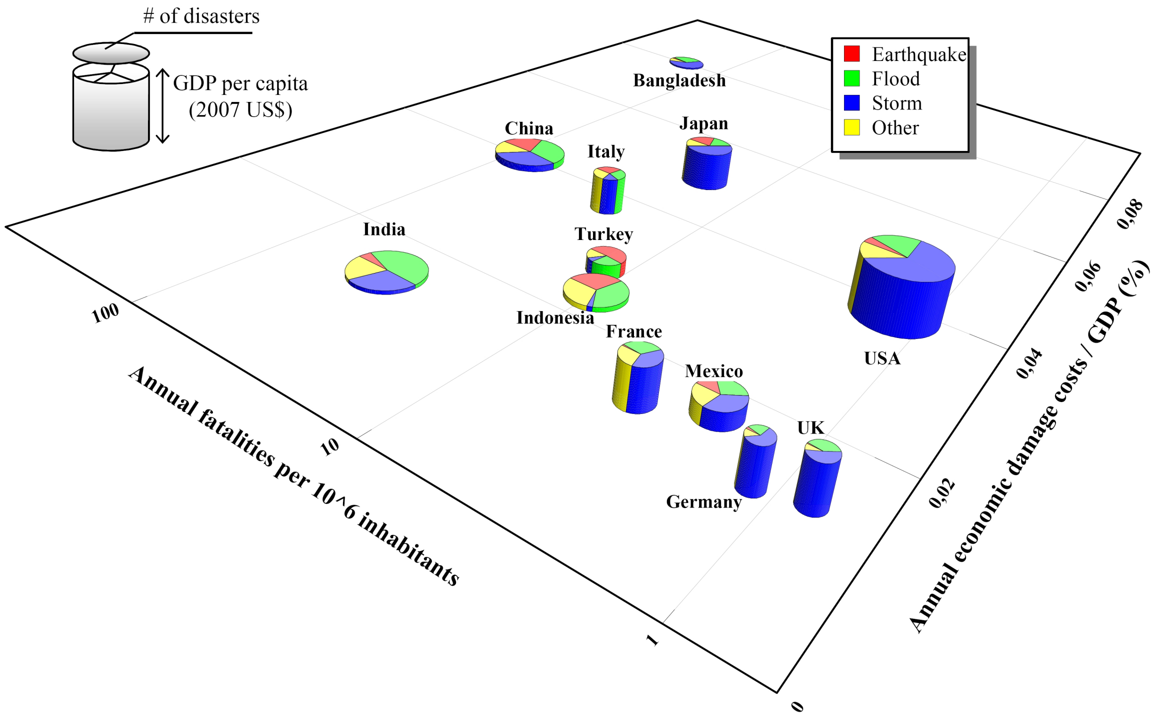


Figure 2.1: Natural disasters country overview 1900-2009

Figure 2.1 compares the impacts of natural disasters on selected countries that occurred since 1900. In the plane region of the graph the annualized fatality rates are standardized to one million inhabitants and the annualized damage costs are set in proportion to the countries' previous year GDPs. Using the year of the disaster might be misleading due to an often observed abrupt change in the country's GDP as a consequence of the disaster. The height of each country's pie chart displays the GDP per capita as an indicator of wealth, whereas the pies' surface sizes indicate the absolute number of disaster occurrences since 1900. The surface of each pie is further subdivided into the diverse disaster types to show the relative importance of particular events on the countries' total disaster exposure. It is distinguished between earthquakes, floods, storms and other events, which subsume droughts, extreme temperatures, volcanic eruptions and wildfires.

In line with the above discussion it is clearly seen that the disaster events arrogate the highest relative death tolls in developing countries such as Bangladesh, China and India, while Japan and Italy also rank comparatively high due to devastating earthquake events that have occurred in the previous century. In terms of relative economic damage cost it becomes obvious that Bangladesh takes the lead with annualized disaster damage costs of 0.08% of total GDP, followed by Japan, China, the U.S. and Italy. The UK, Germany and France in contrast, rank very low in both categories and are thus statistically comparatively weakly affected by natural disasters. With respect to total recorded disaster occurrences the U.S. is mostly hit, followed by China, India and Indonesia.

Scientific predictions and evidence indicate that global climate change will increase the number of extreme events in the future, creating more frequent and intensified natural hazards [228]. In addition, population growth, urbanization and the inability of poor populations to escape from the vicious cycle of poverty makes it more likely that there will be a further increase in the number of people who are vulnerable to natural hazards, with a resulting increase of natural disasters. All these facts highlight the urgent need for developing risk management strategies that make the systematic assessment and treatment of disaster risk more tractable and help to protect people and property in the best possible way.

2.2 The Ambiguity of Disaster Risk

The analysis and management of natural disaster risk is a highly multidisciplinary field of research. It involves the work of natural scientists to determine the hazard characteristic parameters such as probability of occurrence and intensity of an event for a specific location, followed by a profound engineering analysis about the building structure and infrastructural responses due to natural disaster loads. Moreover, investigations of economists are needed to estimate the monetary consequences of the damages and harms to the affected region, resulting in a political discussion about how to handle the peril in order to guarantee an adequate safety level for society.

The necessity to consider disaster management from the perspective of a great variety of sciences has led to the development of various quantitative as well as qualitative approaches towards disaster management. Each field is trying to cultivate its own understanding of the disaster related terms. As a result, communication within the disaster management community is often accompanied by misunderstandings and confusion due to colliding definitions and concepts. Therefore, a homogeneous understanding of disaster management is crucial for an efficient coordination of the important sub-steps and collaboration throughout the various disciplines.

Due to this problem an extensive literature review has been performed. In the following, exemplary disaster risk definitions are provided to demonstrate the far reaching risk terminology existing in literature.

- "The risk associated with flood disaster for any region is a product of both the region's exposure to the hazard (natural event) and the vulnerability of objects (society) to the hazard. It suggests that three main factors contribute to a region's flood disaster risk: hazard, exposure and vulnerability." Hori et al. [117]

- "Risk is the product of hazard (H) and vulnerability (V) as they affect a series of elements (E) comprising the population, properties, economic activities, public services, and so on, under the threat of disaster in a given area." Alexander [7]

- "The probability of harmful consequences, or expected loss of lives, people injured, property, livelihoods, economic activity disrupted (or environment damaged) resulting from interactions between natural and human induced hazards and vulnerable conditions. Risk is conventionally expressed by the equation: Risk = Hazard x Vulnerability." UNDP [251]

- "Risk is the probability of an event multiplied by the consequences if the event occurs." Einstein [72]

- "A combination of the probability or frequency of occurrence of a defined hazard and the magnitude of the consequences of the occurrence. More specific, a risk is defined as the probability of harmful consequences, or expected loss (of lives, people, injured, property, livelihoods, economic activity disrupted or environment damaged) resulting from interactions between natural or human induced hazards." European Spatial Planning Observation Network (ESPON) [183]

- "Risk is an expression or possible loss over a specific period of time or number of operational cycles. It may be indicated by the probability of an accident times the damage in dollars, lives, or operating units." Hammer [101]

Out of these citations basically five widespread classes of disaster risk definitions are extractable:

1. risk := hazard · vulnerability · exposure
2. risk := hazard · vulnerability
3. risk := probability · consequences
4. risk := probability · loss
5. risk := probability · damage

These risk formulae as well as the exemplary verbal definitions evidence that the different understandings of the term risk are mainly due to diverse comprehensions of the terms hazard, vulnerability, exposure, damage and loss. Obviously, the definition boundaries are blurred and intersecting between the authors' grasps. Therefore, there is the need to clearly clarify what is understood by each term. Furthermore, it is evident throughout the definitions that no clear formula is used to define the risk. Whereas some authors define risk as a product of several terms, others even avoid any mathematical deepness by simply arguing that risk is a function of several expressions. This observation has also been made by Thywissen [248], who even goes a step further in arguing:

> "Risk is seen as a function of hazard, vulnerability, exposure and resilience, while the mathematical relationship between the variables is unknown."

In this sense, also the above collected risk formulae (1)-(5) are not to be understood too mathematically, but rather illustratively to emphasize the composition of disaster risk. The only clear mathematical formula to quantify risk, that is known by the author, is the PEER equation for earthquake risk that is provided in Baker et al. [21].

In the next section a fully developed disaster management methodology is presented that clearly outlines the important sub-steps of risk management and supplies unambiguous definitions of the risk defining terms. After this introduction, the theoretical background is sufficient to demonstrate how the above listed definitions interrelate and can be included in the framework.

2.3 The Probabilistic Risk Management Framework

The proposed risk management framework that is presented in this section has been developed in close correlation to Pliefke et al. [201, 208] and is structured in compliance with AS/NZS 4360 [19] that define a risk management process as the:

> "Systematic application of policies, procedures and practices to the task of identifying, analysing, evaluating, treating and monitoring risk."

The additional value of this chapter in comparison to [201, 208] lies in the development of a sound mathematical description of all risk defining terms and steps to be performed within the framework.

As illustrated in Figure 2.2, the three main components of the framework are given through risk identification, risk assessment and risk treatment and are performed sequentially throughout the risk management process, accompanied by a risk review step and continuous risk monitoring.

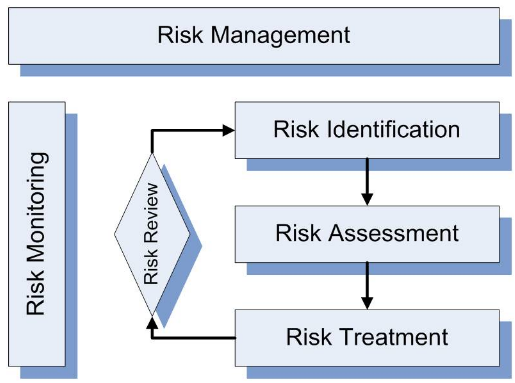


Figure 2.2: The general risk management framework

The risk review process is assigned to the task to constantly include all new information, knowledge and experience about the risk and to indicate its evolution within the process over time. Thus, the risk is updated on a regular basis. It should be emphasized that the risk review process is only performed for risks that have already run through the whole process at least once. Consequently, in each risk review iteration the effectiveness of possibly implemented risk reduction interventions becomes visible. The risk monitoring procedure in contrast, captures the exchange of information of all persons actively or passively involved or participating in the risk management process. This exchange of information is necessary to guarantee a smooth collaboration between interdisciplinary researchers and to discover new hazards due to the ever changing environment.

2.3.1 Risk Identification

The prerequisite for performing the risk identification phase and therefore to initiate the operation of the risk management chain is the condition of being aware of a dangerous situation. If this is met, first of all the boundaries of the model domain have to be circumscribed by defining the **system** under analysis. The system can be composed of a single building or infrastructure element, a city, a region or even an entire country. Next, all sources of events that potentially endanger the functionality of the system have to be identified and are characterized by the term **hazard**. Thus, the risk identification step leads to an answer to the question "what can happen and where?". As soon as this analysis is completed for a particular location, it is proceeded with the risk assessment phase.

2.3.2 Risk Assessment

After having outlined the model domain and identified all possible hazards to the system, the risk assessment phase starts to operate, representing the first crucial step of the risk management framework. The risk assessment itself consists of two sub-procedures, namely risk analysis and risk evaluation, whose tasks are to be seen in quantifying the risk and comparing it to other competing risks, respectively. A brief overview of the risk assessment procedure is provided in Figure 2.3.

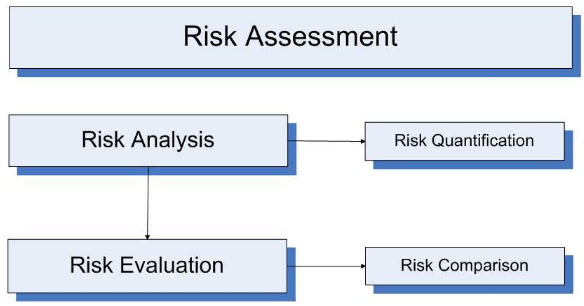


Figure 2.3: The risk assessment phase

2.3.2.1 Risk Analysis

The risk analysis procedure, depicted in Figure 2.4, represents the most sophisticated part of the risk assessment phase, whose major objective lies in the quantification of the risk defining parameters and finally the risk itself, most desirably in monetary units per time unit (i.e. US$ /year). In order to reach this ambition, first of all a hazard assessment is being performed, where the hazard itself as well as its diverse impacts on the different elements of the system are evaluated.

Hazard Assessment The hazard assessment subroutine starts with a hazard analysis where the intensity and frequency parameters of each identified hazard type with respect to the predefined system are estimated. As the calculation of these parameters depends largely on historical data, so called return periods play an important role in hazard analysis. These return periods are computed from a set of recorded past events in combination with expert judgments and sophisticated statistical techniques, which are employed to include changes in environmental conditions as well as rare events for which few observations are available or recorded history is exceeded. Formally speaking, a return period for a certain hazard intensity is the expected mean time (in years) between the occurrences of two hazard events reaching or exceeding the given intensity level.

To exemplify, a return period of 20 years for a hazard of intensity x signifies that during the previous 100 years, a hazard with intensity x or greater has occurred within the system about five times. So the average time span between two hazard events with intensity x or greater is 20 years and therefore the event is often being referred to as a "20-year event".

Return periods are traditionally used to express the frequency of occurrence of an event, although it is often confused with the probability of occurrence or the probability of exceedance of that event. As it is aimed at evaluating the risk on an annual basis, the latter two expressions are clearly advantageous in the following risk calculation as they reflect information about the annual frequency. Formally, the following relationship holds between the return period Tr of a hazard intensity x and the probability P that the actual hazard intensity HI will exceed x within one year [97]:

$$P(HI \geq x) = \frac{1}{Tr(x)} \tag{2.1}$$

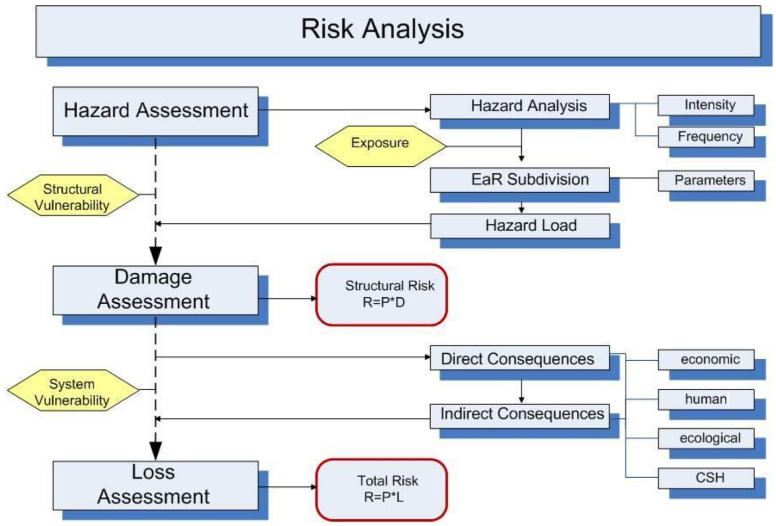


Figure 2.4: The risk analysis phase

Assuming that hazard data are available for all thinkable hazard intensities x, the (continuous) exceedance probability curve ϕ_{HI} is easily obtained. It yields the probability that a hazard with a given intensity will be reached or exceeded during a one year interval. Therefore, the following relation is satisfied:

$$\phi_{HI}(x) = P(HI \geq x) \tag{2.2}$$

From this exceedance probability curve ϕ_{HI} the cumulative distribution function F_{HI} for the hazard intensity x can straightforwardly be obtained:

$$F_{HI}(x) = 1 - \phi_{HI}(x) \tag{2.3}$$

Consequently, both the annual exceedance probability curve ϕ_{HI} as well as the cumulative distribution function F_{HI} contain exactly the same information about the hazard for a given system and differ only in the way of presentation. They can therefore be both taken into account to calculate the risk subsequently. In this context it is often more convenient to include the hazard intensity by means of probability density functions (PDF), which can be either determined on basis of the cumulative probability function or the exceedance probability curve:

$$\begin{aligned} f_{HI}(x) &= \frac{d}{dx} F_{HI}(x) \\ &= \frac{d}{dx}(1 - \phi_{HI}(x)) \\ &= \left| \frac{d}{dx} \phi_{HI}(x) \right| \end{aligned} \tag{2.4}$$

To draw a conclusion, the result of the hazard analysis subroutine is a realvalued function ϕ_{HI}, F_{HI} or f_{HI} respectively, that captures information about the annual probability of occurrence for different hazard intensities. A comprehensive discussion on how to estimate the probability of occurrences specifically for different hazard types is provided in [268].

Once the hazard data for the system have been quantified, it needs to be analyzed which components of the system are **exposed**, i.e. potentially endangered by the impact of the hazard of a given intensity. In this way, a subdivision of the system into **elements at risk (EaR)** and **elements at non risk (EaNR)** is performed, depending on the hazard under consideration:

$$\text{Exposure(x)} := \begin{cases} 0, & \text{if Element is not exposed for hazard intensity } x \\ y, & \text{if Element is exposed for hazard intensity } x \end{cases} \tag{2.5}$$

As the EaNR are by definition not exposed, they are not threatened by the hazard and can therefore be excluded from further analysis. An EaR on the contrary, represents a building or an other arbitrary infrastructure element that is characterized by several parameters that have to be determined. Among these are precise location parameters within the system, information about the functional use (residential, commercial, industrial), occupancy (inventory of contents, number of people living or working inside) and construction type (building material, number of stories, construction year).

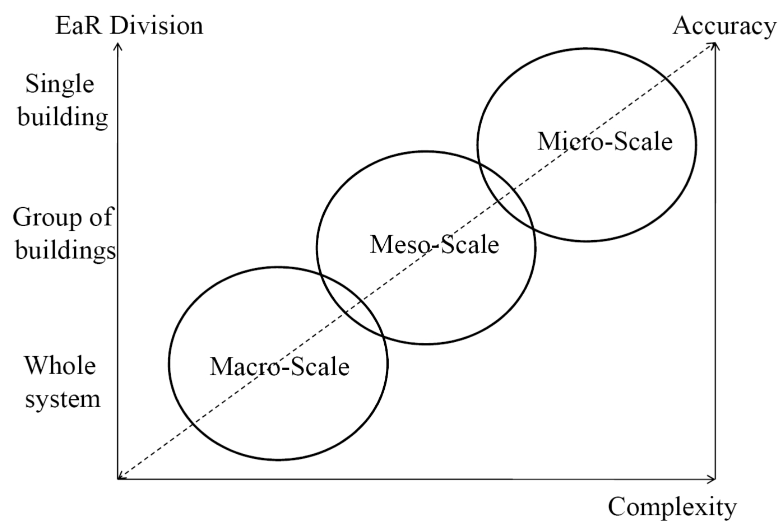


Figure 2.5: Risk analysis performed on micro-, meso- and macro-scales

In this respect, the location parameters serve primarily to forecast the pathways of the hazard, i.e. to convert the impact of a hazard of a given intensity x to the system to a hazard load y affecting the particular EaR situated in a certain area within the system. As the development of the hazard within the system is only assessable probabilistically, a so called pathway function [192] that converts the hazard intensity x to a hazard load y is introduced by means of a conditional PDF $f_{HL|HI}$:

$$\text{Hazard intensity } x \xrightarrow{f_{HL|HI}} \text{Hazard load } y \tag{2.6}$$

According to the law of total probability, that can be reviewed in Appendix B, this pathway function is taken into account to calculate the PDF of the hazard load for the considered EaR:

$$f_{HL}(y) = \int f_{HL|HI}(y,x) f_{HI}(x) dx \tag{2.7}$$

The remaining EaR parameters other than location in contrast, are designed to predict the reaction of the EaR in case of disaster occurrence as well as the immediate and longterm aftermaths that go in line with it. A detailed discussion about the EaR parameters is provided in Grossi et al. [97].

If larger systems such as whole cities, regions or even entire countries are considered, it will hardly be possible to perform the risk analysis on a micro level, i.e. to investigate each EaR individually for its reaction towards disaster load or at least not at a reasonable cost. Therefore, in order to facilitate the analysis, it is possible to group EaRs with similar characteristics into EaR classes, depending on the hazard under consideration. Then, the further analysis can concentrate on one typical representative out of each EaR class, assuming that all other EaR of the same category will show similar behavior. Depending on the size of the EaR classes this approach then results in a micro-, meso- or macro-level risk analysis. Surely, the advantageousness of this decrease in complexity going along with reduced investigation cost comes at the price of a reduced accuracy, as visualized in Figure 2.5.

In order to finally decide on which scale to perform the risk analysis, one might try to maximize the following equation

$$\text{Efficiency} = \frac{\text{Accuracy}}{\text{Complexity}} \tag{2.8}$$

for a given investigation budget, as suggested by [150].

Damage Assessment After all the EaR (classes) have been identified and clearly delineated, the structural behavior of each EaR (class) has to be predicted contingent upon hazard load. The damage module of an EaR is strongly dependent on the structural response of the EaR and captures the physical harm only. It is expressible by a large variety of measures, e.g. water height, crack width, story drift, which are used to derive damage states. It needs to be clearly emphasized however, that damage is not measured in monetary value.

The relation between hazard load and the resulting damage is called **structural vulnerability** and may be interpreted as a measure of building performance. As the damage z for a given hazard load y can only be estimated probabilistically, structural vulnerability is best described on basis of a conditional PDF $f_{DA|HL}$, which has also been suggested by Faizian et al. [78]:

$$\text{Hazard load } HL \overset{f_{DA|HL}}{\longrightarrow} \text{Damage } DA \tag{2.9}$$

Thus, the structural vulnerability is an EaR (class) specific characteristic that indicates the degree of physical susceptibility towards the impact of the hazard. Having determined the structural vulnerability, the law of total probability is employable once again to calculate the PDF of the damage f_{DA} of the considered EaR (class):

$$f_{DA}(z) = \int f_{DA|HL}(z, y) f_{HL}(y) dy \tag{2.10}$$

Now, by taking formula (2.7) into account, the probability of damage can directly be related to the hazard intensity:

$$f_{DA}(z) = \iint f_{DA|HL}(z, y) f_{HL|HI}(y, x) f_{HI}(x) dx dy \tag{2.11}$$

Subsequent to the prediction of the structural behavior of all EaR (classes), the **consequences** for the system that might go in line with a given level of damage of the exposed elements have to be analyzed. For this investigation the above sketched characteristic parameters of each EaR (class) need to be taken into account. It is distinguished between **direct consequences** that occur simultaneously to the time the disaster takes place and **indirect consequences**, which occur with a time shift as a result of the direct consequences.

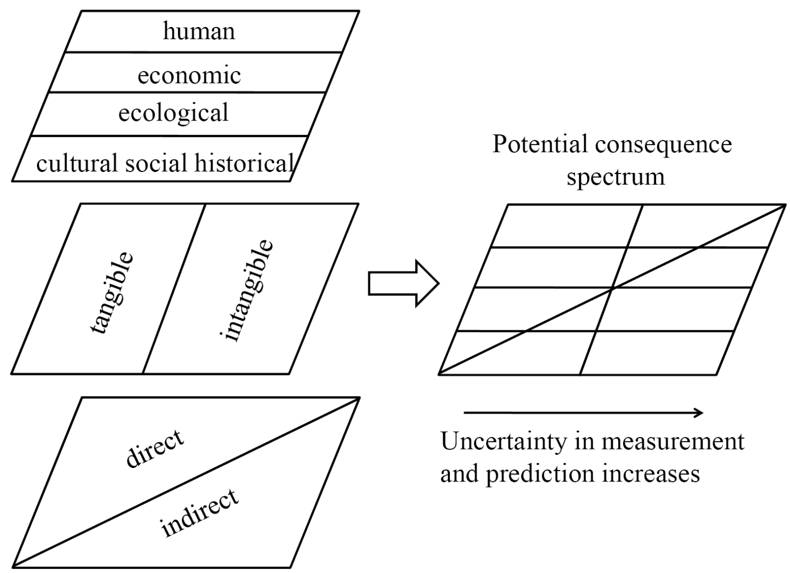


Figure 2.6: Categorization of disaster consequences

Whereas direct consequences are in a straight line linked to the **coping capacity** of the system, i.e. the ability to withstand the natural forces and to provide immediate help, indirect consequences are linked to the **resilience**, i.e. the capacity of the system to remain functional and recover from the disaster. In addition, each consequence class is further subdivided into tangible or economic consequences that are directly measurable in monetary terms and intangible consequences, which are not directly appraisable, e.g. injuries and fatalities, pollution of the environment, loss of cultural social and historical (CSH) values etc.. A comprehensive study about consequence assessment in application to earthquake risk is given in [78]. Figure 2.6 summarizes the above discussion and provides an overview of the consequence classification.

Loss Assessment After all possible consequences for each EaR (class) and thus for the system as a whole have been determined, loss appraises and eventually accumulates all direct and indirect consequences at the time the disaster takes place, as visualized in Figure 2.7. In this respect, the indirect consequences that occur later in time have to be discounted on basis of a properly defined discount rate that is specific for each consequence class. The loss LO is then characterized by means of the net present value (NPV), which represents the sum of discounted expected future consequences and is calculated as:

$$\text{Loss} = \sum_{t=0}^{N} \frac{E\left[\text{Consequence}_{it}\right]}{(1+r_i)^t} \tag{2.12}$$

where E denotes the expected value operator and r_i represents a properly defined discount factor specific for each consequence class i to include time preferences.

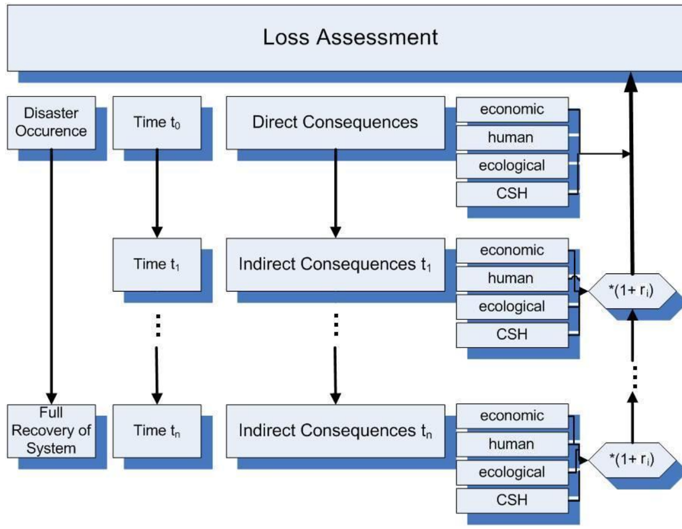


Figure 2.7: Loss assessment

In this context, **system vulnerability** is an EaR (class) specific characteristic, that links damage to loss and indicates the total potential the hazard has on the EaR (class) and in a second step on the system as a whole. Again, as the loss resulting from a given damage state can only be predicted in probabilistic terms, system vulnerability is best described by a conditional PDF $f_{LO|DA}$:

$$\text{Damage } DA \overset{f_{LO|DA}}{\longrightarrow} \text{Loss } LO \tag{2.13}$$

Thus, for a given damage state, system vulnerability contains information about the susceptibility of the EaR (class) and its contents towards that damage as well as the resulting disruption of functionality within the system. In the prediction of system vulnerability interdependencies between EaR (classes) need to be properly accounted for. It is crucially dependent on the latter how severe the consequences to the system will be once a certain EaR (class) damage has occurred. Consequently, system vulnerability allows for an interpretation in terms of system performance.

Having this formulation for the system vulnerability at hand, the PDF for the loss is easily obtained by applying the law of total probability:

$$f_{LO}(v) = \int f_{LO|DA}(v,z) f_{DA}(z) dz \tag{2.14}$$

In analogy to the damage also the probability of loss can directly be related to the hazard intensity by substituting expression (2.11) for the PDF of the damage:

$$f_{LO}(v) = \iiint f_{LO|DA}(v,z) f_{DA|HL}(z,y) f_{HL|HI}(y,x) f_{HI}(x) dx dy dz \tag{2.15}$$

The risk analysis phase terminates with the quantification of risk for the considered EaRs (classes) as well as for the system as a whole. In quantifying the risk, all previously collected information about the single risk components is comprised. It is distinguished between two different types of risk that are introduced subsequently.

Structural Risk Firstly, the risk an EaR is exposed to may be interpreted in terms of physical harm imposed by the hazard and is calculated by taking the product of the annual probability of occurrence of the damage multiplied by the damage that goes in line with it. According to this understanding, risk equals the expected damage per year and is expressed by the following equation:

$$\begin{aligned} R_D &= \text{Damage} \cdot \text{Probability [Damage measure / year]} \\ &= \int z \cdot f_{DA}(z) dz \end{aligned} \tag{2.16}$$

It is being referred to as structural risk. Evidently, the structural risk is of primary importance for civil engineers in order to have a number on hand that subsumes the behavior and the response of a structure or structural element under potential hazard load. In line with the above discussion, structural risk is also directly relatable to the hazard source by substituting the PDF of damage f_{DA} in equation (2.16) with the identity given in (2.11):

$$R_D = \iiint z \cdot f_{DA|HL}(z,y) f_{HL|HI}(y,x) f_{HI}(x) dx dy dz \tag{2.17}$$

From this alternative formulation of the structural risk, which is absolutely equivalent to definition (2.16), the composition of the risk is clarified. It becomes obvious which of the risk defining components has the largest impact on its absolute value. Before the second risk calculation scheme is discussed, it is important to highlight at this point that the damage measure and therefore the structural risk is characteristic for each EaR (class). As the whole system is usually composed of a great heterogeneity of EaR (classes), it is rather unconventional to accumulate the structural risks across EaR (classes) to obtain a structural risk on aggregate level. This is due to the fact that one physical damage measure such as crack width cannot be used equivalently for two different building typologies such as concrete and masonry. This is in sharp contrast to the total risk introduced below, which is based on the loss concept and thus comparable across EaR classes.

Total Risk The second way to express the risk for an EaR (class) is to take the product of the annual probability of occurrence of the loss and the loss that goes in line with it. This calculation scheme sets the risk equal to the expected annual loss and is characterized by the following equation:

$$\begin{aligned} R_L &= \text{Loss} \cdot \text{Probability [Loss measure / year]} \\ &= \int v \cdot f_{LO}(v) dv \end{aligned} \tag{2.18}$$

It is being referred to as total risk. As demonstrated for the structural risk, the total risk can also be directly related to the hazard source, by replacing the PDF of loss f_{LO} with the identity provided in equation (2.15):

$$R_L = \iiiint v \cdot f_{LO|DA}(v,z) f_{DA|HL}(z,y) f_{HL|HI}(y,x) f_{HI}(x) dx dy dz dv \tag{2.19}$$

As outlined above, this formula shows the composition of the total risk in a more exhaustive fashion and is absolutely identical to (2.18).

The total risk may comprise all consequences, both tangible and intangible, if a reasonable way has been found to convert the primary intangible consequences to monetary value. Alternatively, this transformation of intangible outcomes may be omitted and the total risk split with respect to the specific consequence classes. Then, a total risk for each consequence class is obtained and the relative contribution of each consequence class to total harm is quoted separately in units of the respective loss measure (e.g. money, loss of life etc.).

The total risk is a more extensive measure than the structural risk. Not only the instant damage, but the full consequences over time resulting from that damage are evaluated and included in the calculation. As the various consequences have been appraised by means of an adequate loss measure in the context of total risk, it is possible to compare and accumulate the total risk across EaRs (classes). As a result, a total risk for the system on aggregate level is assessable:

$$R_L(System) = \sum_{i=1}^{N} R_L(EaR_i) \tag{2.20}$$

In this expression for the total risk to the system it is implicitly assumed that interrelated losses that occur due to interactions between the single EaRs (classes) within the system are accounted for, by assigning them cautiously to the expected losses of the single EaR (classes).

2.3.2.2 Risk Evaluation

Subsequent to the termination of the risk analysis procedure, the risk evaluation phase is initiated. The purpose of risk evaluation is to make the risk under investigation comparable to other competing risks to the system, by using adequate risk measures. Several commonly applied risk measures are depicted in Figure 2.8.

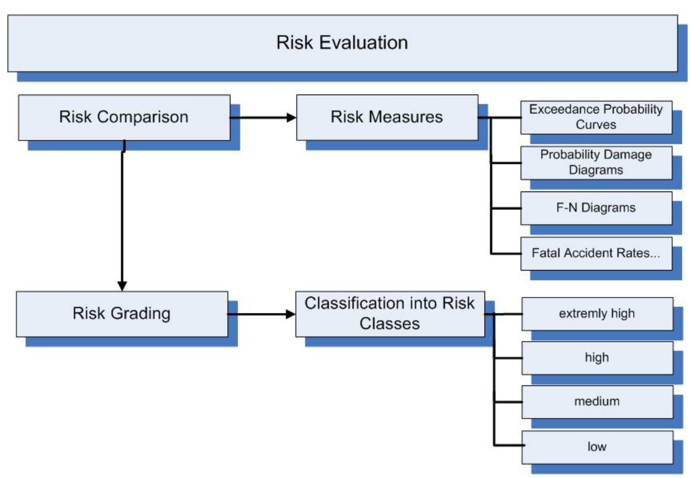


Figure 2.8: Risk evaluation

In this context, so called exceedance probability curves have found wide acceptance as a common tool to illustrate risk graphically. In an exceedance probability curve the probability that a certain level of loss is surpassed within a specific time period is plotted against different loss levels, as shown in Figure 2.9. In line with the above discussion, it is possible to construct an exceedance probability curve EP either by means of the PDF of the loss f_{LO} or the corresponding cumulative distribution function F_{LO} and has the following mathematical representation:

$$EP(v) := 1 - F_{LO}(v) = 1 - \int f_{LO}(v)dv \tag{2.21}$$

Hereby, the loss to the system can be specified in terms of monetary loss, of fatalities or of other suitable impact measures. An insightful overview and more in depth discussion of common risk measures and tools to compare risks is provided in [210].

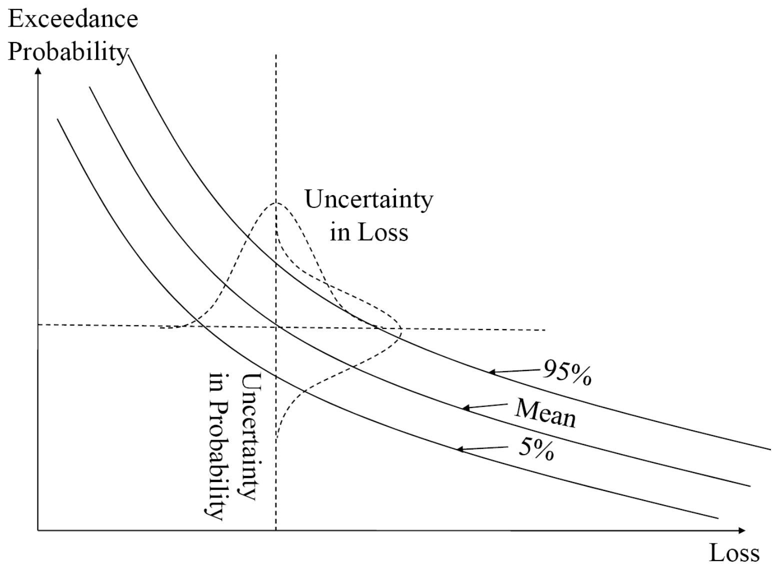


Figure 2.9: Exceedance probability curve

Having analyzed the risk on basis of adequate risk measures, it may be graded into priority classes that highlight, which risks are relatively most threatening to the system and where mitigation measures need to be implemented most urgently. The grading of risks hereby depends on the risk perception of the analyst. Accordingly, the output of the risk evaluation is a prioritized list of risks for further action.

2.3.3 Risk Treatment

After the risk to the predefined system has been analyzed and graded into a risk class, the last procedure of the risk management framework, the risk treatment phase, begins to operate. This procedure is assigned to the task to create a rational basis for deciding about how to handle the risk in the presence of other competing risks. Based on several analytical tools from decision mathematics, finance, economics and public choice theory, a decision whether to accept, to transfer, to reject or to reduce a given risk is derived. In the latter case, risk mitigation initiatives are implemented. Figure 2.10 visualizes the process of risk treatment schematically.

If the risk is to be mitigated, decision makers are able to choose among several alternatives to implement a risk reduction measure. All possible risk reduction strategies have in common that they reduce either the exposure or the vulnerability of the system. Depending on the specific strategy that is chosen, it may either reduce structural vulnerability by increasing the resistance of structures through retrofitting and other improvements or system vulnerability by enhancing the response of the system to the occurring damages and developing strategies to recover from the disaster as quickly as possible. The construction of a dyke to protect properties close to the water might be taken as an example to reduce exposure with respect to flood risk. In the following, the strategies are subdivided according to the time the risk reduction project becomes effective.

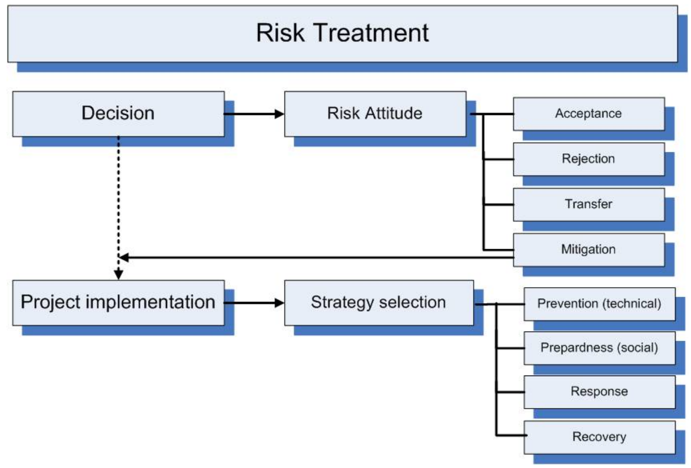


Figure 2.10: The risk treatment phase

Firstly, so called pre-disaster interventions, such as prevention and preparedness, are available. Prevention includes technical measures like structural strengthening that are to be performed with an accurate time horizon before the disaster takes place. Typical examples are dykes against floods or dampers against dynamic actions. Preparedness in contrast, contains all social activities, e.g. evacuation plans and emergency training, that are necessary to limit harm shortly before the disaster takes place.

Secondly, post-disaster strategies may be pursued to reduce the risk. Among these, response covers all activities that are performed immediately after the occurrence of the disaster, such as the organization of help and shelter for the injured and harmed as well as the coordination of emergency forces. Recovery on the contrary, subsumes all activities that need to be undertaken until the pre-disaster status of the system is restored again. Examples for recovery activities might be emergency plans that guarantee the quick reconstruction of damaged sides, the provision of financial aid to the harmed and the quick revitalization of affected economic sectors. Straightforwardly, also a combination of the classified activities is applicable to mitigate the risk.

The process of risk treatment is further discussed in Section 2.6, where public risk reduction projects are studied in depth. Figure 2.11 summarizes the above discussion by reviewing the entire risk management framework schematically. A comprehensive compilation of all introduced risk management terms is provided in Appendix A.

2.4 Integration of Risk Definitions

Following the introduction of the general risk management framework it is discussed in this section, how the risk definitions of Section 2.2 interrelate. Even if the referenced authors might have had

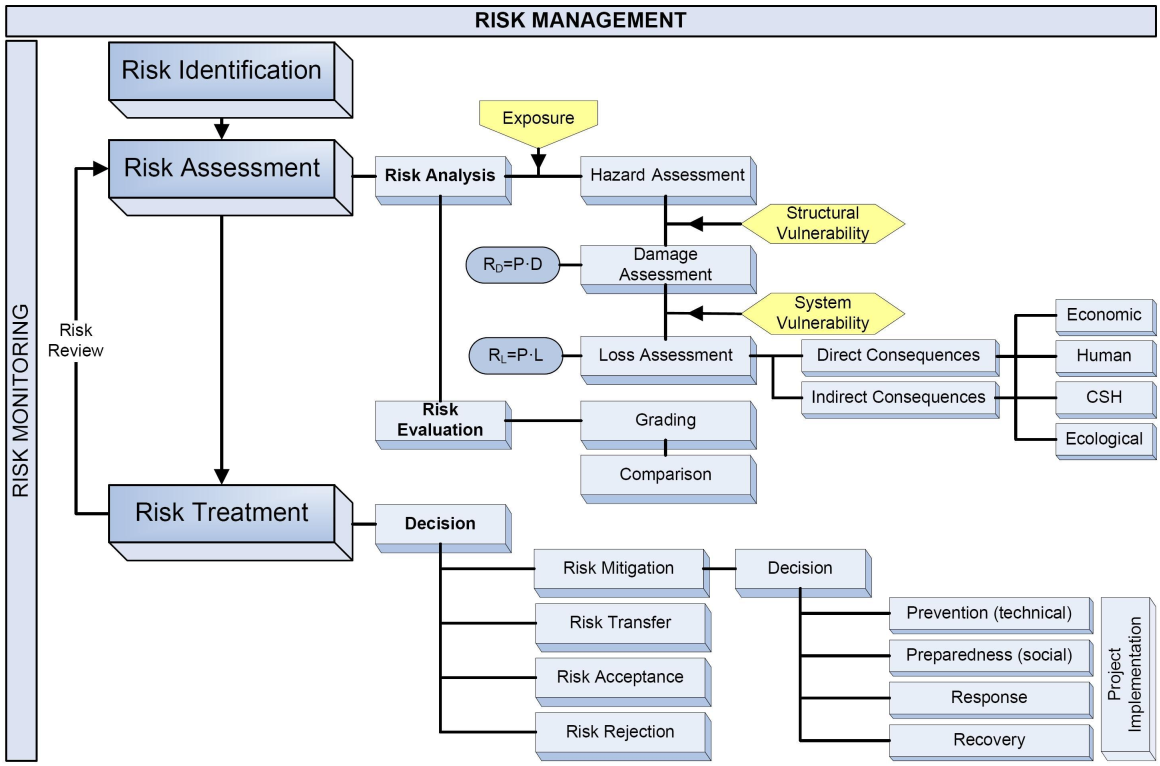


Figure 2.11: The complete risk management framework

different understandings in mind, an interpretation of the risk definitions within the above described methodology is presented. This is done by taking the previously established basal terms and definitions as a baseline for argumentation. In the following, the review of risk Def. (1)-(5) is separated in two segments with respect to the affinity of formulation.

Firstly, risk Def. (1) and (2) are considered together as they both do not include a measure of hazard outcome in their risk formula. Therefore, it might be concluded that these two definitions do not seek to express risk in terms of an expected value but are rather trying to provide a formulation for the full outcome distribution. The two Def. (1) and (2) have the hazard and the vulnerability term in common, while Def. (1) contains an additional exposure multiplier.

Referring to the above derived expressions for the PDF of the loss (2.15) and the damage (2.11) respectively, Def. (1) can be retraced basically in two distinct ways:

$$f_{DA}(z) = \iint \underbrace{f_{DA|HL}(z,y)}_{\text{structural vulnerability}} \underbrace{f_{HL|HI}(y,x)}_{\text{exposure}} \underbrace{f_{HI}(x)}_{\text{hazard}} \, dxdy \tag{2.22}$$

$$f_{LO}(v) = \iiint \underbrace{f_{LO|DA}(v,z) f_{DA|HL}(z,y)}_{\text{total vulnerability}} \underbrace{f_{HL|HI}(y,x)}_{\text{exposure}} \underbrace{f_{HI}(x)}_{\text{hazard}} \, dxdydz \tag{2.23}$$

The reason why there are these two possibilities to interpret risk Def. (1) from the perspective of the provided risk management framework is because it is not obvious from the definition whether structural or system vulnerability is being referred to. If structural vulnerability is taken into account, the definition can be included in the framework by means of formula (2.22) and represents the damage distribution for the considered element. If on the contrary the vulnerability term is intended to reflect also system vulnerability, expression (2.23) is appropriate to embed Def. (1) in the framework. In this case Def. (1) represents the loss distribution for the considered element.

In both (2.22) and (2.23) the hazard pathway function has been declared to reflect the exposure. This can be justified from the point of view that an arbitrary system element becomes exposed (and thus an EaR) to the hazard as soon as the hazard load on the element exceeds a certain threshold level, so that damage is caused. Therefore, Def. (1) is better suited for the analysis of entire systems, that are composed both of endangered objects (EaR) and non endangered objects (EaNR) that are distributed unevenly within the system. Consequently, the exposure term has to be included in the definition in order to firstly analyze the way the hazard is evoluting within the system, and based on this to identify exposed elements for which further analysis is being performed.

In Def.(2) on the other hand, the exposure term is omitted, making the following two inclusions possible:

$$f_{DA}(z) = \int \underbrace{f_{DA|HL=HI}(z,x)}_{\text{structural vulnerability}} \underbrace{f_{HI}(x)}_{\text{hazard}} dx \tag{2.24}$$

$$f_{LO}(v) = \iint \underbrace{f_{LO|DA}(v,z) f_{DA|HL=HI}(z,x)}_{\text{total vulnerability}} \underbrace{f_{HI}(x)}_{\text{hazard}} dxdz \tag{2.25}$$

In line with the above discussion, the vulnerability term can be interpreted in two distinct ways, resulting in two possible representations (2.24) and (2.25). As neither formula contains an exposure term, Def. (2) is specifically tailored for an application of the risk analysis for one single structural element which is then identical to the whole system. In this case, exposure to the impact of the hazard is a prerequisite for initiating the investigation, as outlined in Paragraph 2.3.1. As the system is restricted to one element, there is no hazard pathway and the hazard intensity and hazard load components coincide. Therefore, the outcome distribution is sufficiently described by the product of hazard and vulnerability.

Secondly, risk Def. (3)-(5) are considered jointly as they all contain a measure for the hazard outcome, i.e. consequences, loss and damage, while they have the probability multiplier in common. Obviously, when risk is calculated in this way, an expected value for the respective outcome is obtained. As the three definitions differ in their understanding of hazard outcome, the probability multiplier has to be treated case specifically and therefore differs throughout the definitions. It represents the probability that the respective outcome occurs and can be obtained by taking Def. (1) and (2) into account. Consequently, the probability multiplier comprises all information about hazard intensity, exposure and vulnerability, as illustrated above.

With reference to the outcome specification, the use of the term consequence in Def. (3) is most general and makes a detailed listing of the diverse harms to the system necessary. The depth of analysis cannot be judged upon on basis of the formula. It may either finish with the determination of physical harm to the considered system or include the total spectrum of adverse outcomes over time. In the first case Def. (3) would coincide with expected damage or structural risk, whereas in the second case it would be identical to expected loss or total risk. Def. (3) is most suitable in political decision processes. In this area, it is essential to know precisely which parts of the system are especially endangered by the hazard and to which extent. With this information, specifically tailored risk treatment strategies can be developed.

The use of loss in Def. (4) and damage in Def. (5) as an outcome measure however, entails an evaluation of the consequences on basis of a suitable impact measure, and differs in the depth of analysis. If loss is taken into account, it is implicit in the definition that all possible consequences, both direct and indirect, need to be considered and evaluated, dependent on their occurrence in time. Here, Def. (4) is equivalent to the total risk defined in (2.18):

$$R_L = \iiiint \underbrace{v}_{\text{Loss}} \cdot \underbrace{f_{LO|DA}(v,z) f_{DA|HL}(z,y) f_{HL|HI}(y,x) f_{HI}(x)}_{\text{Probability of Loss}} \, dx dy dz dv \tag{2.26}$$

The use of loss as an outcome measure is advantageous in an economic context, where it is important for instance to relate disaster risk to measures like national income. Furthermore, the effects of risk reduction projects in terms of reduced loss can directly be incorporated in a cost benefit analysis (CBA). Also in insurance industry the calculation of premiums for disaster insurance rely on the loss concept.

Finally, if as in Def. (5) damage is taken into account to express the outcome, the calculation results in the structural risk defined in (2.16):

$$R_D = \iiint \underbrace{z}_{\text{Damage}} \cdot \underbrace{f_{DA|HL}(z,y) f_{HL|HI}(y,x) f_{HI}(x)}_{\text{Probability of Damage}} \, dx dy dz \tag{2.27}$$

In this case, the physical harm of the system elements is in the focus of attention. The expression of risk in terms of damage is of primary importance in civil engineering to express the structural behavior under hazard load. Based on this, the engineer is able to decide for instance whether it is necessary to retrofit a building at a particular risk prone location to improve its performance and guarantee an adequate safety level to its occupants.

2.5 Public Risk Reduction Projects

Risks that have a potentially high impact on the system in relation to other risks are graded in a high priority class in the risk evaluation phase and need to be treated subsequently. One important strategy of risk treatment is to mitigate the risks by implementing risk reduction measures. While each endangered subject of the system, such as households and firms, can take private precaution to reduce their personal risk for instance by retrofitting their property to withstand wind and earthquake loading or transferring a part of the risk by purchasing some form of insurance, it is a matter of their risk perceptions and level of information whether they will do so in the end. Here, in particular private homeowners are among the least active stakeholders in the risk management process. Many homeowners do not take action even if the risk is abundantly clear and loss reducing measures are widely available. The reason for this is that they feel that a disaster will not affect them [97].

Therefore, it is often in the hand of the public authority, i.e. the federal, state and local governments, to take the lead in managing disaster risk on large scales [145]. There are a variety of measures at disposition for the government to engage in risk reduction, both legislative and administrative, to enhance the level of protection of people and property and guarantee an adequate safety level that applies to all [196]. Building codes that include criteria for wind or earthquake resistance and legislation for land use might serve as examples of public risk reduction. Also, incentive programs instituted to reduce losses from disaster events might be followed. California's Proposition 127 [20] enacted in 1990 is a good illustration of the later category. It guarantees to private homeowners that those who perform seismic retrofits to their property will not face increased property tax until ownership changes. Further examples for public risk reduction projects might be seen in the construction of dykes or rain storage reservoirs to limit flood risk, the retrofitting and regular surveillance of public buildings and infrastructure elements, such as bridges, tunnels, pipelines and many more.

In all these public activities that might be undertaken to reduce disaster risk, the government has mainly two incentives to allocate resources to risk reduction:

1. As the government in its role as the insurer of the last resort [164] often takes on the responsibility of providing funds to cover losses from catastrophic disasters, it has a direct economic incentive to mitigate the risk from these events. If preventative measures have been implemented with adequate time horizon before a disaster takes place, it directly pays for the government in case of disaster occurrence.

2. The government aims at serving the public interest. Risk reduction investments might provide the public authority with the opportunity to increase the welfare of its citizens and of society as a whole.

As public resources are limited, the decision to devote resources to disaster risk reduction must be continually appraised in light of other competing needs. Allocating resources in disaster risk reduction means taking resources away elsewhere, such as health care, education and social services that also have the potential to enhance social welfare [195]. There is an increasing cost to a fixed increment of risk reduction. Absolute safety cannot be obtained and even less at a reasonable cost. Therefore, it is necessary to strike a balance between the cost of risk reduction and its benefits, making trade-offs in the decision process unavoidable.

CBA now constitutes a valuable economic tool to systematically access and evaluate the cost and benefits that are associated with a risk reduction project in order to assist the public authority in making informed and rational decisions. The application of CBA to public risk reduction projects however, brings along certain difficulties. As a great part of the benefits in terms of reduced disaster consequences of the various categories are of intangible nature, such as prevented fatalities or reduced environmental and CSH losses, a reasonable way has to be found to convert these preserved values to monetary value in order to relate them meaningfully to the project cost. Here, the willingness to pay (WTP) concept is an important economic tool to appraise intangible values, as illustrated in particular in Chapters 4 and 5 in its special application to safety. Furthermore, as many different individuals are affected by the project and to different extents, there is often no consensus about the project decision and a conflict of interest results. Here, welfare economic serves as the theoretical basis to identify policy changes which are beneficial for society as a whole, as outlined in Chapter 3.

Outline - Considered Risk Reduction Projects

Before it is proceeded with characterizing social risk reduction projects economically, it is first necessary to specify the type of risk reduction projects that are considered in this thesis within the risk management framework. As public risk reduction projects are in the focus of attention that are evaluated by means of a social CBA, it is obvious that comparatively large projects are the subject of study. Such an analysis only makes sense, if a rather large number of individuals is potentially affected and the impacts of the intervention are measurable on aggregate level. Therefore, the system under consideration is assumed to be an entire country, a region, a city or at least a suburb, including a substantial number of structural elements and people exposed to the risk and where the final decision about project appraisal is in the hand of the public authority.

Furthermore, the performance of a social CBA requires some time to collect all the data and is therefore of particular importance within preventative disaster management. Disaster reduction activities of this category are performed with considerable time horizon before the disaster takes place where its occurrence is a pure statistical issue.

Strategies of the categories preparedness in contrast, might not leave enough time to carry out cost benefit considerations as the disaster is already about to strike the system. A quick decision about how to evacuate people and protect property in the short term is needed. The same holds for response activities. As the disaster has already taken place, an immediate decision about how to allocate emergency forces and provide shelter to the victims of the disaster is necessary to prevent the disaster aftermaths from becoming even more devastating. Also for the evaluation of disaster risk reduction strategies of the category recovery there is the urgency to regain the functionality of the system as quickly as possible. Only in this way, the occurrence of further indirect disaster consequences such as business interruption is avoidable, making a time consuming investigation unfeasible.

At this point it has to be noted however, that the distinction between the different risk reduction strategies with respect to their implementation relative to the time of disaster occurrence is often not as sharp as it may seem. Also preparedness, response and recovery activities require attentive planning with adequate time horizon before a possible disaster. Consequently, their implementation is shortly before, immediately after and shortly after the disaster respectively, while their planning is to be considered as preventative. Nevertheless, it is focused on structural risk reduction measures in the following that either reduce the exposure of the system's elements or their structural vulnerability.

2.6 Public Risk Reduction Projects - The Economic View

From an engineering point of view there are many possibilities to engage in disaster risk reduction, with examples provided above. All the different measures that might be undertaken differ greatly in their technical implementation and require an analysis of diverse physical processes. Nevertheless, they have in common that they all reduce either the exposure or the vulnerability of single EaRs and thus of the system as a whole, resulting in a reduction of damage and in a second step of loss in case a disaster occurs.

From an economic point of view in contrast, the technical details of the risk reduction measures are of minor importance. Here, the risk reduction project decomposes in a series of cost and benefits occurring at different points in time. As an initial expenditure is necessary to implement the measure and the benefits in terms of reduced disaster consequences flow back uncertainly over the effective period, the public risk reduction intervention meets the basic characteristics of a financial investment [148]. Furthermore, these investments in preventative disaster risk reduction are largely characterized by the following three properties:

1. **Uncertainty:** The cost and even more the benefits are associated with a tremendous amount of uncertainty. There is uncertainty about the occurrence of a natural disaster as well as its intensity, about the evolution of its impact within the system, about the damages that might go in line with a certain hazard load as well as the loss that results from the damaged elements over time. Also uncertain is the loss reducing potential of the considered risk reduction initiative.

2. **Irreversibility:** Once the decision has been made to implement the project, the invested financial means cannot be recovered in case of adverse project development. Consequently, at least a part of the investment cost is lost or "sunk".

3. **Flexibility:** There is the possibility to postpone the investment to some later point in time. This may be advantageous in order to improve the information basis, finally making cost and benefit estimations more reliable. Certainly, the postponement of the investment entails the possibility that a disaster occurs in the meantime that could have been confined by an implementation of the measure in time.

The importance of these three investment characteristics will become clear in Chapter 6, where a real option approach is developed to account for the flexibility involved in the project decision under uncertainty. As a result of the above discussion, the social risk reduction project constitutes an investment that is economically characterized by the cost and benefit occurrences over time. Therefore, these two driving forces shall be illustrated more in detail subsequently.

2.6.1 Cost of Risk Reduction

The cost of the risk reduction project depends on the specific measure under investigation. To take into account all cost that accrue over the service life of the measure, the cost assessment is based on a life cycle cost approach. Accordingly, the total cost C^T can be split into investment cost C^I, operations cost C^O, maintenance & repair cost C^{MR}, replacement cost C^R and a residual value C^{RV} [189]:

$$C^T = C^I + C^O + C^{MR} + C^R + C^{RV} \tag{2.28}$$

The investment cost represents the initial expenditure that is necessary to implement the risk reduction project. As it is due at the very beginning of the measure's service life, it may be estimated quite reliably under cautions planning. The operations, maintenance & repair cost is a regularly reoccurring expenditure that is required to maintain its functionality over the design life and to prevent the risk reduction measure from deteriorating. Whereas the operations cost can usually be determined quite reliably on an annual basis, the maintenance & repair cost are subject to uncertainties in the time of occurrence as well as in their absolute amounts. The replacement cost covers the expenditure that is needed to replace single elements of the structure or the structure itself in case of failure or at the end of their service lives. Normally, the replacement cost is due at rather advanced ages of the measure's service life and there is a considerable amount of uncertainty involved in its estimation. The residual value is the net worth of the measure at the end of the study period n and constitutes the only negative cost component. It is especially important when evaluating project alternatives that have different life expectancies. Table 2.3 provides and exemplary overview of the different cost components and their possible occurrences over the study period n.

Cost component	$t = 0$	$t = 1$	$t = 2$	**...**	$t = n - 1$	$t = n$
Investment	C_0^I	C_1^I	0	...	0	0
Operation	0	C_1^O	C_2^O	...	C_{n-1}^O	0
Maintenance & Repair	0	C_1^{MR}	C_2^{MR}	...	C_{n-1}^{MR}	0
Replacement	0	0	0	...	C_{n-1}^R	C_n^R
Residual Value	0	0	0	...	0	C^{RV}
Annuity	0	AC	AC	...	AC	AC

Table 2.3: The cost of a risk reduction investment

In order to avoid double counting, the estimation of the different cost components has to be carried out cautiously in accordance with the characteristics of the special risk reduction measure under investigation. Here, it has to be distinguished between strengthening measures such as retrofit that are applied to existing structures and measures that are newly constructed and serve entirely for protection, such as flood defenses. Whereas the investment cost needs to be fully accounted for throughout all measures, in the estimation of the other cost components a distinction has to be made. In case of strengthening measures only the additional operations, maintenance & repair and replacement cost in comparison to status quo[1] have to be considered, while for measures that are newly implemented, the full cost need to be taken into account. If this cost difference in comparison to status quo should be negative, it enters the calculation on the benefit side. The same holds for the estimation of the residual value. In the estimation of the various cost components expected values are to be taken as a baseline for calculation [216].

The reliable assessment of the various cost components is a complex engineering issue that has to be carried out case specifically with focus on a particular risk reduction measure. As this thesis aims at providing a general framework for judging on public risk reduction projects, it is assumed throughout this thesis that the cost of the risk reduction project are given and at hand. For a more in depth discussion on life cycle costing it is referred to [85]. What remains to be done from an economic point of view to is to meaningfully relate the cost to the benefits with respect to their occurrence in time. In this respect, one possibility is to discount the diverse cost components back to the decision point $t = 0$ by means of a properly defined discount rate r to obtain the NPV of the total cost:

$$NPV_{C^T} = \sum_{t=0}^{n} \frac{C_t^I + C_t^O + C_t^{MR} + C_t^R}{(1+r)^t} + \frac{C^{RV}}{(1+r)^n} \tag{2.29}$$

The NPV corresponds to the value of the total cost at the time the decision is made. The second strategy that might be followed is to transform the stream of cost over time into a constant yearly cost by means of the annuity:

[1] Refers to the condition of an EaR or the whole system in the initial situation, before a risk reduction measure has been implemented.

$$AC = \frac{r}{1-(1+r)^{-n}} \cdot NPV_{C^T} \tag{2.30}$$

The transformation of the total cost in a constant annuity cost is sometimes convenient in the application of social CBA to disaster risk reduction projects, because the benefits are obtained in terms of reduced annual disaster consequences. Therefore, a comparison by means of the annuity cost becomes directly possible. Alternatively, the annual benefits may be converted to their present value and compared to the NPV of the cost.

2.6.2 Benefits of Risk Reduction

Whereas the cost estimation of risk reduction interventions already poses a considerable amount of difficulties, the assessment of their benefits is an even more challenging task. In the benefit estimation, the above introduced risk analysis procedure plays a central role. Being more precise, the procedure is employed to firstly estimate the particular disaster risk in status quo situation and in a second step under the new conditions that are expected to hold after project implementation. Then, by performing a risk review iteration, the annual disaster risks before and after the risk reduction project are compared to net out the measure's annual benefit potential. It is assumed in the following that the benefits are formulated in terms of reduced expected annual disaster loss, quoted separately for each consequence category.

Benefit component	$t=0$	$t=1$	$t=2$	**...**	$t=n$
Reduced economic loss	0	$B_1^{econ}[\$]$	$B_2^{econ}[\$]$	...	$B_n^{econ}[\$]$
Reduced human loss	0	$B_1^{hum}[m]$	$B_2^{hum}[m]$	...	$B_n^{hum}[m]$
Reduced environm. loss	0	$B_1^{env}[\$]$	$B_2^{env}[\$]$	...	$B_n^{env}[\$]$
Reduced CSH losses	0	$B_1^{CSH}[\$]$	$B_2^{CSH}[\$]$	...	$B_n^{CSH}[\$]$

Table 2.4: The benefits of a risk reduction investment

In order to relate these reduced losses meaningfully to the cost of the project, they need to be translated into monetary value. In this sense, the evaluation of reduced CSH as well as reduced environmental losses is beyond the scope of this thesis and they are assumed to have been priced by the application of some other method available in literature [38, 40, 204, 252]. What remains to be priced are the reduced annual economic and human losses. Whereas the reduction in economic losses is already given in monetary terms, the reduced human losses are formulated in terms of prevented fatalities and injuries per year. The derivation of a social value for the prevented fatalities is best done by means of the WTP concept, which constitutes one of the main focuses of this thesis and is discussed in Chapters 4 and 5.

As a result, the data input for the social CBA from the benefit side is summarized in Table 2.4, where it is assumed that the measure becomes effective from period one onwards and saves m lives annually.

Probabilistic Benefit Assessment

After the benefits of a public risk reduction measure have been outlined, it is now analyzed, how the measure impacts the loss distribution and thus total disaster risk mathematically. Depending on the specific preventative measure that is chosen to reduce the risk, it either causes a reduction in exposure of the system's elements or reduces their structural vulnerability. In the present context it is rational to assume that the measure leads to a higher or at least the same level of protection for all considered hazard intensities. Statistically, this implies that either the hazard pathway function $f_{HL|HI}$ or the structural vulnerability function $f_{DA|HL}$ need to be substituted by new functions $g_{HL|HI}$ or $g_{DA|HL}$ respectively, that are first order stochastically dominated by the old functions that characterize status quo condition.[2] Thus, if exposure has been reduced by the measure, the following condition holds for the hazard pathway functions before and after the intervention:

$$\int_0^a f_{HL|HI}(y,\overline{x})dy \leq \int_0^a g_{HL|HI}(y,\overline{x})dy \quad \text{for all } a \text{ and each given } \overline{x} \tag{2.31}$$

This equation implies that the probability that the actual hazard load falls below an arbitrary level a is higher after the implementation of the measure than in status quo condition for each fixed hazard intensity $\overline{x}$. Stated more simply, the probability that lower hazard loads will occur is higher after the implementation of the measure than before for each fixed hazard intensity.

If structural vulnerability has been reduced in contrast, the status quo and new structural vulnerability functions are related in a similar way:

$$\int_0^a f_{DA|HL}(z,\overline{y})dz \leq \int_0^a g_{DA|HL}(z,\overline{y})dz \quad \text{for all } a \text{ and each given } \overline{y} \tag{2.32}$$

Analogously, this equation implies that the probability that the level of damage falls below an arbitrary level a is higher after the implementation of the risk reduction project than before for each fixed hazard load $\overline{y}$. Consequently, the resistance of the structure has been improved.

Now, as the PDF of damage depends both on the conditional PDFs characterizing exposure and structural vulnerability (as indicated by equation (2.11)), a reduction of either exposure or structural vulnerability also causes a shift in the damage PDF from f_{DA} to g_{DA}. Therefore, the new damage PDF g_{DA} is first order stochastically dominated by the damage PDF f_{DA} representing the status quo as well:

$$\int_0^a f_{DA}(z)dz \leq \int_0^a g_{DA}(z)dz \quad \text{for all } a \tag{2.33}$$

This means that no matter whether exposure or structural vulnerability has been reduced, the damage PDF after the intervention will deliver higher probabilities that the actual level of damage falls below an arbitrary level a. This implies that lower damage states are more likely to occur.

Before the final step from damage to the loss is undertaken, it has to be outlined, that the system vulnerability $f_{LO|DA}$ is not directly affected by the preventative risk reduction activities under consideration. Thus, its functional form is maintained and still valid under the new conditions. Nevertheless, as the PDF of loss, which is provided in (2.15), is both dependent on the system vulnerability and on the damage PDF, also a shift in the loss distribution from f_{LO} to g_{LO} is imposed by the risk reduction intervention:

$$f_{LO}(v) = \int f_{LO|DA}(v,z)f_{DA}(z)dz \quad \overset{f_{DA}\rightarrow g_{DA}}{\longrightarrow} \quad g_{LO}(v) = \int f_{LO|DA}(v,z)g_{DA}(z)dz \tag{2.34}$$

[2]The basics about stochastic dominance are provided in Appendix B.2.

Now, from the first order stochastic dominance relation between new and old damage PDF given in (2.33) it follows, that also the new loss PDF g_{LO} is first order stochastically dominated by the loss PDF f_{LO} that has been valid before the risk reduction intervention:

$$\int_0^a f_{LO}(v)dv \leq \int_0^a g_{LO}(v)dv \quad \text{for all } a \tag{2.35}$$

Again, this relation implies that the new probability that the actual level of loss will fall below an arbitrary level a will be higher after the intervention than before. Thus, lower levels of loss are more likely to occur.

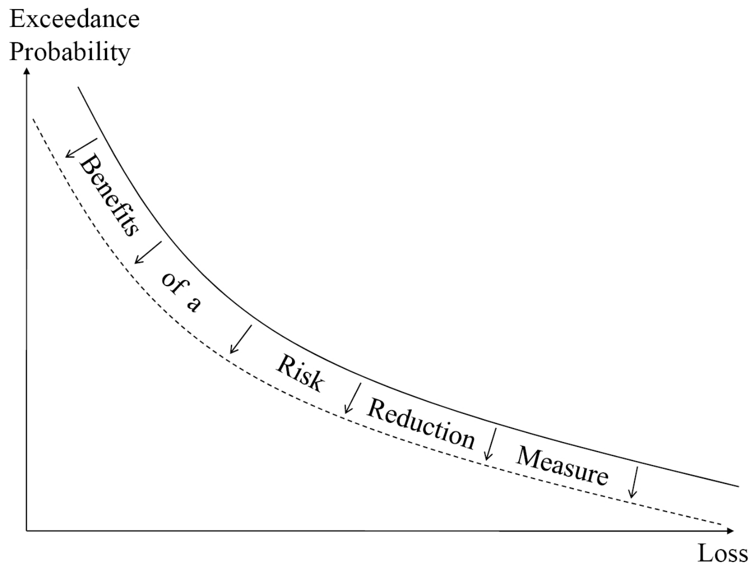


Figure 2.12: Annual benefit of a disaster risk reduction measure

According to the discussion in Section 2.3.2.2, disaster risk is often visualized by means of exceedance probability curves as they are easy to interpret. With the results derived above, it becomes possible to visualize the effect of a disaster risk reduction intervention on the exceedance probability curve, as is common practice in literature [97]. This is done by referring to equation (2.21) which captured the relation between the loss PDF and the corresponding exceedance probability curve. If φ_L and ψ_L represent the loss exceedance probability functions before and after the intervention respectively, the following condition holds:

$$\varphi_L(a) = 1 - \int_0^a f_{LO}(v)dv \qquad \text{and} \qquad \psi_L(a) = 1 - \int_0^a g_{LO}(v)dv \tag{2.36}$$

From this equation in combination with (2.35) and the fact that probability density integrals are always bounded between 0 and 1 it can be concluded that $\varphi_L(a) \geq \psi_L(a)$ holds for all possible losses a. This result is displayed graphically in Figure 2.12 and is in line with the above discussion about stochastic dominance.

The two types of losses that are of particular importance in this thesis are the economic loss and the loss of life. To distinguish between these two loss types mathematically, the so far general loss distributions f_{LO} and g_{LO} before and after the risk reduction intervention need to be split in order to indicate the economic and human loss potential conditional on a certain damage state. In this respect, the functions f_{LO}^{econ} and g_{LO}^{econ} as well as f_{LO}^{hum} and g_{LO}^{hum} are introduced to describe the PDF of the economic and human losses before and after the intervention, respectively. As the functions f_{LO}^{econ} and f_{LO}^{hum} are related to the same damage PDF f_{DA}, this split in economic and human losses straightforwardly also implies a split of the system vulnerabilities in $f_{LO|DA}^{econ}$ and $f_{LO|DA}^{hum}$, that predict economic and human losses for a given damage state. Obviously, the same holds for the functions of the g family that describe the situation after risk reduction.

With this notation in hand, the benefits of the risk reduction intervention are ready to be formalized. It is clear that the benefits of the risk reduction intervention represent the prevented losses in the change from status quo to final state. If $LO_{initial}$ denotes the total loss before the intervention and LO_{final} the loss after the intervention, these two variables are random variables following the loss distributions f_{LO} and g_{LO}, respectively. Therefore, the total benefit of the risk reduction intervention given by

$$B^T = LO_{initial} - LO_{final} \tag{2.37}$$

must also be a random variable following a certain benefit distribution. Without knowing which forms the functions f_{LO} and g_{LO} will concretely take however, the benefit distribution cannot be further specified analytically. What can be done instead is to determine the expected value of the benefits, due to the linearity property of the expected value operator. Distinguishing between human and economic benefits, the expected annual benefits of the disaster risk reduction project are determined to be

$$\begin{aligned} E(B^{hum}) &= E(LO_{initial}^{hum}) - E(LO_{final}^{hum}) = \int v \cdot f_{LO}^{hum}(v)dv - \int v \cdot g_{LO}^{hum}(v)dv \\ E(B^{econ}) &= E(LO_{initial}^{econ}) - E(LO_{final}^{econ}) = \int v \cdot f_{LO}^{econ}(v)dv - \int v \cdot g_{LO}^{econ}(v)dv \end{aligned} \tag{2.38}$$

which represents exactly the delta in total risk in comparing the situations before and after the intervention. As the loss PDFs of the g family are first order stochastically dominated by the loss PDFs of the f family, it follows that the expected benefits must be positive, as illustrated in Appendix B.2. Accordingly, the expected benefits represent the prevented human and economic losses per year. If it is assumed that the risk reduction measure is equally effective over the effective period, these benefits reoccur statistically every year as illustrated in Table 2.4.

To complete the probabilistic description of disaster risk reduction benefits, the variance of the two benefit categories is given by

$$\begin{aligned} Var(B^{hum}) &= Var(LO_{initial}^{hum}) + Var(LO_{final}^{hum}) - 2 \cdot Cov(LO_{initial}^{hum}, LO_{final}^{hum}) \\ Var(B^{econ}) &= Var(LO_{initial}^{econ}) + Var(LO_{final}^{econ}) - 2 \cdot Cov(LO_{initial}^{econ}, LO_{final}^{econ}) \end{aligned} \tag{2.39}$$

and contains valuable information about the variability of the annual benefit stream. This is an important measure to be included in economic investment decisions as discussed in the sequel.

2.6.3 Disaster Risk and Risk of Misinvestment

In this paragraph, a short review about the different understandings of risk in the engineering and financial context is presented. In civil engineering the total disaster risk is defined by the probability of loss times loss itself, as outlined in (2.18) above. The calculation of disaster risk by means of (2.18) reduces the whole bunch of information that is contained in the loss distribution to its mean value. This is justifiable in the disaster risk management context because disaster risk is a pure downside risk, meaning that there is just the possibility to lose [165]. Therefore, the expected value may serve as an approximation with how much annual loss on average has to be calculated with.

Nevertheless, it does not indicate whether the main contribution of the risk comes from frequent and comparatively small events that only cause moderate loss or if it is mainly driven by rare events that are devastating and go in line with tremendous amounts of loss throughout all consequence categories. Therefore, it seems advisable to include measures of dispersion around the mean by means of second moments and measures of skewness through the incorporation of third moments in the analysis of the loss distribution. By means of these statistical tools it is seen more clearly how the probability mass under the loss PDF is distributed across different loss levels.

Furthermore, the loss potential of extreme events can be assessed by the Value at Risk (VaR) or Expected Shortfall (ES), which are specifically designed risk measures that operate on the tails of the loss distribution F_{LO}. Hereby, the VaR represents the level of loss that will potentially not be exceeded with a given probability α over a specific time horizon

$$VaR_{\alpha}(LO) = F_{LO}^{-1}(\alpha) \tag{2.40}$$

whereas the ES measures the expected value of loss given that the Value at Risk has been exceeded:

$$ES_{\alpha}(LO) = E[LO|LO \geq VaR_{\alpha}(LO)] \tag{2.41}$$

These two measures, originating from financial risk management literature, may consequently be employed to characterize disaster risk more precisely. Further details about these two measures are provided in [116, 157, 219].

Financial risks in contrast, where the risk of misinvestment implicit in social risk reduction investments can be integrated in, belong to the more general class of speculative risks. Here, the probability to lose is accompanied by the chance to win. In this context, the initial estimation if the project or investment under investigation will be profitable is based on the expected values of the returns. These are then confronted with the investment cost. If the expected difference of the two is positive, the project is expected to be profitable and is therefore attractive to the investor at first sight. Nevertheless, because the returns on the investment are uncertain, there is always the risk of misinvestment, which describes the possibility that returns occur that fall short of the expected value below the investment cost. Consequently, the financial risk is to be seen more like the possibility that the expected value will be missed, which actually constituted the basis of planning.

Therefore, financial risks are often characterized by statistic parameters that describe also the scattering around the mean, among these variance σ^2, standard deviation σ or volatility $\overline{\sigma}$ [206]. Higher scatterings around the mean imply a higher risk of misinvestment. But this is only one side of the coin. Higher scatterings are also accompanied by higher chances of deviating from the mean in the other direction, i.e. the chances to receive higher than expected returns also increase. Therefore, the final decision about the investment associated with a certain risk of misinvestment depends on the risk attitude of the investor. Is she risk neutral, will she decide on basis of expected returns only. Is she risk averse or risk seeking in contrast, will she subjectively prefer investments with lower or higher volatility respectively, given that they have equal expected returns. To conclude, the dominating influences on the investment decision are the expected returns, the scattering around the mean as well as the risk attitude of the decision maker.

2.6.4 Structural Failure and Risk of Misinvestment

To finally close this section, a nice analogy between structural engineering and finance is presented. An important concept within structural reliability analysis in civil engineering constitutes structural failure analysis. In this field of research, the event of structural failure is assessed by investigating the interactions between the two driving forces load S and resistance R. The load and resistance are subject to substantial uncertainties and are thus described probabilistically. Structural failure is then defined as the event where the load exceeds the resistance. Accordingly, the failure probability may be characterized by introducing a random variable M, which is often being referred to as safety margin [77]:

$$M = R - S \tag{2.42}$$

The probability of failure eventually corresponds to the event where M becomes negative.

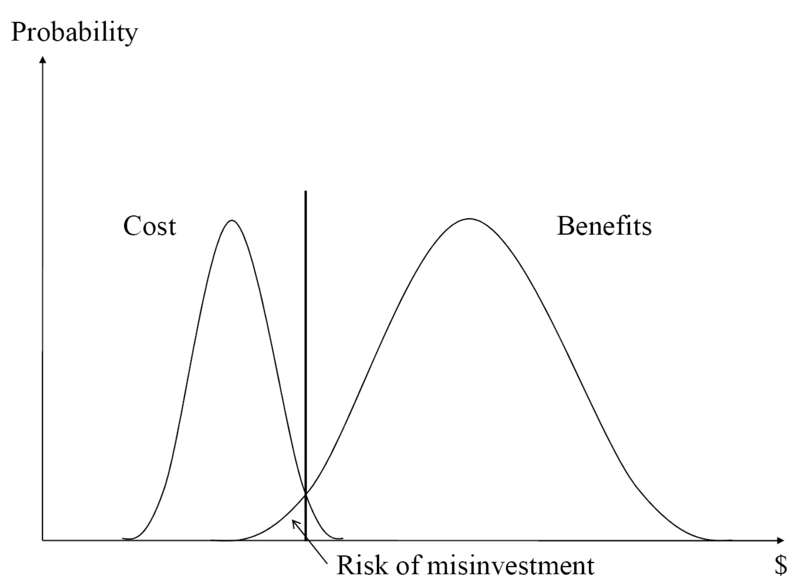


Figure 2.13: The risk of misinvestment

In the evaluation of public risk reduction projects by means of a social CBA there is the risk that the investment will lead to a misinvestment. The total cost C^T and even more the total benefits B^T are subject to substantial uncertainties and are assessed probabilistically. A misinvestment is then defined as the event where the cost exceed the benefits. Accordingly, the misinvestment probability may be characterized by introducing a random variable NB, which is often being referred to as net benefits:

$$NB = B^T - C^T \tag{2.43}$$

The probability of misinvestment eventually corresponds to the event where NB becomes negative. This analogy is depicted in Figure 2.13 where the well-known load-reliability graph is transformed into the financial context.

2.7 Discussion

In this chapter, an innovative probabilistic risk management framework has been introduced that enables a systematical assessment, evaluation and treatment of natural disaster risk. The consecutive steps to be performed have been discussed in detail and a mathematical description of the important, in literature often ambiguously defined terms has been provided. Furthermore, possible interpretations for various risk definitions that are often encountered in risk related publications have been presented.

In a second step, the necessity for the public authority to engage in risk mitigation has been emphasized and the problem of evaluating public risk reduction projects economically has been discussed. By focusing on preventative disaster risk reduction activities, the general approach to estimate the cost and benefits of a risk reduction investment has been introduced and a probabilistic benefit assessment has been carried out. To describe the benefits with sufficient statistical accuracy, the loss distributions before and after the implementation of the risk reduction project turned out to be of particular value. Concentrating on mean values only, the annual benefits equal the delta in total risk. Eventually, by comparing the different understandings of risk in an engineering and economic context it has become clear, that the risk encountered in disaster management constitutes a pure downside risk, while the risk inherent in financial project appraisal belongs to the class of speculative risks. In both cases however, the necessity to include statistical measures beyond the mean value has been highlighted.

The theory assembled throughout this chapter enables the sound identification and assessment of the cost and benefits of public risk reduction projects. It has been assumed that the cost of the project as well as the benefits in terms of reduced environmental and CSH losses have already been appraised and are on hand. As the benefits in terms of reduced economic loss are directly measured in monetary terms, the major remaining challenge at this point is to economically evaluate the prevented fatalities in order to include them in a CBA. This is subject of the three upcoming chapters. In Chapter 3 the welfare economic theory to perform this appraisal is provided and based on this, Chapters 4 and 5 concentrate on the economic evaluation of increased human safety by discussing the WTP concept both on individual and aggregate level.

Chapter 3

Cost Benefit Analysis as Applied Welfare Economics

Societies must make choices on how to use their scarce resources such as labor, capital, land and so on. At any point in time the total amount of these resources has to be considered as fixed, making trade-offs in the decision processes on how to allocate these resources inevitable. Consuming more of one thing usually goes in line with the necessity to consume less of another thing. In addition, as society is composed of numerous heterogeneous individuals, changes in allocation often result in a conflict of interest. A social risk reduction investment is such a change. It uses resources now and causes an increase in the social safety level over the effective period of the measure. This in turn results in the prevention of economic and human losses over the design period, which will eventually increase the capital and labor endowments of society long term. Therefore, it is necessary to rely on certain criteria that help to decide under which conditions the economic situation is better in the final state after the implementation of the measure than in the initial state before. Welfare economics now is that branch of economics that aims at characterizing social welfare levels in different economic states [31]. Consequently, it delivers the theoretical foundation for managing risk in public interest and is at the heart of public policy and at the core of cost benefit analysis (CBA) [35]. CBA is often interpreted as applied welfare economics [130] as it aims at quantifying individual and social welfare changes and thus transforms the welfare analysis in the monetary scale. By briefly discussing the essentials of welfare economics and CBA, this chapter will provide the essentials for social decision making and constitutes the theoretical basis of this work.

3.1 A Broader View of Welfare Economics

Economics is a discipline of social sciences that studies the consumption, production and distribution of goods and services within an economy. By examining the behavior of economic agents that interact within the economy on markets, theories are derived that aim at explaining how the forces of supply and demand allocate scarce resources. Whereas microeconomics studies the behavior of single individuals, firms and the government typically on markets where particular goods are traded, macroeconomics on the other hand examines effects such as economic growth, inflation, unemployment on aggregate level of the entire economy. Therefore, in the field of macroeconomics aggregated indicators such as GDP, unemployment rates, life expectancy are of special importance to investigate how the whole economy is working and developing over time. In this sense, a great part of macroeconomics has so called microfoundations, implying that microeconomic models of consumer and firm behavior are employed and aggregated on social level to study the relationships between these macroeconomic indicators.

Positive economics is to be interpreted as that branch of economic theory that deals with understanding and predicting economic behavior. The theorems of positive economics are logically derived from axioms or assumptions about consumer and producer behavior, technology and availability of scarce resources. The theoretical parts of positive economics, insofar as they possess empirical implications, provide hypothesis that can be tested statistically. Furthermore, there might be controversy whether or not the assumptions behind a positive model are sufficiently realistic, but moral or ethical assumptions are completely outside the scope of positive economics. Therefore, it is only concerned with "what is" [139]. Normative economics, on the other hand, incorporates value judgments and is concerned with what "ought to be". It tries to evaluate the desirability of certain aspects of the economy and is designed to support public decision making to achieve particular desirable goals.

Welfare economics is normative economics. It focuses on using resources optimally to achieve the maximum wellbeing for the individuals of society as well as for society as a whole. However, there is a great deal of controversy of what is to be considered as optimal. As with positive economics, the propositions of welfare economics are also logical derivations from a set of axioms and assumptions, but they differ from those of positive economics in the sense that they are ethical assumptions and value judgments with which people may legitimately disagree. Welfare is not an observable entity such as market prices or profits [138]. Two assumptions that are fundamental to welfare economic are:

- The welfare status of society must be judged solely by the members of society, which recognizes the traditional emphasis on the importance of the individual in the western society and
- The notion that society is only better off if any member of society is made better off without making anyone worth off.

From these two principles it becomes obvious that welfare economics constitutes a science that operates between micro- and macroeconomics. Based on investigations of individual behavior and preferences, statements about total social welfare on aggregate level are derived. Furthermore, it is important to outline at this point that welfare economics itself is not about decision making. The essential objective in contrast may be seen in enabling the political decision maker representing her constituency to make informed decisions. The effects of the public project are therefore analyzed and quantified to provide an understanding of the extent and magnitude of the economic consequences, the final decision whether to implement the project or not however constitutes the politician's task. This is due to the fact that political factors such as political orientation, international relations or related economic considerations such as trade deficit management may play a crucial role influencing the project decision [139].

3.2 Preferences and Utility Functions

A crucial step in welfare economics constitutes the measurement of wellbeing of individuals that comprise society. In order to judge if a particular social state is preferred to another it must be assessed how the individuals would decide with respect to the proposed change. Consequently, each individual contributes to the final social judgment with her personal decision, in line with the individualistic principle. In this sense, the preference construct is of particular importance in welfare economics. Preferences are taken into account to explain people's choices in the presence of several alternatives. In a second step then, under certain requirements, utility functions are constructed to represent preferences in a mathematical way. Therefore, a utility function rationalizes the preference relation.

3.2.1 Preferences and Rational Consumer Axioms

Preferences are a theoretical construct that help to explain consumer behavior. As they are not directly observable, their assessment is all but an easy task and crucial for the outcome of an economic analysis. In this respect, several methods have been developed that are classified in behavioral and non-behavioral approaches [136]. Behavioral approaches are discussed in detail in Chapter 4 and non-behavioral approaches constitute the focus of attention in Chapter 5 in the context of safety.

Starting point for the preference analysis are so called consumption bundles. A consumption bundle defines a certain combination of goods and services the individual chooses to consume and their respective amounts. If X denotes the total consumption space that subsumes all goods and services that are available for consumption, a consumption bundle is best described by a vector $x \in X$ of the following form [104]:

$$x = (x_1, \dots, x_n) \tag{3.1}$$

Hereby, the vector components $x_i, i = 1, ..., n$ describe the amount that is consumed of good or service i. Now, if the considered individual in her role as a consumer has the free choice to select between two different consumption bundles x and x', she will intuitively choose the one that brings her subjectively more satisfaction or will be indifferent between the two. A decision for consumption bundle x leads to the conclusion that she prefers x over x' and thus, $x \geq x'$ holds. Therefore, the preference relation is a binary relation on X that reflects the consumer's subjective evaluation of different consumption bundles. In order to include preferences in economic analysis it is common to impose some assumptions about their consistency that facilitate their treatment. In this sense, it is presumed that the individual behaves as a "rational consumer", implying that her preferences are in line with the following four axioms [256]:

1. **Reflexivity:** Every consumption bundle is at least as good as itself, implying $x \geq x$.
2. **Transitivity:** If consumption bundle x is preferred to x' and x' is preferred to x'', then also x is preferred to x''.
3. **Completeness:** For any two possible consumption bundles x and x' out of consumption space X the consumer must be able to say if she prefers one to another $x \geq x'$ or $x' \geq x$ or is indifferent between the two $x \geq x'$ and $x' \geq x$.
4. **Continuity:** The preference relation must be continuous over consumption space X.

The assumption that these axioms hold and the consumers are rational in their preference expressions enables the graphical and mathematical representation of preferences through continuous utility functions, as discussed next.

3.2.2 Utility Functions

A rational consumer's preferences satisfying the four stated axioms may conveniently be represented by a utility function U. Utility is closely related to the satisfaction, happiness or wellbeing the consumer derives from consuming a certain bundle of goods, whereas non economic aspects of wellbeing are either ignored or assumed to remain unchanged [35]. A utility function constitutes a tool used by economists to analyze and understand individual market behavior. In this respect, whenever a consumer prefers a commodity bundle x to a commodity bundle x', it is inferred that a consumer's utility is greater in the former case than in the latter, so that the following condition holds:

$$U(x) \geq U(x') \Leftrightarrow \text{ commodity bundle } x \text{ is preferred to commodity bundle } x' \tag{3.2}$$

Thus, if a consumer behaves in accordance with the rationality assumptions, it can be said that she behaves as if she was equipped with a well-defined utility function. Whether she will do so at the end is ultimately an empirical question [129].

As the requirements for the utility function are rather weak, there are generally many utility functions that match a given preference ordering. Nonetheless, it is sufficient to select one arbitrary utility representation out of all possibilities as they lead to similar conclusions and serve equally well. Within utility theory two conceptually distinct ways to describe utility have emerged over the years, splitting welfare economic theory in two branches:

Cardinal Utility: Classical welfare economists like Marshall, Walras, Sidgwick and Edgeworth were partisans of the cardinal branch of utility theory, being convinced that utility was a measurable quantity and fully interpersonally comparable, provided that enough information about preferences had been collected. In the cardinal sense, utility itself has a significance and utility differences are meaningfully comparable. Thus, it may be stated not only that an individual prefers one good to another, but also how much she prefers it over the other. Cardinal utility functions are equivalent up to positive linear transformations, as they preserve the second derivative of the function. Moreover, they are generally assumed to be increasing in each argument and fulfill the diminishing marginal utility condition, indicating that an individual derives less and less additional utility from increments in commodity consumption at higher absolute levels of consumption. Nowadays, cardinal utility functions are of particular importance in life cycle consumption models and in the analysis of risk attitudes.

Ordinal Utility: As a reaction to the constructive critics of many economists, such as Robbins [223], that argued that there is no objectively justifiable way to measure utility on a cardinal scale and even more to support interpersonal comparisons of utility, the development of the new welfare economics approach was initiated. Mostly influenced by the works of Pareto [199], Hicks [111] and Kaldor [140], it could be shown that ordinal utility is sufficient to derive the fundamental economical results that have been obtained under cardinality. For this reason, there is nowadays general agreement among economists that the analysis of consumer behavior should be based on ordinal utility. In ordinal utility theory it is assumed that a consumer is only able to rank different bundles of consumption goods and to integrate them into a preference ordering. It can only be said that one particular consumption bundle is preferred to another but not how much it is preferred. Thus, the absolute level of utility has no meaning and utility differences are of no significance.

Therefore, the utility functions are equivalent up to any positive monotone transformation, as they maintain the preference ordering. In order to work with ordinal utility functions, they must be at least two dimensional. The central value of information that is contained in an ordinal utility function is how much of one good is exchangeable for a certain amount of another good so that the absolute level of utility remains constant. This exchange rate is referred to as the marginal rate of substitution (MRS) and is at the core of indifference curves analysis.

3.3 Welfare Economic Principles

Based on the individualistic principle of social welfare, welfare economics is concerned with two major objectives. The first aim is to allocate society's scarce resources efficiently in order to supply the maximum achievable wealth and prosperity for society as a whole. The second goal lies in the fair distribution of the efficient allocations among the members of society. Both of these sub disciplines rely on the utility concept and are discussed in the following.

3.3.1 Efficiency

In contemporary welfare economics questions of economic efficiency are generally assessed on basis of the Pareto criterion. This criterion is an efficiency norm that describes the conditions necessary to achieve optimality in resource allocation.

The Pareto principle:

A social state is defined as Pareto optimal when no action can be undertaken that makes one individual of society better off without making at least one other person worse off. In this connection, the terms better off and worse off refer to the individual utility functions. It is said that a person is better off (worse off) from a change in resource allocation when the utility level is higher (lower) after than before the change. Accordingly, a reallocation of resources constitutes a Pareto improvement if at least one individual of society is made better off in terms of a utility gain and all other members of society maintain at least their preliminary level of utility. A Pareto optimal social state is a situation where no further Pareto improvement is possible. Although the Pareto concept clearly is a value judgment, it is a weak one in the sense that most people would accept it [129]. In most real world situations however, the Pareto criterion reaches its limits rather fast as public projects usually involve both winners and losers. In addition, a Pareto optimal state might be very unfair from a distributional point of view. If for instance one individual possesses all society's wealth, any intervention that makes this individual worse off and all others better off cannot be judged upon by the Pareto criterion.

In a free market economy where consumers and producers interact with the intention to maximize utility and profits respectively, a set of marginal conditions have to be fulfilled to ensure that Pareto optimality holds. In this respect, it is important to emphasize the marginality property of these optimality rules, meaning that they refer to the last unit consumed or produced, respectively. The condition that no one can be made better off without making somebody else worse of must hold between consumers (exchange efficiency), in production (production efficiency) and between consumers and producers (allocative efficiency). The rules which guarantee these efficiency conditions are as follows [119]:

1. **Exchange efficiency:** The MRS between any two goods x and y must be equal across all consumers of the two goods.

2. **Production efficiency:** The marginal rate of technical substitution (MRTS)[1] between any two factors of production, say labor L and capital K, must be equal across all producers in the production of x and y.

3. **Allocative efficiency:** The marginal rate of transformation (MRT)[2] for producers must be the same as the MRS between the goods for consumers.

If any of these three conditions is violated, a Pareto improvement will be possible. A detailed discussion on this topic is provided in [139] and the optimality conditions are readopted in the discussion of Chapter 4 and 5.

The strength of the Pareto criterion lies in the fact that it does not require the utilities of different individuals to be compared or weighted and that it requires only ordinal utility. Its weakness lies in the fact that it yields an incomplete ordering, i.e. there are many Pareto optimal allocations that satisfy these three marginal conditions. In addition, different Pareto optimal solutions are not comparable solely by relying on the Pareto criterion and the Pareto criterion per se does not rank every Pareto optimal allocation superior to any non optimal allocation [31]. What is therefore needed are further criteria that allow to identify a path towards a Pareto optimal state and more demandingly, to select one out of the set of all Pareto optimal solutions that is most desirable for society. Such concepts are given by compensation tests and the formulation of a social welfare function.

Compensation Tests and Potential Pareto Improvements:

In almost all practical situations both winners and losers are involved, so that the Pareto principle is of limited value for the evaluation of welfare changes. Addressing this limitation, compensation tests have been developed that are specifically designed for the evaluation of welfare changes due to public projects. The most common compensation test originates from a work of Kaldor [140] and Hicks [111] and is often being referred to as Kaldor-Hicks compensation test or potential Pareto improvement. This test requires for a social change to be approved that the gainers are able to compensate the losers and still have some net benefit left over.

In this respect, it has to be emphasized that the test considers only hypothetical compensation, whether compensation is actually carried out in the end is regarded to be an important but separate decision [129]. In practice, it may be too costly to ensure that there are actually no losers, because the administrative expenses for compensation are often greater than the gain in net benefits [35]. In addition, if the redistribution was actually carried out however, the entire exercise would be a direct application of the Pareto principle itself [139]. The purpose of considering hypothetical redistribution is to try to separate efficiency and distributional aspects of a policy change. Consequently, the Kaldor-Hicks compensation test focuses only on the efficiency aspect of a welfare change and provides

[1] The MRTS measures the rate at which one factor can be substituted for another at constant levels of output.

[2] The MRT measures the rate at which good x can be transformed in good y in the production process. It is given by the slope of the production possibility frontier, which represents all efficient output combinations for any two goods x and y.

the theoretical justification for CBA. The main tool to carry out these compensation tests in practice is the willingness to pay (WTP) concept that is discussed in Section 3.5.

By means of the Kaldor-Hicks compensation test, a Pareto optimal allocation is always consistently ranked superior to any non Pareto optimal allocation. However, problems arise when the criterion is applied to judge on any other type of change, as demonstrated for instance by Scitovsky [231]. Therefore, the compensation principle cannot be employed to obtain a complete order of social states and many social states still remain non comparable. A comprehensive and more in depth discussion on this issue is provided in [168]. To draw a conclusion, for the construction of a complete social welfare ordering, it is necessary to make stronger normative assumptions through the specification of a social welfare function [31].

3.3.2 Distribution

Whenever a utility conflict between at least two different individuals concerning a change in resource allocation is observed, more than the Pareto principle is required to decide between different alternatives. Such a complete ranking of the numerous economical efficient allocations of goods is called a social welfare ordering. It is comparable to the individual preference orderings that are formulated in terms of utility functions to rank different bundles of consumption goods. If the social welfare ordering is assumed to be continuous, it may be represented by a social welfare function (SWF) that takes the utility functions of the individuals as its arguments, such that a higher value of the function is preferred to a lower one:

$$W = W(U_1, \ldots, U_m) = W(U_1(x_1, \ldots, x_n), \ldots, U_m(x_1, \ldots, x_n)) \tag{3.3}$$

A SWF of this type is known as Bergsonian welfare function [27]. Unfortunately, a priori there is not much more to say about the concrete form the SWF will take. The form of the SWF depends on the subject behind the function. It may represent the views of the parliament or that of the social decision maker. In literature however, it is generally assumed that the SWF satisfies certain reasonable requirements [129]:

1. The SWF should depend only on the utility levels of the individuals and thus, fulfill "welfarism".

2. In order to satisfy the Pareto criterion it should be monotonically increasing in its arguments, meaning that an increase in any individual's utility function ceteris paribus[3] leads to a higher level of social welfare. This condition implies that the indifference curves of a SWF in utility space are negatively sloped. If one individual is worse off, another individual must be better off to maintain a given level of social welfare. The intensity of this trade-off is given by the slope of the social welfare indifference curve and depends on the degree of inequality. In this sense, it is generally assumed that social welfare indifference curves are convex to the origin, just like the utility indifference curves of the consumers.

3. The SWF should satisfy the principle of anonymity, indicating that it is insensitive to who actually receives a high or low level of utility. Therefore, any permutation of the individual utility arguments within the SWF must lead to the same result.

[3] All other things being equal.

Under the premise that a social welfare function possessing all these desirable properties has been constructed, the maximum social welfare point is identified where the grand utility possibility frontier, that represents the set of all Pareto efficient combinations in utility space, is just tangent to the highest social welfare indifference curve. A comprehensive discussion on the properties of social welfare functions is provided in [232].

Beside the strong normative assumptions that are required in the construction of a SWF, the formulation of a SWF only makes sense if utility is cardinal and comparable across individuals. This result has been proven mathematically by Arrow [12] and is widely known as Arrow's impossibility theorem. If utility functions are ordinal and not interpersonal comparable, the only possible consistent social welfare ordering is a dictatorship, i.e. the SWF must coincide with the utility function of some individual. For these reasons questions about distribution often play only a minor role in practical welfare economic analysis. They are often being declared as a political issue that can be accomplished for instance through a progressive tax transfer system and are frequently considered to be beyond the scope of economics [105]. Instead, it is propagated to focus on efficiency considerations only. This view is also adopted for the purpose of this thesis. For an in depth discussion on the construction of SWFs and the inclusion of distributional issues in welfare analyses it is referred to [34, 35, 139, 203, 233]. An interesting essay on the interrelations between efficiency and equality goals is provided in [191].

3.3.3 Fundamental Theorems of Welfare Economics

In Paragraph 3.3.1 the concept of Pareto optimality has been introduced and characterized by a set of marginal conditions that must be fulfilled in order to guarantee an efficient resource allocation. Rather than following some optimization strategy to reinforce the validity of the conditions "manually", most professional economics put their faith in the "invisible hand"[4] of the markets. The first welfare economic theorem derived by Arrow and Debreu [13] delivers the justification for this attitude:

First fundamental theorem of welfare economics: A perfectly competitive market equilibrium is Pareto optimal.

A perfectly competitive market is characterized by the following four conditions [180]:

1. With freely available and perfect information about a market transaction, both consumers and firms show utility and profit maximizing behavior respectively, choosing the outcome that yields the greatest possible net benefits to themselves.

2. Firms and consumers are price takers, i.e. they are too small in relation to the entire market to influence the market prices. Therefore, price fixing, leadership and monopolistic practices constraining output levels are ruled out by assumption.

3. There are no market barriers that prevent firms from entering into or from exiting a market. Resources are completely mobile, and firms are able to enter freely if a market is profitable or leave if losses are expected.

[4] Famous formulation of Adam Smith, introduced in his work on "The Wealth of Nations IV" [242], who has been a pioneer of political economy and is often being cited as the godfather of modern economics.

4. All factors of production are privately owned.

Given that these four conditions are met, the market mechanism generates a set of outputs and prices through the interaction of supply and demand. A perfectly competitive market is in equilibrium where demand equals supply and represents the point where demand and supply curves intersect. The demand curve displays the amount the consumers are willing to pay for each unit of output. The WTP may be interpreted as the subjective benefit they derive from the consumption of an additional unit of output. The supply curve in contrast shows the producers' or resource owners' marginal cost for each unit of output. It is equal to the value of resources that have to be forgone in order to produce the extra unit of output in perfect competition. Therefore, in equilibrium the WTP is equal to the marginal cost and defines market price and quantity. Moreover, supply and demand are perfectly accordable, indicating that there is no excess supply and no excess demand so that the market clears. As a consequence, an equilibrium corresponds to a Pareto optimal solution and any other price-output combination is inefficient. Any move towards the equilibrium price-output level constitutes a Pareto improvement.

In line with the above discussion, there is an infinite number of Pareto optimal allocations which may be for most instances very unfair. The outcome actually depends only on the initial distribution of endowments. If this distribution is altered, a new Pareto optimum is obtained. This essence is subject of the second welfare economic theorem:

Second fundamental theorem of welfare economics: Each possible Pareto optimum can be achieved as a competitive equilibrium after a redistribution of initial endowments if preferences and technology are convex.

In other words, all that is required to reach a particular more desirable outcome is a redistribution of initial endowments after which the market can be left alone to do its work. This suggests that the two objectives of efficiency and distribution may be followed separately and need not involve a trade-off. A formal mathematical proof of the two welfare economic theorems is provided in [11, 79, 255].

3.4 Market Failures

Certain limitations have to be taken into account when relying on the market mechanism to obtain the social value of goods. There are several real world conditions that might impose market failure, preventing the market from producing efficient prices. They distort the working of the price mechanism and cause the market to deviate from its ideal social efficiency level. These imperfections generate a divergence between observed prices and production cost in both factor and product markets, causing an inefficient use of resources. Therefore, in case of market failure, observed prices do not reflect the real scarcity values of society's resources and corrective measures need to be applied. This provides the government with a justification to intervene in the market process to correct the imperfections as well as to redistribute resources by transfer payments and progressive taxation. There are a couple of reasons why markets may not operate efficiently, which can be integrated predominantly in the following classes [225]: imperfect information, imperfect competition, externalities and public goods. For a comprehensive discussion on market failure mechanisms and their treatment it is referred to [54, 79]. The last mentioned reason for market failure, public goods, shall further be exemplified as they are of particular relevance in the subsequent discussion.

Public Goods

A public good as opposed to a private good is generally characterized by two properties [152]:

1. **Non-Rivalry:** A public good is non-rivaled, meaning that one individual's consumption does not reduce the availability of the good for others. Thus, they are simultaneously consumable by everyone.

2. **Non-Excludability:** A public good is non-excludable, indicating that no one can effectively be excluded from consuming that good.

These two properties collectively imply that actually all individuals of society benefit from the provision of a public good, which is available to everyone to equal amounts. Therefore, in practice the pricing of public goods is often being carried out on basis of representative consumer models, that allow to analyze welfare effects on the average individual of society [31]. This is further discussed in Chapter 4. Especially the non-excludability property of public goods makes it difficult to provide such goods on the market. People do not have the incentives to reveal their true WTP for the good because they can possibly consume it at no cost. This is commonly known as the free rider problem. If suppliers in turn cannot succeed to collect all the benefits of a public good they have produced, there won't be enough incentives for them to produce it. This leads to the fact that private markets generally underproduce public goods [32], i.e. the supply of public goods is below its socially optimal level. As a consequence, it remains the government's task to correct this imperfection and to provide the public good in efficient amounts, by making use of its power to reinforce a payment via taxation [96].

Examples of public goods that are often being referred to are military, clean air, environmental goods and many more. Also, many examples exist where the definition boundaries between public and private goods are blurred. Safety constitutes an example of the latter category: there is a basic infrastructure that is supplied to all, while each individual may take additional private precaution to elevate her personal safety level. For the efficient provision of public goods certain tools are at disposition for the government such as the definition of user fees, property rights and taxation [35]. As no direct prices for public goods are observable, their valuation is often being carried out by extracting a WTP measure out of the consumer's behavior on the markets or by survey techniques [171]. This is further illustrated in Chapter 4.

3.5 Cost Benefit Analysis as Applied Welfare Economics

In the previous sections it has been demonstrated, that welfare economics is about judging on changes to evaluate if one economic situation is preferable to another. The individualistic postulate of welfare economics dictates that individual preferences represented by utility functions are to be taken as the baseline to judge on changes. In order for the change to be approved on social level, it must fulfill the Kaldor-Hicks efficiency test and be in accordance with certain distributional value judgments. As the whole welfare economic discussion is based on ordinal utility functions that are in itself neither measurable nor interpersonal comparable, they cannot be taken into account to judge if the welfare gain of one individual is higher than the welfare loss of another individual due to the implementation of a public project. This is exactly the point where CBA comes into play. CBA is about measuring changes in monetary terms and thus serves as an instrument to translate the reasoning behind the Kaldor-Hicks criterion into practice. In the CBA context the WTP becomes the primary means of measuring utility changes and the measuring rod that permits aggregation of preferences on social level [200]. For potential losses, the willingness to accept (WTA) measure might also be used, as further illustrated in Chapter 4.

In the present thesis, changes in social welfare result from the implementation of a public risk reduction project. To carry out the project, resources are diverted away from other sources and cause opportunity cost. On the other hand, benefits occur that individuals derive from increases in consumption of the project's output, which is mainly given in terms of increased safety henceforth. In other words, these two effects result in a welfare loss on markets where resources are diverted away and a welfare gain on markets where increased output is produced. The central question that is tackled by CBA is under which conditions the overall effect of the project is beneficial to society.

To find an answer to this question it is crucial to identify and record all changes that occur due to the implementation of the public project. Immediate adjustments in the quantities of the provisioned goods and services as well as secondary modifications in individual consumer behavior that might occur due to changes in the relative prices of the goods are to be priced and included in the economic analysis. But as the supply with consumption goods does not constitute the ultimate purpose of economic activity and rather serves to satisfy the needs of the consumer, the final evaluation of changes is carried out by measuring changes in individual utility by means of the WTP concept. Therefore, individual utility levels are not only the basis of welfare economics but in the center of CBA [3, 104].

	Social CBA	**Financial CBA**
Viewpoint	Society as a whole	Individual, household or firm
Objective	Increase in welfare	Increase in income or profit
Benefit	Any kind of increase in wellbeing or utility including monetary revenue	Monetary revenue
Benefit Measure	WTP, WTA	Monetary revenue
Cost	Any kind of decrease in wellbeing or utility including monetary cost	Monetary cost
Cost Measure	Opportunity cost	Monetary cost
Value	Net change in welfare	Net change in revenue
Measure	Dollars	Dollars

Table 3.1: Differences between social- and financial CBA [Binning [28]]

As a consequence of the preceding discussion it follows that social CBA is to be conceptually distinguished from private or financial CBA that is carried out by a private investor. The economist engaged in social CBA is not asking a different kind of question from that being asked by the private investor. Rather the same kind of question is evaluated for a larger group of people. Instead of analyzing if the project will be profitable to the single investor, the economist asks whether society as a whole will become better off by undertaking the project. For the more precise concept of revenue to the investor, the economist substitutes the less precise concept of WTP to measure the individual and in a second step the social benefit [170]. For the cost to the private firm the economist substitutes the concept of opportunity cost, i.e. the social value forgone when resources are diverted away from alternative economic activities into the specific project. The general project decision rule that holds for both CBA disciplines is that the benefits must outweigh the cost for the project to be approved. The conceptual differences between social and financial CBA are summarized in Table 3.1.

3.5.1 The Concept of Willingness to Pay (WTP)

CBA aims at recording and pricing changes in individual welfare levels that are imposed by a public project and in a second step at aggregating them on social level. Utility functions serve as the basis to price changes in individual welfare. It has been emphasized previously, that the ordinal utility concept has been adopted for analyzing consumer behavior, implying that absolute utility levels as well as utility changes have no meaning themselves and are not interpersonally comparable. Therefore, it is not admissible to characterize welfare changes by changes in individual utility. What has to be done instead is to convert individual utility changes to monetary equivalents, that then in turn enable the evaluation of changes in wellbeing across individuals. This is exactly where the WTP concept begins to operate, making the utility concept functional to evaluate policy changes.

The basic value of information that is contained in an ordinal utility function is how much of one good an individual is willing to trade off against a certain amount of an other good so that she remains equally well off. This analysis of trade-offs is generally known as indifference analysis and illustrated graphically by means of indifference curves. Accordingly, a gain in an individual's utility can be measured implicitly by the maximum amount of money or income that she is willing to give up in order to obtain a certain change, so that the initial level of utility is exactly maintained. This amount of income in turn defines the WTP.

To demonstrate this more illustratively, consider an individual in an initial state of wellbeing given by utility level U_0 that she achieves with a money income y_0 and a safety level s_0, so that

$$U_0 = (y_0, s_0) \tag{3.4}$$

holds. Suppose further that there is an increase in the safety level from s_0 to s_1 due to a public risk reduction project. This change would increase the individual's wellbeing to:

$$U_1 = (y_0, s_1) \tag{3.5}$$

What needs to be known is how much the wellbeing of the individual has increased by this safety improvement, i.e. $U_1 - U_0$. Since utility is not measurable it is sought for an indirect measure, namely the maximum amount of income the individual would be willing to pay for that change. The individual is now hypothesized to be considering two combinations of income and safety that both yield the same level of wellbeing U_0. One by which her income is reduced and safety increased, and a second where her income is not reduced and safety not increased:

$$U_0 = (y_0 - WTP, s_1) = U_0(y_0, s_0) \tag{3.6}$$

Therefore, the individual implicitly adjusts the WTP to the point at which these two combinations yield equal wellbeing. Therefore, the WTP is defined as the monetary equivalent or value of the change in wellbeing $U_1 - U_0$ resulting from the safety change.

In a last step, in order to arrive at a social willingness to pay (SWTP), the individual WTPs have to be aggregated over all members of society:

$$SWTP = \sum WTP_i \tag{3.7}$$

This formula often appears augmented by distribution weights that are assigned to the individual WTPs to elevate the relative position of economically comparatively weak individuals, in order to incorporate distributional considerations in the CBA. As this entails placing a rather strong normative value judgment in analogy to the definition of a social welfare function, the determination of these weights is not considered in the following and it is referred to [35, 139] for an in depth treatment of this issue. The WTP concept in general though, is extended and discussed in detail in Chapter 4.

In particular, the WTP concept plays an important role in the evaluation of non market goods, that are by definition not traded on markets and therefore have no direct prices. Once the preferences of an individual with respect to a non market good relative to a market good have been assessed, it becomes possible to infer the above illustrated trade-off and assign an implicit WTP, representing the individual's valuation of the non market good.

3.5.2 Steps to be Performed within a Social CBA

In line with the above discussion, a comprehensive CBA for social project evaluation requires the execution of the following steps [35, 162]:

1. The status quo state of society has to be assessed. This entails an analysis of the current level of social welfare, i.e. individual preferences, incomes, current prices, safety level and market imperfections have to be recorded.

2. The potential effects of the public project have to be estimated. In this respect, it is important that all the effects on all individuals are assessed, both private and social, direct and indirect, tangible and intangible.

3. All recorded effects are to be priced. Here, for marginal effects on traded goods in many cases market prices may serve of a proxy, noting the illustrated limitations though. For large effects on traded goods as well as for non traded goods in contrast, a derivation of the WTP on basis of utility functions is crucial to assess the true social price.

4. The social discount rate needs to be specified to discount the stream of cost and benefits, accounting for the fact that cost and benefits accrue at different points in time.

5. Distributional effects are to be recorded and incorporated by weighting individual WTPs with distribution weights or by means of inequality measures on aggregate level.

6. The impact of uncertainty has to be evaluated and included in the final project recommendation.

In this work, in particular steps 1 and 2 are being covered by the above introduced risk management framework. By means of the latter, the current level of disaster risk may systematically be assessed and the effectiveness of potential risk reduction projects over the effective period estimated.

With special emphasis on the pricing of public safety, Chapters 4 and 5 are concerned with the implementation of step 3. Furthermore, Chapter 5 also contains a discussion on how to specify the social discount rate. As already stated previously, step 5 is not being accounted for in the present thesis. Chapter 6 deals with the evaluation of uncertainty inherent in the project's cost and benefit estimations and therefore covers step 6. Eventually, Chapter 7 provides a practical application of all illustrated steps.

3.5.3 CBA Decision Rules

As soon as all consecutive steps of the social CBA have been executed, a final decision whether to implement or to reject the project under evaluation must be made. If the total cost C^T and the total benefits B^T of the project have been identified and priced at the different points in time they are expected to occur, there are several tools available to demonstrate cost efficiency. These are in particular:

1. Net Present Value (NPV)
2. Benefit to Cost Ratio (B^T/C^T)
3. Internal Rate of Return (IRR)

Each of these project evaluation criteria has its advantages and drawbacks that can be reviewed in detail in [162, 200]. Instead of providing a detailed discussion on these decision rules, only the Net Present Value rule is readopted, as it is most intuitive and predominantly applied in practice [238].

Net Present Value: The net present value criterion (NPV) measures the difference between the discounted flow of benefits and the discounted flow of cost and is therefore adequate for both private and social perspective. The rules that need to be accepted for using the NPV criterion are the following [123, 124, 249]:

1. Only projects with positive NPV are to be accepted.
2. Under a fixed budget constraint the subset of available projects is to be chosen so that the NPV is maximized.
3. Without fixed budget constraint the project with the highest NPV is to be chosen.
4. Only investment strategies with approximately the same length of life are to be compared.

$$NPV = \sum_{t=0}^{n} \frac{B_t - C_t}{(1+r)^t} \tag{3.8}$$

In this equation B_t and C_t represent the monetized cost and benefits occurring at period t, respectively and r denotes the discount rate indicating time preferences. In this sense, the NPV represents the amount of money or magnitude that may be earned by a project at decision point $t = 0$.

3.6 Discussion

This chapter reviewed the fundamental aspects of welfare economics, which serve as a baseline to guide public policy decisions. The two central goals of welfare economics have been outlined to be the efficient allocation of resources and their fair distribution among the members of society. Whereas efficient allocations are characterized by means of Pareto optimality, the normative judgment of an allocation to be fair is implicitly included in the formulation of a social welfare function.

In order to verify if a public project drives an economy towards a more efficient allocation of resources, the Kaldor-Hicks compensation test constitutes a valuable tool for practical purposes and provides the theoretical basis for CBA. Social CBA transforms the theoretical underpinning of welfare economics into practice, by converting individual welfare changes to monetary value through the WTP concept. By means of a clear structure to systematically identify and analyze social welfare changes that accrue due to the implementation of a public project and a set of decision rules, CBA provides clear guidelines to assist public policy decisions.

A public disaster risk reduction investment is to be seen as a public project that has an impact on social welfare. While on the one hand it causes cost in terms of material and labor that are needed to implement the project and diverted away from other uses in the economy, on the other hand it brings benefits to the members of society in terms of reduced disaster losses. Referring to Chapter 2, the benefits occur over a wide range of categories which have been subdivided into reduced economic, human, cultural social historical (CSH) and environmental losses. For a great fraction of the avoided losses no direct market prices exist, which could be taken into account as an indicator for their scarcity values. Among these are potentially saved lives.

Instead of placing a value on human life directly, which would obviously contradict with our ethical understanding that human life is innumerable and priceless, the conventional way in economics is to convert these "statistical lives" saved to reductions in the risk of death, which are in turn enjoyed by any member of society. Put in this perspective, the necessity to value life in order to guarantee an efficient resource allocation becomes more acceptable in public view. Furthermore, safety may be interpreted in terms of an economic good that brings utility to individuals and any changes in safety that result from reductions in mortality risk are consistently appraisable by means of the WTP concept. This in turn enables a coherent incorporation of safety related aspects into CBA.

The derivation of a SWTP to price reduced human disaster losses constitutes the focus of study in the two upcoming chapters. After a conversion of the public risk reduction project decision problem to a pure safety problem, the next chapter concentrates on deriving a SWTP for safety in a bottom up fashion: based on individual preferences for safety, the WTP for safety is assessed for each economic agent individually and in a second step aggregated on social level. In Chapter 5 in contrast, a top down approach for directly obtaining a WTP on social level is presented, which is based on the Life Quality Index (LQI).

Chapter 4

Pricing Safety - Bottom Up Approaches

A profound decision on the profitability of a public risk reduction investment depends crucially on two aspects: the reliable estimation of the measure's cost and benefits and the assessment of their social scarcity values. After the risk management framework has been introduced addressing the first aspect, it is now focused on the derivation of a social price for benefits that occur in terms of prevented fatalities, i.e. the social willingness to pay (SWTP) for safety. In line with the individualistic postulate of welfare economics, a bottom up approach is followed to estimate the SWTP: based on each economic agent's preferences, firstly a willingness to pay (WTP) is derived on an individual level and then, in a second step aggregated over all individuals that comprise society to obtain a SWTP for safety. To do this consistently, at the outset a literature review on existing WTP estimation approaches is being performed and exemplary values are listed. Then, a theoretical model is developed, that discloses the influence of safety in a market economy. Here, safety is being modeled as a public good that enters the utility functions of the consumers and production functions of the producers as a separate argument, but is outside their control. No direct market for safety exists. Only the government has the ability and power to impose changes in the publicly provided safety level that uniformly affect all economic agents. These in turn reply to safety changes by modifying their behavior. The resulting welfare changes are then priced by means of the WTP concept, taking into account all possible effects. Afterwards, by introducing a representative consumer model it is departed from the pure individualistic safety pricing strategies and general equilibrium safety pricing rules are developed. These in turn serve as a basis for the analysis of the Life Quality Index (LQI) in Chapter 5.

4.1 Willingness to Pay for Safety and Value of Statistical Life

One of the most contentious issues arising in the field of cost benefit analysis (CBA) of public projects that impact human safety has been the valuation of reductions in mortality risk. The conceptual basis for most valuation approaches constitutes the WTP for safety and the closely related value of statistical life (VSL). Despite almost four decades of both theoretical and empirical research in this field, still no consensus concerning the validity of the results it produces in actual applications has been achieved [41]. In this introductory section, the historical development of the WTP approach as well as commonly applied strategies to assess the WTP for safety in public policy are briefly reviewed and exemplary VSL values obtained from different studies are provided.

The essentials for valuing increases in safety[1] by means of the WTP concept were introduced in 1968 by Schelling [229] and in 1971 by Mishan [169]. Both of these two early works in this field highlighted the conceptual weaknesses of the human capital method[2] to valuing lives, which corresponds to the present value of an individual's expected future income [221, 264]. The two authors pointed out that consistency with the theoretical foundations of CBA and welfare economics requires to value benefits in the form of prevented fatalities in terms of small changes in the risk of death for any particular individual on basis of the individual's WTP. Mishan states in this respect:

> "It is the only economically justifiable concept...And this assertion does not rest on any novel ethical premise. It follows as a matter of consistency in the application of the Pareto principle in cost-benefit calculations."

Schelling similarly highlights the importance of the individual concept in the evaluation of an increased chance of survival:

> "A common way to place a value on life is to determine the dollar amount that an individual would be willing to pay to reduce the probability of his death."

As a consequence of those two early influential works, most real world applications are based on the view of this statistical concept [180].

Hence, it is commonly accepted that the price for safety is to be determined by the WTP for reductions in mortality risk. As already discussed in Chapter 3, the WTP is defined as the maximum amount of income an individual is willing to give up in order to obtain a certain good or service and is considered as a measure of how much the individual values that good. In a conventional economic context it is assessed by investigating prices of goods and services that are traded on the markets. In the particular case of safety however, such market transactions cannot be directly observed because the reduction of mortality risk is not directly purchased. Therefore, in an economic sense, safety or mortality risk reductions constitute a non market good.

Once obtained, the estimations on the WTP for a particular small change in mortality risk $\Delta\mu$ can be normalized to some common level of risk reduction. The standard way has become to employ the VSL as a normalization of the WTP concept. Rather than the value for any particular individual's life, the VSL represents the amount that the entire society is willing to pay to reducing each member's risk by a marginal amount [61]: if for instance a group of 1,000,000 individuals is willing to pay \$10 each to reduce their individual risk of death by $2 \cdot 10^{-6}$, the SWTP sums up to \$10 million. Statistically, two lives are saved in this society so that the VSL is calculated to be \$5 million. Consequently, the VSL standardizes the WTP concept to the reduction in mortality risk that corresponds to exactly one prevented fatality in society. Mathematically, the WTP and the VSL are related in the following way

$$VSL = \frac{\sum_{i=1}^{N} WTP_i}{N \cdot \Delta\mu} \tag{4.1}$$

where WTP_i represents individual $i's$ WTP for a reduction in mortality risk of $\Delta\mu$ and N denotes total population size.

[1] The expressions 'increase in safety' and 'reduction in mortality risk' are used synonymously henceforth.

[2] The human capital approach is further discussed in Chapter 5.

Despite the statistical nature of the VSL concept, valuing safety and life remains a controversial and problematic issue. Much of the discussion concerning the role of the economist in this matter stems from the reluctance to trade off dollars for lives. The necessity to value life in public decision making becomes more acceptable in public view when it is put in a slightly different perspective. What is done in project evaluation is not to value the worth of any particular identifiable individual whose life is at stake ex post but rather the value of preventing a statistical death ex ante, i.e. the avoidance or reduction of a small probability of death [222]. It is investigated how resources should be devoted to programs which reduce the probability of death from a specific cause for a specific group of people. In order to do this comprehensively, economists are forced to place a value on an expected life saved, so that the mysterious personal nature of life valuation is removed and resources may be allocated most efficiently throughout projects [91]. Viscusi [257] summarizes the controversial issue of valuing life in the following manner:

> "Ignoring the issue of valuation of life and limb may circumvent the problem of offending people's sensitivities by making the trade-offs explicit. But at the same time it may be very costly in that it sacrifices lives that could have been improved or saved by a more systematic allocation process. An important issue for society as a whole, and one that many people are unwilling to face, is whether lives will be sacrificed in an effort to maintain the illusion that we will not trade off lives for dollars."

As a consequence, the crucial step in the safety pricing procedure is to assess how people are willing to trade off changes in their consumption and labor provision pattern against changes in the publicly provided safety level according to their preferences.

In general, two classes of economic approaches for measuring the WTP for the non market good safety are available in literature. Firstly, certain real world situations make it possible to reveal people's WTP for mortality risk directly. Approaches of this category rely on observations of actual behavior on the markets and are therefore termed revealed preference approaches. The second category makes use of survey techniques and choice experiments to create a hypothetical situation designed to have people disclose their WTP for safety. They are known as stated preference approaches.

4.1.1 Revealed Preference Approaches

The revealed preference method has been applied extensively to estimate the VSL [61]. The central assumption underlying the approach is that people reveal their preferences through their market behavior. The information on the WTP is acquired by identifying situations in which people actually make trade-off decisions between income or wealth and mortality risk, either implicitly or explicitly.

Labor Market Studies

The major part of revealed preference approaches has been conducted on the labor market by means of wage-risk studies. Here, wage premiums associated with greater mortality risk on the work place are estimated. These premiums are deduced by regressing the wage on the risk of death. Regression analysis is applied to factor out influences other than risk of death that are reflected in the observed wage differentials. The extracted wage premium then indicates that there is a trade-off between income and mortality risk. The amount of additional wages people are paid per unit of additional mortality risk serves as an indicator for the monetary value of the risk increment to those individuals who voluntarily accept the risk in exchange for a given wage increment. This goes in both directions. If the wage premium is positive and people are exposed to a higher risk on the job, it corresponds to the peoples' willingness to accept (WTA). If the wage premium is negative on the contrary, it represents peoples' WTP for increased safety. In both cases, the wage premium can be taken into account to compute a VSL in the above sketched fashion. Thaler and Rosen [247] were the first to apply hedonic pricing on the labor market in 1975.

Authors	**Year**	**VSL (US$ 2007)**	**Country**
Labor market studies			
Weiss et al. [265]	1986	8,936,585	Europe
Moore and Viscusi [173]	1988	7,577,561	U.S.
Moore and Viscusi [174]	1988	11,288,780	U.S.
Moore and Viscusi [175]	1989	12,062,439	U.S.
Knieser and Leeth [143]	1991	927,805	U.S.
Cousineau et al. [56]	1992	5,040,650	Canada
Martinello and Meng [161]	1992	7,317,074	Canada
Elliot and Sandy [73]	1996	1,756,098	UK
Meng and Smith [166]	1999	6,016,260	Canada
Arabsheibani and Marin [10]	2000	17,232,878	UK

Table 4.1: VSL estimates from labor market studies

Exemplary VSL estimates from several labor market studies have been collected and are summarized in Table 4.1. All estimates have been converted to US$ 2007[3]. The table contains ten distinct labor market studies all published in peer reviewed journals, which should warrant their analytical credibility. Even if the studies originate from different sources and have been obtained by labor market observations in different countries, they have all been conducted in industrialized countries with comparable income. Nevertheless it is astonishing, how far the range of the values, i.e. from US$ 927,805 to US$ 17,232,878 goes. The mean value of the ten sample studies is US$ 7,815,613, with a median of US$ 7,447,317 and a large standard deviation of US$ 4,892,657. It is anticipated at this point, that labor market studies usually lead to significantly higher VSL values than the other approaches and show the greatest variance across all studies. A possible explanation for this might be, that the trade-offs involved in wage-risk studies refer to before tax wage premiums, while studies of the other categories base their calculation on after tax income.

The wage-risk method to price safety on the labor market relies on several assumptions to guarantee a correct reflection of a person's subjective WTP in the market conditions [91]:

1. The labor market under investigation operates freely and is in equilibrium.
2. Workers possess correct information about the risk that is inherent in different kinds of jobs.

[3]If necessary, prices have been converted from local currency to US$ using the purchasing power parity of the given year and then an inflation adjustment based on the consumer price index has been carried out to obtain US$ 2007, as suggested by [47].

A violation of the first assumption will render biased WTP estimates as the wages do not reflect true scarcity values. The second assumption that workers are able to correctly calculate the actual risk level of potential jobs is necessary for observed (i.e. market) wage-risk premiums to mirror individuals' marginal values of safety. The assumption that perceptions of risk and actual risk coincide in real world applications however, is at best very suspect [91].

Consumer Market Studies

Consumer market studies constitute the second category of revealed preference approaches and are by far not as prominent as wage risk studies in safety literature. Here, trade-offs people make with respect to income or wealth and mortality risk are evaluated in their everyday consumption decisions. On basis of sophisticated econometric approaches, such as hedonic regression and conjoint analysis, market prices for safety attributes implicit in certain composite goods that are sold on real product markets are derived. For example, data on the market price of smoke detectors and their effects in reducing the probability of dying in a fire might serve to extract people's WTP for safety, as performed by [58]. Another illustration for consumer market studies to derive a VSL is the investigation of different safety features in cars and the corresponding reduction in mortality risk, see for instance [18]. In the special application to derive a WTP for enhanced disaster safety, the method has been applied in the analysis of real estate prices with similar characteristics but with different property exposures to disaster risk [87].

Authors	**Year**	**VSL (US$ 2007)**	**Country**
Consumer market studies			
Garbacz [89]	1989	4,082,618	U.S.
Atkinson and Halvorsen [18]	1990	5,839,024	U.S.
Carlin and Sandy [44]	1991	996,098	U.S.
Garbacz [90]	1991	5,675,457	U.S.
Blomquist and Miller [30]	1992	4,541,463	U.S.
Dreyfus and Viscusi [70]	1995	5,238,049	U.S.

Table 4.2: VSL estimates from consumer market studies

Table 4.2 provides an overview about a collection of VSL estimates from consumer market studies that have been extracted from literature. Here, all studies have been conducted in the U.S.. The mean value of the sample is US$ 4,395,451, the median amounts to US$ 4,889,756, which corresponds to only around 60% of the labor market result and is thus significantly lower. The standard deviation is US$ 1,794,830, so that the distribution of values is more compressed.

In the application of consumer market studies to derive a WTP for safety several limitations have to be outlined. Also here, in order to obtain reliable VSL estimates that reflect people's WTP correctly, the safety related markets under investigation must be in equilibrium and perfect consumer information must be guaranteed. This means for instance, that consumers of smoke detectors must perfectly understand the probability of dying in a fire as well as the amount of risk reduction or safety increase associated with installing a smoke detector. This assumption about complete knowledge also entails that consumers mentally calculate whether the cost of buying the smoke alarm is worth the minute risk reduction that is provided. Therefore, the valuation of increased safety due to the implementation of the smoke detector must in turn also equal the money paid for an increase in traffic safety through the purchase of an airbag in a car, if compared per unit of increased safety.

In real world situations however, people typically underestimate the probability of higher probability death events such as cancer and smoking and overestimate the chances of lower death causes such as earthquake, lightening and so on [83]. Consequently, the assumption of perfect information is unlikely to hold in safety related markets. In addition to this limitation there are also significant difficulties in reliably extracting the price for one attribute out of a composite good, because it is hardly possible to find two comparable goods that differ only in the attribute to be valued. Also critical is the assumption that the consumer may consume exactly the amount of safety she wants, even if in reality this is only possible in combination with consuming the composite good.

Despite the stated limitations of revealed preference approaches, their primary advantage is that they are based on actual behavior that is observed in real world situations. The presumption is that people's own choices are the best reflection of their WTP values [47].

4.1.2 Stated Preference Approaches

Stated preference approaches use survey techniques in which people are confronted with a hypothetical situation that involves a trade-off between income or expenditures and a change in the risk of death. The main example for the stated preferences approach is contingent valuation, which was firstly applied by Davis [59] in 1964. In a direct contingent valuation approach, individuals are asked to state in specifically designed interviews or questionnaires, how much they would be willing to pay to change their mortality risk by a specific, generally small amount. In special application to disaster risk a typical question could be: how much would you willing to pay to reduce your risk of dying in an earthquake from $2 \cdot 10^{-7}$ to $1 \cdot 10^{-7}$?

A second variant of this approach is to disclose people's preferences towards safety by selling various goods with different risk exposures on artificial test markets and extracting the amount that is voluntarily spent for goods with enhanced safety level out of a fixed virtual budget. In the NOAA[4] Panel [16] guidelines are presented for performing contingent valuation consistently. An extensive listing of performed contingent valuation studies can be found in [45, 171] for the U.S. and in [95, 182] for Europe.

Several VSL estimates originating from stated preferences have been collected and are summarized in Table 4.3. All studies have been carried out in western countries with comparable incomes. The mean value of the selected studies is US$ 4,948,065, the median amounts to US$ 5,141,463 and the standard deviation is the lowest of all studies, namely US$ 1,282,778. It becomes obvious, that the selected stated preference approaches lead to results which are quite comparable to those of consumer market studies and significantly lower than the labor market values.

Certain limitations have to be taken into account when relying on contingent valuation to reveal peoples preferences towards safety. Biases occur because it can hardly be assured that respondents will provide honest answers to the questions they are asked and often reply as socially desired.

[4]US National Oceanic and Atmospheric Administration (NOAA)

Authors	**Year**	**VSL (US$ 2007)**	**Country**
Contingent valuation			
Jones-Lee et al. [135]	1985	6,516,098	U.K.
Gerking et al. [92]	1988	5,160,976	U.S.
Viscusi et al. [260]	1991	4,640,000	U.S.
Johannesson et al. [125]	1996	5,847,805	Sweden
Corso et al. [55]	2001	4,165,854	U.S.
Ludwig and Cook [153]	2001	6,427,317	U.S.
Persson et al. [207]	2001	3,145,366	Sweden
Alberini et al. [5]	2004	2,764,228	Canada/US
Chestnut et al. [48]	2004	5,691,057	Canada
DeShazo and Cameron [60]	2004	5,121,951	U.S.

Table 4.3: VSL estimates from stated preference approaches

Furthermore, the responds depend strongly on the way the questions are formulated and on phrasing. Hypothetical bias occurs because no real money is involved. Respondents know that they are not actually required to expend anything for their choices and therefore, the reaction is different in hypothetical markets than in real markets. However, certain strategies may be followed to mitigate the effects of hypothetical biases [1, 178]. Finally, respondents may have difficulties in understanding the small impact of changes in mortality risk properly. Contingent valuation studies strongly depend on the design of the study and even more on the way individuals understand and perceive the characteristics of the hazard under consideration [240, 241].

The major advantage of stated preferences approaches is that they allow to tailor questionnaire and sample to exactly elicit the information that is needed. They are applicable to the general population, while labor market studies are restricted to workers and consumer market studies to consumers of the particular safety related good. In addition, the availability of the responses and the characteristics of the respondent allow for an identification of the major determinants of the WTP [61].

In summary, both of the approaches discussed can be taken into account to directly obtain a WTP for safety or to process the collected preference information towards safety in the formulation of a utility function. Either of the approaches suffer from conceptual weaknesses. Neither approach is inherently superior and, indeed, they are almost certainly best viewed as essentially complementary rather than competing estimation procedures [134].

4.1.3 WTP and VSL for Public Policy

The previous section impressively highlighted how far the WTP and VSL estimations can range and vary across different application approaches and contexts. As the real preferences are unknown, it is not clear which of the approaches delivers the most accurate estimates. On an individual level the following factors certainly impact and might explain the observed variance in the WTP estimations to a certain extent: age of the respondent, background risk, altruistic concerns, income/wealth, education, risk preferences, context of risk, the question whether the study has relied on the WTP or the WTA to derive the VSL and many more [126].

With increasing sample size however, these variations should be less emphasized. Other possible explanations might be that market distortions have been present in the markets where labor or consumer market studies have been conducted or that perceived risk differed significantly from the actual risk involved.

The large variation in the VSL estimates raises the question which value to employ for public policy decisions or risk reduction project appraisal that have an effect on a potentially large number of people. One assumption that could be made is that the WTP for a unit of change in mortality risk is the same regardless of the type of risk. Another assumption could be that the average WTP for a population is always identical regardless of the characteristics of the population at risk. One or both of these hypothesis is implicit whenever a VSL value is applied in any context that differs from the context in which it was originally estimated. It becomes a concern when the population whose mortality risk is affected by a policy is considerably different from the population from which the VSL estimates have been obtained. It is also a concern if the type of risk is different from the context of the original WTP study.

Recognizing that evaluating the risk reduction benefits of a project by simply counting the number of lives saved and multiplying them by a single VSL is potentially inadequate for valuing changes in mortality risks in all circumstances, analysts have been investigating the dimensions of mortality risk that may be relevant to monetary valuation in project appraisal for projects that impact human safety. The central question is how the WTP for mortality risk reduction may vary in different application fields. Economic theory suggests that WTP may vary, but it does not answer the question of how precisely it varies. Thus, it remains largely an empirical question. Empirical work has been carried out, but the results are not yet sufficiently robust to provide a fully adequate basis for a new valuation approach for public project appraisal. Some alternatives have been proposed, and their basis may soon be sufficient for policy applications when additional research is undertaken to further explore these issues [47].

A desirable goal for public policy analysis would be the ability to extrapolate WTP estimates from one context and one population to another population and context and use standard VSL values. Such a procedure will become admissible only if numerous WTP studies have been performed on a variety of risk reductions contexts and on a variety of populations, allowing the estimation of an overarching WTP function that links WTP values with the characteristics of the risk and of the affected population. As it is usually quite costly and time consuming to conduct a SWTP analysis any time before a public decision is made, in practice policy analysis often relies on meta analyses. Meta analysis is a methodology comprising a vast array of statistical techniques, developed to systematically analyze differences between outcomes of WTP studies, ultimately leading to a synthesis of results [29]. In this sense the law of large numbers suggests, that the obtained results factor out systematical inconsistencies of single studies and converge to the true social mean WTP.

Table 4.4 provides an overview of recent meta studies that have been identified in literature. Also here, the results are not uniform and significant deviations are observable. The mean value over all studies is calculated to be US$ 5,809,360, while the mean value weighted by the number of observations included in each study is US$ 5,425,471. The standard deviation amounts to US$

Authors	**Year**	**VSL (US$ 2007)**	**Country**	**Number of studies included**
Meta Analyses				
Viscusi [258]	1992	7,950,000	Multiple	36
Miller [167]	2000	5,080,000	Multiple	68
Dionne and Lanoie [61]	2002	4,878,049	Multiple	86
Mrozek and Taylor [177]	2002	3,008,130	US	91
Viscusi and Aldy [259]	2003	8,374,000	Multiple	64
Kochi et al. [144]	2006	5,565,979	Multiple	78

Table 4.4: VSL estimates from meta analyses

2,023,124. It follows that even if there are not as many outliers in meta studies in comparison to single studies, the question which value to employ for policy decisions remains unanswered.

The US EPA's[5] Guidelines advise analysts to use a uniform VSL estimate of US$ 4.8 million in 1990 dollars for policy analysis. Based on the consumer price index to adjust for inflation this converts to approximately US$ 7.61 million in 2007 dollars [269]. This value is derived from a meta analysis of 26 estimates assembled for EPA's first retrospective analysis of the Clean Air Act [75], being all but two from the Viscusi 1992 [258] review. Each estimate is from a different study, with 21 of the estimates from hedonic wage studies and the remaining five derived from contingent valuation studies. The estimates range from US$ 0.9 million to US$ 20.9 million in 2002 dollars and the studies were published between 1976 and 1991. The estimates are fitted to a Weibull distribution that is often used in probabilistic assessments of uncertainty in EPA benefit calculations [69].

To conclude, for the derivation of a social price for safety to be employed in the evaluation of public disaster risk reduction projects that have an impact on a potentially large number of individuals, it might not be appropriate to rely on VSLs that have been determined in differing contexts and populations. It is generally not admissible to apply a value that has been obtained by artificially isolating one particular market and assessing the WTPs of a subset of individuals for a particular type of risk only. Rather it is necessary for public policy analysis to explore the influence of safety in the whole economy, taking into account its effects on both consumer and producer behavior on a great diversity of markets simultaneously. The theoretical model for this approach is developed in the remainder of this chapter, where safety pricing rules in a general equilibrium setting are derived. This methodology then in turn serves as the microeconomic foundation of the LQI that is introduced in Chapter 5.

4.2 Modeling Safety in a Market Economy

After the general problem of deriving a social price for safety has been introduced and the diverse practices to estimate the WTP and VSL as well as their price ranges have been documented, this section is designed to analyze the influence of safety in a market economy. Based on the economic theory to be developed in the sequel, safety pricing rules in general equilibrium are derived that are

[5]US EPA = United States Environmental Protection Agency.

appropriate for the analysis of public risk reduction projects potentially affecting a large number of individuals. In order to do this comprehensively, the entire problem of deciding upon a public risk reduction project is firstly converted to a pure safety problem that concentrates on safety related aspects only.

4.2.1 The Pure Safety Problem

To facilitate the subsequent discussion the problem of evaluating public risk reduction projects is transformed into a second problem, that allows to focus on safety related aspects of project appraisal only. It is assumed at this point that the benefits of all consequence categories, except for the saved lives, have already been priced by the application of some other procedure and are on hand. Subtracting the monetized benefits of all other consequence categories from the total cost of the investment yields

$$NCHS = C^T - B^{econ}[\$] - B^{env}[\$] - B^{CSH}[\$] \tag{4.2}$$

which is being referred to as the net cost into human safety (NCHS) . Accordingly, the NCHS describes the residual of the project's total cost that is attributed to enhance human safety. By means of the latter, the question about project efficiency is convertable to a pure safety problem, focusing only on cost and benefits that are related to human safety. Efficiency in safety is granted accordingly, if the benefits that society derives from the enhanced human safety standards, measured by the SWTP, are higher than the NCHS, leading to the following efficiency criterion for the pure safety problem:

$$SWTP \geq NCHS \tag{4.3}$$

By construction of the pure safety problem it is easily verified that the fulfillment of this condition implies not only efficiency in safety but also total project efficiency. In the remainder of this chapter and of Chapter 5 it is therefore focused only on the pure safety problem of a public risk reduction project. All changes in individual welfare levels that are being evaluated in the following are directly or indirectly induced by changes in the publicly provided safety level. Other changes that result from the implementation of the project, such as price changes on markets where material and labor are used by the project, need not be considered in the pure safety problem.

Furthermore, it is assumed that the NCHS are on hand, so that issues related to project financing are not being accounted for. This is justifiable by pointing out that questions related to project financing make it necessary to specify the risk reduction project under consideration and can hardly be treated generally. An update of building codes for example carries over the cost directly to the citizens, whereas a construction of a flood protection is largely financed by the government out of its fixed budget or by raising taxes. Safer public infrastructure elements in contrast might be financed through user fees. An elaborate discussion of issues related to project financing is included in [32]. For this reason it is exclusively focused on pricing the benefits of changes in the publicly provided safety level by means of the WTP concept.

4.2.2 A Simple Economy's Workflow

This section briefly explores the behavior of consumers and producers within a simplified economy model and serves as a basis for an understanding of how safety impacts the economy. As visualized in

Figure 4.1 the simplified economy consists of two classes of economic agents, namely the consumers and the producers that interact on two distinct types of markets, the product markets and the factor markets.

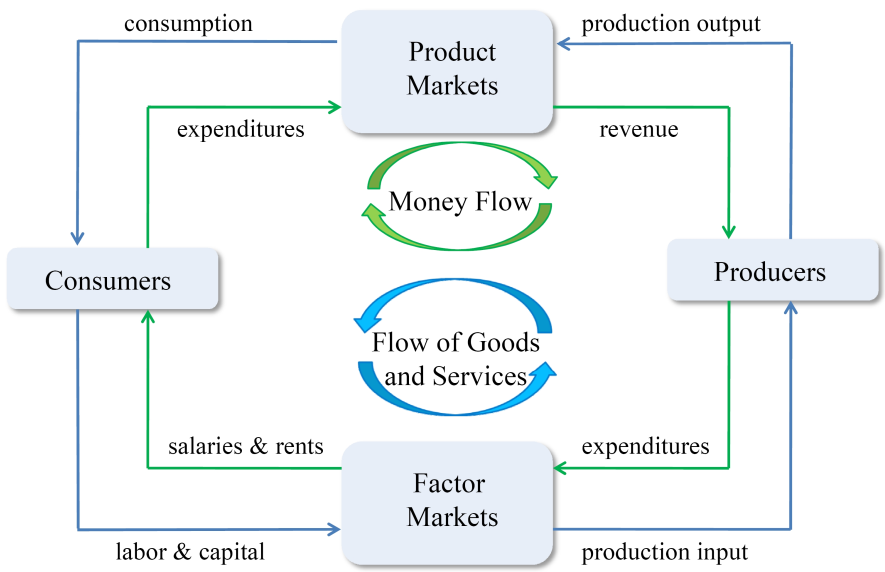


Figure 4.1: A simplified economy model

The fundamental goal of the consumers is to maximize their utility. They derive utility from consuming goods and services that are obtained on the product markets. But their capacities to buy these goods and services are limited by the amount of income they gain. The income is earned by selling parts of their endowment in terms of labor and capital on the factor markets. The provision of labor has a negative impact on consumer utility, i.e. it brings disutility to her as labor is a bad. This is because most work is dull, troublesome and sometimes dangerous [213].

The producers buy labor and capital on the factor markets for the production processes of goods and services. Producers follow the aim of profit maximization. They supply their products on the product markets and receive a revenue from the consumers' purchases. If the expenditures for labor wages and rents on capital that the producers have to expend for demanding the factors of production from the consumers is subtracted from this revenue, the profits are obtained. The relation between the outputs producers are able to produce from a certain amount of input factors is captured by the production function, which therefore limits their profit maximizing ambitions by indicating what is technically feasible.

Thus, there are essentially two flows observable in this simplified economy. The first one constitutes the stream of goods and services, whereas the second one is the money flow. They work in opposite directions. Furthermore, it is clear that the consumers derive their utility from the flow of goods and services, whereas their consumption ability is limited by the money flow. The opposite holds for producers. Their objective to maximize profits is defined over the money flow, while their ability to produce is limited by their production function that is defined over the flow of goods and services.

The link between consumers and producers is given by consumer income. In a private ownership economy firms are ultimately owned by consumers. Thus, the firms' profits are transmitted to the consumers via shares and can therefore be included as part of consumer income. This essential step ensures the fulfillment of Walras' Law[6] and hence provides for the existence of a general equilibrium [243]: under perfect competition, where all economic agents are price takers[7], the forces of supply and demand will result in a set of prices so that all markets clear. This set of prices is being referred to as general equilibrium and balances supply and demand on each market, so that the utility maximizing strategies of the consumers are perfectly accordable with the profit maximization strategies of the producers. Thus, at the general equilibrium solution social welfare is maximized.

From this brief introduction in an economy's work flow it follows that there are basically two types of markets at disposition where safety could principally be priced. The product market and the factor market. Obviously, a direct market for safety does not exist. In the absence of clearly defined markets, the value of safety can be derived from information acquired through surrogate markets: there are goods sold on the product markets that have a direct impact on safety and whose prices therefore contain a value of safety implicitly. Also, a price for safety is included in wages that are paid on labor markets for jobs with different risk exposures, as already discussed above.

4.2.3 Characterizing Safety as a Public Good

Safety has a considerable influence on our daily decisions and our daily decisions influence the safety level we are exposed to. Many actions like buying a smoke detector, selecting a particular job, tightening regulations on air quality or building codes may be viewed as activities that determine our personal safety. Most of these important decisions are made by individuals on their own behalf. However, society and in particular the government as the authority to represent the public interest undertakes a variety of decisions on behalf of all of us [236]. These include allocations of public resources into traffic safety, flood control, natural disaster mitigation programs, public health and the like and designate the circumstances we are living in and provide the framework under which we make our decisions. As a consequence, the safety level that is enjoyed by each individual of society is the result of a twofold process: there is a basic infrastructure that applies to all and there is some superstructure that allows for individual variability [51].

Within the latter category the market perspective is of particular importance. In the market perspective the value of safety is a residual that results from a series of other decisions. Each individual evaluates the value of safety herself, determines her WTP for protective, safety increasing products or services (such as smoke detectors, seat belts etc.) and tries to find that service at that price and in the desired amount to increase her own personal safety. In short, she has control over the situation as far as her financial capabilities admit [91]. The market in turn provides these products or services as long as it is profitable to do so. The final equilibrium price is then a reflection of all this wealth of information on individual preferences and contains a price for safety implicitly. The initially introduced labor and consumer market studies to determine a VSL are approaches to price safety increments of this category.

[6] The Walras' Law [261] states that if each individual satisfies her budget constraint, so that the value of goods sold equals the value of goods bought, then the total value of all sales by all individuals equals the total value of all purchases by all individuals. This holds at any given prices.

[7] i.e. neither single consumers nor producers are influential enough to affect prices.

The basic safety level in contrast, is externally provided to the individual and the evaluation whether the prevailing safety level is subjectively worthwhile is outside her control. It is imposed to her and determines the circumstances under which she makes her decisions. Here, it is the government's task to provide a basic safety level to protect people and property regardless of their personal financial background. Any change in the publicly provided safety level, which is induced for instance by the construction of safer roads, the provision of flood defenses or an improvement of building codes, indiscriminately makes life safer for all of us and thus possesses to some extent the non-rivalry and non-excludability property of public goods, introduced in Section 3.4. Jones Lee goes even a step further in stating: "the vast majority of safety improvements are non-marketed public goods" [133].

The publicly provided safety that is affected by the public risk reduction project under investigation is therefore considered as a public good henceforth, in line with [91, 121, 133]. It is clear however, that the project induced safety change will not affect all individuals in exactly the same way and to equal amounts. Some will benefit from the project to larger extents, while others in turn might be affected only marginally because their financial background already allowed for sufficient private precaution prior to the intervention. Nevertheless, the introduction of a uniform safety variable s to represent the public good safety that is equal for all individuals of society is justifiable at least in good approximation.

The recognition that the basic safety level constitutes a public good provided by the government, potentially affecting all individuals raises the question how much society as a whole should spend on safety provision. There is a solution that provides an answer to this question in a simple and straightforward way, guaranteeing the Pareto optimal provision of public goods, which is known as the Samuelson condition. Essentially, it states that the public good should be provided in such amounts that the aggregate WTP is equal to the marginal cost of providing the good. But as in the present context of project appraisal it is not focused on the provision of an overall optimal safety level to society but rather on the question if the single safety change induced by the project is socially efficient, the Samuelson condition is not further elaborated in this work. For a detailed discussion on the latter it is referred to [130].

Measuring Safety

The first step in the mathematical derivation of a SWTP for public safety is to specify a unit in which safety is measured [35]. This is done in the sequel by means of the society's crude mortality rate

$$\mu := \frac{M}{N} \tag{4.4}$$

which equals the number of deaths M per year in relation to total population size N. In Figure 4.2 the development of crude mortality rates over time is depicted for selected countries. For developed economies it is observable that the crude mortality rate clusters at around 0.01, whereas in developing countries the rates are significantly higher due to differences in the population structure as well as lower safety standards. Accordingly, safety s can be defined as the annual survival probability:

$$s := 1 - \mu \tag{4.5}$$

If multiplied by one year, the annual survival probability allows for an interpretation in terms of annual survival time, measured in years, that is on average enjoyed by each individual of society. It

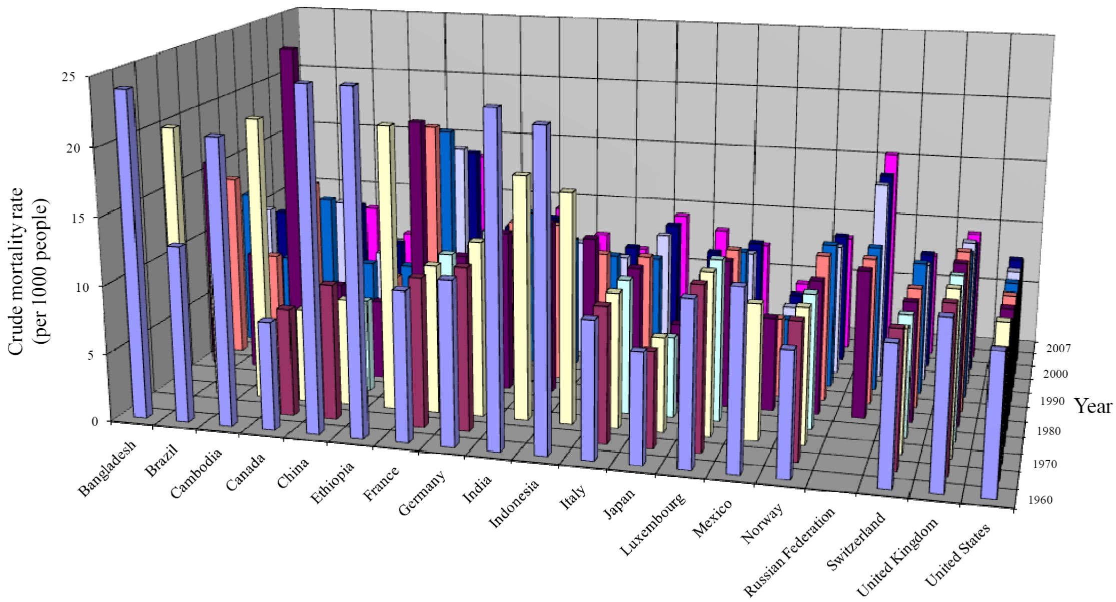


Figure 4.2: Crude mortality rates for selected countries [data source: WDI [262]]

is clear that the so defined safety level is a result of the twofold safety structure and reflects both private and public precaution. If a public risk reduction project now saves m lives annually, the change in crude mortality in the move from status quo condition μ_0 to post project condition μ_1 amounts to

$$\Delta\mu = \mu_1 - \mu_0 = \frac{M-m}{N} - \frac{M}{N} = -\frac{m}{N} \tag{4.6}$$

which by definition yields a safety improvement of:

$$\Delta s = \frac{m}{N} \tag{4.7}$$

As a consequence, the prevented fatalities due to the implementation of the public risk reduction project convert in a reduction in mortality risk, which in turn leads to an increase in safety, that is statistically enjoyed by any member of society. Accordingly, the benefits take the form of lives extended rather than lives saved [41]. Therefore, the ethically questionable issue of pricing human life is transformed to the problem of evaluating reductions in mortality risk or changes in the public good safety. For later convenience the concept of $\Delta\mu$-units is introduced at this point:

$$\Delta\mu - \text{unit} := \frac{1}{N} \tag{4.8}$$

It is easily verified, that a reduction in mortality risk or an increase in safety equal to one $\Delta\mu$-unit corresponds to saving exactly one statistical life in society. Accordingly, equation (4.1) indicates that the SWTP for one $\Delta\mu$-unit coincides with the VSL.

Size of safety changes

Before it is started with the derivation of WTP estimations for safety changes that are induced by the public risk reduction project it is necessary to outline the size of safety changes that are under consideration. As illustrated above, overall crude mortality in industrialized countries roughly amounts to 0.01. Only three deaths in 10,000 however are not owing to natural causes. If deaths due to voluntary risk activities such as sports, smoking and traffic accidents as well as unavoidable accidents such as house accidents are subtracted from this number, a reduction in crude mortality of around $2 \cdot 10^{-4}$ or less constitutes the focus of study. It may be a little higher as certain risks typical for industrialized countries, like air pollution, are not separated out in usual statistical records [217]. To draw conclusion, only marginal changes in the publicly provided safety level are attainable by the implementation of a public risk reduction project and therefore only marginal safety changes are to be accounted for in the pure safety problem.

4.2.4 Consumer Side - Utility Maximization

This section explores the consumer's utility maximization strategy in the presence of the public good safety in mathematical detail. The consumer is assumed to consume n different private goods x_i, where $i = 1, ..., n$. These are purchased on the product markets in non-negative quantities at exogenously given fixed positive prices p_i. Moreover, the consumer supplies k different kinds of labor l_j, where $j = 1, ..., k$ and obtains an exogenously given fixed wage rate w_j for each particular labor unit supplied on the labor market. Furthermore, the publicly provided safety level s is exogenously given and changes can only be imposed by governmental safety projects. The consumer's preferences are captured by an ordinal utility function

$$U = U(x, l, s) \tag{4.9}$$

where x is the n-dimensional vector of consumption goods, l is the vector of labor supply of order k and safety s enters the utility function as a separate argument which is outside the consumer's influence. If not stated otherwise, the vector notation is maintained in the following. The utility function U is assumed to be continuous, twice continuously differentiable and well-behaved in the sense of Section 3.2.2. Thus, the utility function possesses the following properties with respect to the first partial derivatives:

$$\frac{\partial U(x, l, s)}{\partial x} > 0, \qquad \frac{\partial U(x, l, s)}{\partial l} < 0, \qquad \frac{\partial U(x, l, s)}{\partial s} > 0 \tag{4.10}$$

In this sense, it is hypothesized that the consumer always prefers a higher level of consumption and of safety to a lower one respectively, so that the utility function is increasing in each of these two arguments. As labor constitutes a bad that brings disutility to the consumer, she always prefers less work to more, other things equal, so that the utility function is negatively sloped in the labor argument.[8] Furthermore, the utility function is assumed to exhibit diminishing marginal rates of substitution between safety and consumption: if an individual's safety level is on a quite low standard, she would be willing to trade off more units of consumption to obtain an additional unit of safety than if the individual's safety level was already on a comparatively advanced level. Empirical evidence seems to confirm this hypothesis [4], which leads to convex indifference curves in safety consumption space.

[8] As discussed below sometimes it is more convenient to work with the complement of labor time, i.e. leisure time, in the consumer utility function as leisure constitutes a good that is consumed.

Utility maximization problem: If the economy consists of C distinct consumers, the problem of utility maximization for consumer c ($c = 1, ..., C$) is formulated as follows:

$$\begin{aligned} &\max_{x^c, l^c} \quad U^c(x^c, l^c, s) \\ &\text{s.t.} \quad y^c + wl^c = px^c \qquad\qquad \forall c = 1, ..., C \end{aligned} \tag{4.11}$$

Here, the variable $y^c = k^c + \pi^c - \tau^c$ represents the consumer's non labor income, that is treated as fixed in the following as the focus is on the short term perspective. This income y^c is composed of capital income k^c and profit income π^c, received by the consumer from holding firm shares. Furthermore, the income y^c may be interpreted as after tax τ^c income as the influence of the government is not explicitly accounted for in the model. Since safety is modeled as a public good, it is consumed in equal amounts by all consumers implying that $s^c = s$ for all c.

The direct utility function of consumer c is therefore augmented by the argument s, while her budget constraint remains unchanged. This is justifiable by the assumption, that any cost for the provision of the public good are covered by tax payments that have already been collected, so that the government finances the project out of its fixed budget. Normally, it would be necessary to add an additional time constraint to the consumer's utility maximization problem, requiring that work time plus leisure time sum up to total time available to the consumer. This constraint is suppressed at this point however, in order to keep things simple.

As the utility maximization problem stated in (4.11) constitutes a constrained optimization problem it may conveniently be solved by Lagrange multipliers. The Lagrange function for the above stated constrained maximization problem is written as:

$$\Lambda^c = U^c(x^c, l^c, s) - \lambda^c(px^c - wl^c - y^c) \tag{4.12}$$

Then, the first order conditions for an interior solution to the utility maximization problem (4.11) are easily calculated:

$$\frac{\partial \Lambda^c}{\partial x^c} = \frac{\partial U(x^c, l^c, s)}{\partial x^c} - \lambda p = 0 \tag{4.13}$$

$$\frac{\partial \Lambda^c}{\partial l^c} = \frac{\partial U(x^c, l^c, s)}{\partial l^c} + \lambda w = 0 \tag{4.14}$$

$$\frac{\partial \Lambda^c}{\partial \lambda^c} = y^c + wl^c - px^c = 0 \tag{4.15}$$

Dividing equation (4.14) by equation (4.13) and rearranging terms leads to

$$\frac{\frac{\partial U(x^c, l^c, s)}{\partial l^c}}{\frac{\partial U^c(x^c, l^c, s))}{\partial x^c}} = -\frac{w}{p} \tag{4.16}$$

showing that each consumer maximizes her utility at the point where the marginal rate of substitution (MRS) of labor for consumption equals the negative of their ratio of prices. As all consumers in the economy face the same set of prices it follows straightforwardly, that when all consumers follow the strategy of utility maximization, the MRS must be equal across all consumers in the economy [237]. Thus, the Pareto optimality condition in an exchange economy is fulfilled, as outlined in Chapter 3.

Now, if the first order conditions for utility maximization (4.13),(4.14) and (4.15) are solved for the three unknown variables x^c, l^c and λ^c as functions of prices p, wages w, non labor income y^c and safety level s, the Marshallian[9] demand functions for goods, the Marshallian supply functions for labor and the marginal utility of income function are obtained:

$$x^c = x^c(p, w, y^c, s) \tag{4.17}$$
$$l^c = l^c(p, w, y^c, s) \tag{4.18}$$
$$\lambda^c = \lambda^c(p, w, y^c, s) \tag{4.19}$$

The demand functions for consumption goods and the supply functions of labor indicate how much the consumer demands on the product markets and how much labor she supplies on the labor market in order to maximize her utility under the given exogenous variables p,w,y^c and safety level s. Thus, it becomes obvious that in particular the publicly provided safety level has an influence on all consumer decisions in the ambition to maximize her utility.

The demand and supply functions can easily be used to derive the Marshallian demand and supply curves for consumption and labor, respectively. The Marshallian demand and supply curves are directly observable on the markets and display the amount of goods consumed and labor supplied at varying prices p and wages w, keeping all other exogenous arguments of the demand and supply functions constant. They represent a useful tool in policy analysis in particular to investigate the impact of price changes. Along the Marshallian demand and supply curves income is kept constant, whereas the utility level is allowed to vary. According to the law of demand, consumers consume more at lower prices and the demand curve is downward sloping. The contrary holds for the labor supply curve.

It has to be emphasized at this point that there is a difference between a change in demand/supply and a change in quantity demanded/supplied. A change in quantity demanded/supplied is caused only by a price change and constitutes a movement along the demand/supply curve. A change in demand/supply in contrast, occurs when more or less is demanded/supplied at any price of the good and is mainly caused by a change in the exogenous variables excluding the own price or alternatively, a change in preferences, i.e. the utility function. Therefore, as safety s constitutes a public good that enters the demand/supply functions as an exogenous variable, a change in safety will result in demand/supply curve shifts on several markets at once.

Although safety is not explicitly included in the budget constraint, it affects the demand for goods and labor supply through the marginal utilities for such goods, which in turn causes the consumer to adjust her economic behavior. The direction of change is ambiguous in general and depends on the way the marginal utilities change when the safety level is elevated. If $\frac{\partial^2 U}{\partial x_i \partial s} > 0$ is fulfilled, the consumer will expand her consumption of good x_i to maximize her utility. This in turn causes her demand curve for x_i to shift outwards. An example for such a situation in context of disaster risk might be the increased demand for houses close to the sea, where safety has been increased by the construction of a nearby flood protection. If $\frac{\partial^2 U}{\partial x_i \partial s} < 0$ holds in contrast, the consumer will consume less of the good and the demand curve will shift in the opposite direction. The reduced demand for water isolation devices in the above example might serve as an illustration. Finally, if $\frac{\partial^2 U}{\partial x_i \partial s} = 0$ is

[9] Named after the famous economist Alfred Marshall [160]

given, safety has no influence and the consumer will not modify her consumption pattern for good i. The direction of shift in response to a safety change is easily verified by means of the optimality conditions of the utility maximization problem (4.11). For labor supply it is argued analogously, keeping in mind that labor is a bad though.

The marginal utility of income λ^c on the other hand indicates, how much additional utility the consumer derives from a marginal increase in income under the given circumstances. Thus, it is to be interpreted as the shadow price of income [128]. As shown below, the marginal utility of income function represents a useful tool to convert marginal changes in utility to monetary value and vice versa. Reinserting the demand and supply functions (4.17) and (4.18) in the utility function U^c yields the indirect utility function V^c:

$$\begin{aligned} V^c(p, w, y^c, s) &:= \max_{x^c, l^c} \{U^c(x^c, l^c, s) : y^c + wl^c = px^c\} \\ &:= U^c(x^c(p, w, y^c, s), l^c(p, w, y^c, s), s) \end{aligned} \tag{4.20}$$

The indirect utility function V^c displays the maximum utility level that is attainable for the consumer as a function of the exogenously given variables p, w, y^c and safety level s. It is a convenient tool to analyze how the consumer's optimal utility level will change when the exogenous variables are modified for instance due to the implementation of a public project. Without going deeper in the mathematical properties of the indirect utility function it is important to emphasize, that it is increasing in wages, income and safety level, while it is decreasing in commodity prices.

4.2.5 Producer Side - Profit Maximization

On the complementary side of the market, the producer's or firm's primary objective is to maximize profits. If it is assumed that there are $f = 1, ..., F$ producers on the market and labor L and capital K are required in the production of the consumption goods x, at wages w and rents r respectively, and F represents the production function, each firm's constrained profit maximization problem can be stated as follows:

$$\begin{aligned} &\max_{x^f, L^f, K^f} \pi^f = px^f - wL^f - rK^f \\ &\text{s.t.} \quad x^f = F^f(L^f, K^f, s) \qquad\qquad \forall f = 1, ..., F \end{aligned} \tag{4.21}$$

The profit π of the firm f is given by the revenues obtained on the product markets subtracted by the variable cost of labor and the fixed cost of capital expended on the factor markets to carry out the production. The firm's profit maximizing ambition is limited by the production function F^f, that indicates how much goods x can maximally be produced out of a certain amount of labor L and capital K at a given and fixed publicly provided safety level s. Clearly, the production function F^f is a vector of production functions, that contains n different sub production functions F^{fi} as its arguments to produce each of the n distinct commodities x_i. For notational simplicity it is furthermore assumed, that each producer f produces all n commodities that are consumed by the consumer. The safety level enters the production function because it has an impact both on the quality of the hired labor force and on the production process itself that has to be carried out in compliance with safety regulations.

Furthermore, it has to be noted that the capital K represents all capital goods including factories, machinery, tools, equipment and various buildings which are used in the production process and are therefore not variable in the short term optimization. The labor component on the other hand, is the accumulated labor time of all employees that is needed in the production process and is adjustable also short term. To solve the constrained profit maximization problem the production function F^f is substituted in the profit function π^f :

$$\max_{L^f,K^f} \pi^f = pF^f(L^f, K^f, s) - wL^f - rK^f \tag{4.22}$$

The first order conditions for this transformed unconstrained profit maximization problem are easily obtained:

$$\frac{\partial \pi^f}{\partial L^f} = p\frac{\partial F^f(L^f,K^f,s)}{\partial L^f} - w = 0 \tag{4.23}$$

$$\frac{\partial \pi^f}{\partial K^f} = p\frac{\partial F^f(L^f,K^f,s)}{\partial K^f} - r = 0 \tag{4.24}$$

From the first order conditions (4.23) and (4.24) it is easily seen, that the firm maximizes its profits at the point, where the marginal product of labor (MPL) $\frac{\partial F^f(L^f,K^f,s)}{\partial L^f}$ and the marginal product of capital (MPK) $\frac{\partial F^f(L^f,K^f,s)}{\partial K^f}$ equal the real wage and the real interest rate, respectively. Dividing equation (4.23) by (4.24) and rearranging terms shows that the producer maximizes profits at this combination of labor and capital, where the marginal rate of technical substitution (MRTS) between the two factors of production equals their ratio of prices:

$$\frac{\frac{\partial F^f(L^f,K^f,s)}{\partial L^f}}{\frac{\partial F^f(L^f,K^f,s)}{\partial K^f}} = \frac{w}{r} \tag{4.25}$$

As the prices for labor and capital are exogenously given it follows, that the MRTS must be equal across all producers in the economy, yielding the Pareto optimality condition in production as outlined in Chapter 3. Solving these first order conditions for L^f and K^f respectively, yields the two factor demand functions

$$\begin{aligned} L^f &= L^f(p, w, r, s) \\ K^f &= K^f(p, w, r, s) \end{aligned} \tag{4.26}$$

which capture the amount of labor and capital the producer demands on the factor markets to maximize her profits under given prices p, wages w, rents r and safety level s. If these factor demand functions are substituted in the production function F^f, the supply function for consumption goods is obtained:

$$x^f = x^f(p, w, r, s) = F^f(L^f(p, w, r, s), K^f(p, w, r, s), s) \tag{4.27}$$

The producer's supply function discloses the amount of commodities the producer offers on the product market in order to maximize her profits as a function of exogenously given prices p, wages w, rents r and safety level s. Thus, also on the producer side in particular the publicly provided safety level has an impact on all producer decisions. In analogy to the discussion on the consumer side, the factor demand functions and the product supply function can be taken into account to construct the factor demand and product supply curves, that display the firm's optimal quantities demanded and supplied in dependency of their prices. Also here a change in the exogenous variables other than the own price causes the curves to shift. In particular, this holds for the publicly provided safety level s. Again, the direction of the shift is implicit in the demand and supply functions and may be analyzed by means of the first order conditions to the profit maximization problem.

Substituting the labor and capital demand functions into the objective function gives the profit function

$$\Pi^f = pF^f(L^f(p,w,r,s), K^f(p,w,r,s), s) - wL^f(p,w,r,s) - rK^f(p,w,r,s) \tag{4.28}$$

which constitutes a value function, indicating the firm's maximum attainable profits under the given exogenous variables. Thus, in analogy to the consumer's indirect utility function, the firm's profit function displays the producer's optimal outcome under the circumstances the firm operates. The profit function is increasing in commodity prices p and safety level s and decreasing in factor prices w and r.

4.2.6 Single Market Equilibria and General Equilibrium

It was demonstrated in the previous sections how consumers and producers behave ideally in a market economy to maximize their welfare. Each of the two economic agents acted under exogenously given prices that were outside their influence in a competitive market setting. Now, in the market perspective it is the job of the price mechanism to coordinate the forces of supply and demand so that the independent and decentralized utility maximizing strategies of the consumers and profit maximizing ambitions of the producers become accordable [243]. It is important to outline that even if the market price for a good is independent on the single agents' actions, the final market price is the result of the aggregate behavior of all economic agents collectively [256].

The aggregate supply and demand on a market level for a given price is obtained by summing up the quantities supplied and demanded over all consumers and producers respectively, taking the supply and demand curves into account. In order to facilitate the presentation, in the following the influence of capital on the product supplies and factor (labor) demands is suppressed and assumed to be constant over the considered period. This is common practice in economics, especially if it is focused on the short term perspective where firms are unable to modify their capital structures [129].

Equilibrium on Product Markets Under this assumption, firstly the aggregate supply curve is constructed by focusing on an arbitrary product market i, where commodity x_i is traded. Adding up the above derived Marshallian supply curves over all firms operating in the economy yields

$$S_i(p_i) = \sum_{f=1}^{F} x_i^f(p_i, p^-, w, s) \tag{4.29}$$

where $S_i(p_i)$ denotes aggregate supply on product market i at product price p_i and p^- represents the price vector of all remaining products excluding i. For the aggregate demand on product market i it is proceeded analogously, by summing the demand curves over all consumers in the economy

$$D_i(p_i) = \sum_{c=1}^{C} x_i^c(p_i, p^-, w, y^c, s) \tag{4.30}$$

where $D_i(p_i)$ represents aggregate demand on product market i at price p_i. This notation in hand, the excess demand function on market i is determined by the following equation:

$$ED_i(p_i) = D_i(p_i) - S_i(p_i) \tag{4.31}$$

The ith product market now is in equilibrium at a price p_i^* and quantity x_i^* where the aggregate supply and demand curves intersect and supply equals demand, as shown in Figure 4.3. Here, the excess demand function (4.31) is equal to zero and the market clears. At equilibrium price and quantity the utility maximization strategies of the consumers and the profit maximization strategies of the producers become perfectly accordable and the consumers' marginal WTP equals the producers' marginal cost for the product. Therefore, the allocative efficiency condition of Pareto optimality is met and social welfare is maximized. This is the essence of the first welfare economic theorem, presented in Section 3.3.3. Markets converge to equilibrium prices over time unless particular exogenous events occur in the economy [205] and the interaction of supply and demand determines price and quantity of the good that will clear the market [176].

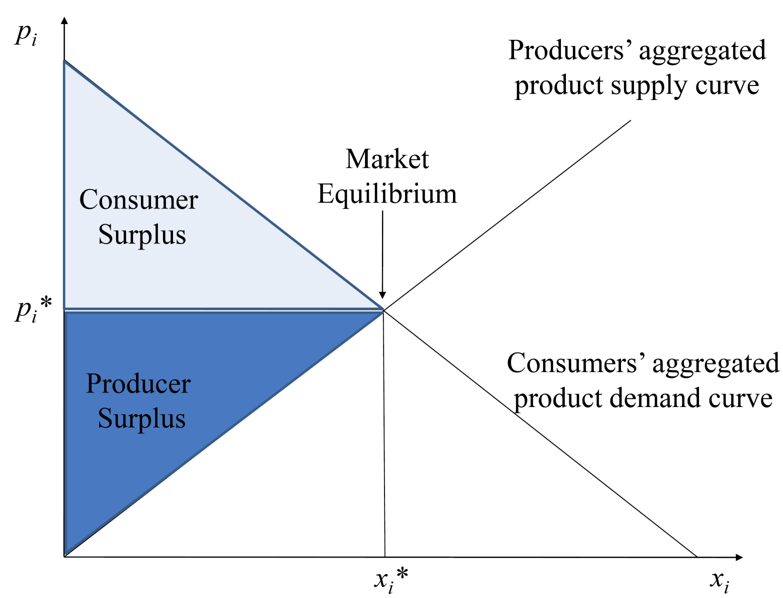


Figure 4.3: Product market equilibrium

The total benefits that all consumers collectively derive from the provision of product x_i on the market is given by the area below the aggregated demand curve and above the price line. This area corresponds to the aggregate consumer surplus and represents the amount the consumers are willing to pay less the price they actually pay. The producer surplus in contrast, equals the region below the price line and above the aggregated supply curve. It represents the amount that producers actually earn from selling the product at the market price less their marginal cost of production. It is to be interpreted as the total benefits the producers derive from supplying product x_i on the market. The overall social benefit from market i is given by the sum of consumer and producer surplus.

It has to be emphasized at this point, that the equilibrium price and quantity on product market i have been obtained for a given publicly provided safety level s, that determined the circumstances under which consumers and producers made their decisions of demanding and supplying the good. It has been outlined in the description of supply and demand curves, that a change in the public safety level s due to the implementation of a risk reduction project causes the individual supply and demand curves to shift, as both producers and consumers respond by a modification of their economic behavior.

Hence, as the equilibrium constitutes the intersection between aggregate supply and demand curves, it will no longer be stable after a safety improvement and will move to a new equilibrium price and quantity. The move from status quo to the new equilibrium goes in line with changes in both consumer and producer surpluses, which are solely attributed to the safety increase. Converting these changes in monetary units yields a price for safety, ceteris paribus. Such an approach to derive a price for safety is called partial equilibrium analysis. In the relevant context of disaster safety, such partial analyses are being carried out for instance in the attempt to value safety on basis of changing real estate prices that are differently exposed to disaster risk and thus provide the theoretical basis for consumer market studies in the VSL assessment.

Many public projects however, can reasonably be expected to impact a large number of markets both directly where the policy is applied, and indirectly through spillover and feedback effects on those and other markets [76]. Therefore, the safety prices obtained from partial equilibrium analysis might not be adequate and strongly biased when relying on the ceteris paribus assumption. A strength of general equilibrium models in contrast, is their ability to account consistently for the linkages between all sectors of the economy, as illustrated further below.

Equilibrium on Labor Markets In principle, the above described methodology to define price and quantity that equilibrates supply and demand holds also on the labor markets. One important difference however is that the role of consumers and producers is reversed in the sense that consumers are suppliers of labor, whereas producers demand labor on the markets. Accordingly, by focusing on an arbitrary labor market j, the aggregated labor supply curve is given by

$$S_j(w_j) = \sum_{c=1}^{C} l_j^c(p, w_j, w^-, s) \tag{4.32}$$

i.e. the consumers' labor supply curves are summed over all consumers. As before, w_j denotes the wage rate on the considered labor market j, whereas w^- constitutes the $k-1$ dimensional vector of wages on all other labor markets except for j. For the aggregated labor demand curve it is obtained

$$D_j(w_j) = \sum_{f=1}^{F} L_j^f(p, w_j, w^-, s) \tag{4.33}$$

i.e. it represents the sum of all firms' labor demands. The excess demand function on labor market j is represented by:

$$ED_j(w_j) = D_j(w_j) - S_j(w_j) \tag{4.34}$$

The labor market is in equilibrium where the excess demand function (4.34) is equal to zero. As the above derived results and properties concerning equilibrium price and quantity hold also for the labor market, no further explanation is needed.

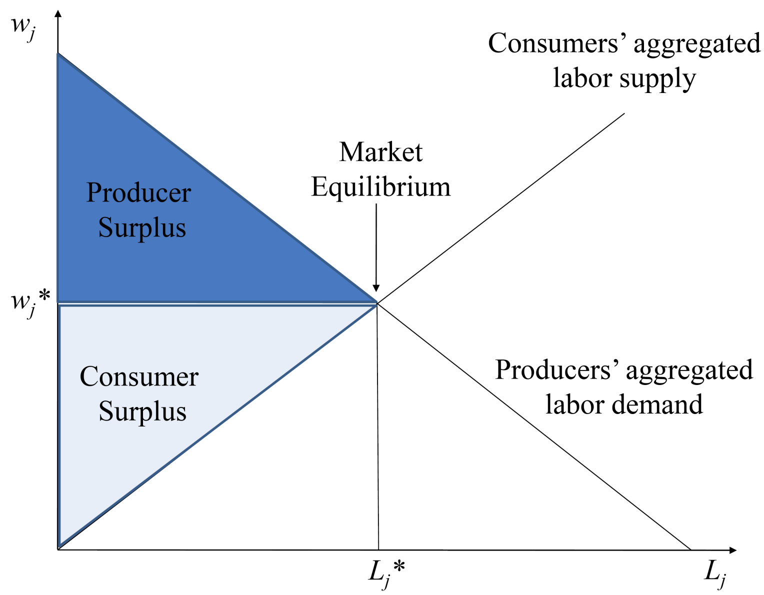


Figure 4.4: Labor market equilibrium

In Figure 4.4 the labor market equilibrium is visualized graphically. The crucial difference to the product market here is that aggregate consumer surplus on labor market j is given by the area above the aggregate labor supply curve and below the wage rate, whereas the aggregate producer surplus is represented by the area below the aggregated labor demand curve and above the wage rate. Total social welfare that results from labor market j is the sum of consumer and producer welfare.

Also with respect to the equilibrium on labor market j it has to be emphasized that the optimal wage w_j^* and quantity L_j^* are dependent on the publicly provided safety level s in general. In line with the above discussion, the equilibrium will shift in response to a publicly imposed safety change so that a new equilibrium solution is reached. Therefore, also here a partial equilibrium approach for pricing safety may be performed, by relying on the ceteris paribus clause. This constitutes the theoretical basis of labor market studies in VSL assessments. The above stated limitations must be kept in mind though.

General Equilibrium In a modern economy single markets are often closely interrelated, making it necessary to analyze welfare changes beyond the other things being equal assumption and look at multiple markets simultaneously. Interrelations between markets are essential to forecasting and understanding economic activity. This holds because there are distinctive interactions across markets and therefore it is important that the equilibrium concept includes simultaneous determinations of equilibrium prices across markets [243]. This is of particular importance in the analysis of public goods as they are not traded on particular markets and can generally be expected to have an impact on diverse markets simultaneously. The general equilibrium consideration is able to present a solution strategy for the economy as a whole, taking into account all market interdependencies.

Formally, a general equilibrium in the short term is defined as a set of allocations for the consumers $\{x^c, l^c\}_{c=1}^{C}$, a set of allocations for the producers $\left\{x^f, L^f\right\}_{f=1}^{F}$ and a set of prices (p, w), such that all consumers and firms maximize their utility and profits, respectively and all markets in the economy are simultaneously in equilibrium [2, 146]. In line with the above notation, the general equilibrium in the considered economy can be characterized by the following $n+k$ equations:

$$\begin{aligned} ED_i(p_i) &= \sum_{c=1}^{C} x_i^c(p_i, p^-, w, y^c, s) - \sum_{f=1}^{F} x_i^f(p_i, p^-, w, s) = 0, \qquad \forall i = 1, ..., n \quad (4.35) \\ ED_j(w_j) &= \sum_{f=1}^{F} L_j^f(p, w_j, w^-, s) - \sum_{c=1}^{C} l_j^c(p, w_j, w^-, y^c, s) = 0, \qquad \forall j = 1, ..., k \quad (4.36) \end{aligned}$$

Thus, at the general equilibrium allocations and prices, each single market in the economy clears and has an excess demand equal to zero. At the general equilibrium it is impossible to improve the situation of one economic agent without worsening that of another and therefore social welfare is maximized in the whole economy. This is the essence of the first welfare economic theorem that was stated in Chapter 3. In order to analyze the properties of the general equilibrium the Walras' Law [261] is of particular importance:

$$\sum_{i=1}^{n} p_i(D_i(p_i) - S_i(p_i)) + \sum_{j=1}^{k} w_j(D_j(w_j) - S_j(w_j)) = 0 \qquad (4.37)$$

The Walras' Law states that at each given set of prices, which need not necessarily be equilibrium prices, the price weighted sum of excess demands over all markets of the economy is equal to zero. This relation holds whether the economy is in equilibrium or not [243]. Therefore, the Walras' Law implies that in a general equilibrium model there are $n+k$ prices to determine, but there are only $n+k-1$ linear independent equations at disposition that characterize these prices [256]. If supply equals demand on $n+k-1$ markets of the model economy and each individual's budget constraint holds with equality as a result of utility maximization, it follows from (4.37) that the $(n+k)-th$ market must be in equilibrium as well [244]. Therefore, only $n+k-1$ independent prices exist.

As a consequence, the equilibrium price vector (p, w) is not unique and the general equilibrium model cannot be taken into account to determine absolute prices. Some particular price must be kept constant, so that the corresponding good serves as a so called numeraire good. In this vein, the system is being normalized with respect to the numeraire: prices of all other goods on all other markets are expressed relative to this numeraire price, which is commonly set equal to one [256]. This approach may be clarified by recalling that all demand and supply curves on the market are homogeneous of degree zero in prices and income

$$\begin{aligned} x^c(p, w, y^c, s) &= x^c(tp, tw, ty^c, s) \\ l^c(p, w, y^c, s) &= l^c(tp, tw, ty^c, s) \\ x^f(p, w, s) &= x^f(tp, tw, s) \\ L^f(p, w, s) &= L^f(tp, tw, s) \qquad \forall t > 0 \end{aligned} \qquad (4.38)$$

i.e. multiplying all prices and income by some arbitrary positive factor $t > 0$ will not result in any adjustment in the behavior and serves equally well. Therefore, only relative prices count and all prices and income can legitimately be multiplied by the reciprocal of the numeraire price, which implies setting the latter equal to one.

On the other hand it is not necessarily true that an equilibrium price vector (p, w) exists. Certain assumptions about the firm's production technology in combination with certain requirements concerning the consumption set and consumer preferences, that can be reviewed in [14, 243], in contrast, ensure the existence of a general equilibrium. Alternatively, there might be multiple equilibria in the sense that more than one price vector results in a general equilibrium, given the initial distribution of endowments [130]. But the discussion of these rather complex issues is outside the scope of this thesis and it is henceforth assumed that there is one unique equilibrium price vector.

As in the case of single market equilibria, also the general equilibrium in the above described model has been determined for one particular safety level only and will respond to safety changes that are imposed by a public risk reduction project in moving to another general equilibrium. Therefore, the status quo general equilibrium serves as the starting condition for investigating the impacts of changes in the publicly provided safety level on the whole economy by means of the WTP concept henceforth. To justify this approach, it needs to be verified that there are generally two ways to depart from a general equilibrium without suffering a decrease in efficiency.

One way is to change the initial distribution of endowments so that after utility and profit maximization a new general equilibrium is obtained. This new equilibrium goes in line with a different distribution of welfare across individuals. Nevertheless, from a social point of view the status quo and the new equilibrium are equally efficient, they only differ in distribution. The Kaldor-Hicks compensation test and its practical realization in terms of the WTP concept is only applicable to identify increases in efficiency, while it is of limited use in the comparison of two Pareto optima. A selection for one of these two equilibria needs to be made by means of a social welfare function, as discussed in Chapter 3.

The second reason that leads to the formation of a new general equilibrium in the considered closed economy model is given by technological progress or an increase in the available resources [105]. In these cases, a new equilibrium solution is obtained that is comparable to the status quo equilibrium by means of the Kaldor-Hicks criterion. This is due to the fact that either of the two cases mentioned lead to an increase in the economy's efficiency. Therefore, the status quo equilibrium becomes inefficient under the new conditions and the welfare change in the move from status quo to the new equilibrium can be evaluated meaningfully by means of the WTP concept. It is crucial to note that welfare changes due to these reasons are the only ones for which estimated first order income effects must be included in the welfare change measure. Such effects are not generated by solely price changes taking place within the competitive economy, unless there are distortions present. Here, the benefits to consumers from a fall in price are canceled out by the cost to producers, and vice versa in the case of a price increase [105]. These considerations are further elaborated in Section 4.4.

Now, from the definition of safety in terms of annual survival probability it is easily verified, that an increase in the public safety level resulting from the implementation of a risk reduction project leads to a situation, where any individual of society has statistically more time at disposition. Accordingly, the total time endowment of society is enhanced, which in turn can be interpreted as an increase in economic resources [33, 109]. Consequently, public safety improvements are to be integrated in the second category of general equilibrium changes and the evaluation of safety increases by means of the WTP concept, departing from the general equilibrium under status quo safety level, finds good theoretical support.

4.3 Evaluating Safety Induced Welfare Changes

This section analyzes, how marginal changes in the publicly provided safety level affect both single consumer and producer welfare. In a second step, these changes are transformed into the money scale and aggregated on social level to finally arrive at a social price for safety that is employed in the social CBA. How the government finances the intervention is not explicitly considered here, but it is possible to interpret y^c as after tax income as discussed above.

4.3.1 Consumer Welfare Change Measures

A change in the publicly provided safety level has an impact on potentially all consumers of the considered economy. In order to analyze how the improved safety conditions affect consumer welfare concretely, it is assumed that the public project causes a change in the publicly provided safety level from s^0 to s^1, where superscript 0 represents the safety standard in status quo condition and 1 refers to the safety level after project implementation. Thus, the safety change

$$\Delta s = s^1 - s^0 \tag{4.39}$$

is the subject of study that affects all consumers in society uniformly. For simplicity it is hypothesized at this point, that the considered safety change leaves all prices p, wage rates w and consumer income y^c unaffected. Now, by means of the indirect utility concept

$$V^c(p, w, y^c, s) = U^c(x^c(p, w, y^c, s), l^c(p, w, y^c, s), s) \tag{4.40}$$

that yields the optimal utility level of the consumer as a function of prices p, wages w, non-labor income y^c and safety level s, the change in utility that results from the safety increase is:

$$\Delta V^c = V^c(p, w, y^c, s^1) - V^c(p, w, y^c, s^0) \tag{4.41}$$

Since the utility of the consumer is defined on an ordinal scale only, the utility change to the consumer in absolute terms is of no significance as any monotone transformation of the utility function serves equally well to describe consumer behavior. What is needed in contrast, is a measure that is invariant under these transformations and captures the utility change in monetary units. Although a great variety of different measures have been proposed in literature [104, 163, 185] attention is focused on the Hicksian measures compensating variation (CV) and equivalent variation (EV) henceforth, that have been introduced by Hicks in a series of articles [112, 113, 114].

Originally, the two measures have been developed to evaluate welfare changes that result from price changes. In this thesis the two measures are primarily applied to evaluate quantity changes in the public good safety. In the context of quantity changes the welfare measures are often being referred to as compensating and equivalent surplus and can be interpreted as an extension of the measures for price changes [103]. But as the distinction seems increasingly artificial [152, 172] the term compensating and equivalent variation is maintained in the following.

In thinking about compensating and equivalent surplus as opposed to variations, it is always useful to remember what is public and what is private. In case of market goods, prices are public and the demand for goods varies across individuals. For the considered non market good safety, the safety level is publicly provided and shared by all, while the marginal values vary among individuals. These considerations help to differentiate between compensating variations and surpluses [46].

The Compensating Variation (CV)

The CV is equal to the maximium amount of money that can be taken away from the individual after an improvement in the publicly provided safety level so that she is exactly as well off as before the change. Formally, the CV is implicitly defined by means of the following equation:

$$V^c(p, w, y^c - CV^c, s^1) = V^c(p, w, y^c, s^0) \tag{4.42}$$

Thus, the maximal achievable utility level of the individual under enhanced safety conditions and a by means of CV reduced non labor income is equal to the maximal achievable utility level before the change. As a consequence, the consumer will remain on the same indifference curve by paying the compensation payment CV and enjoying the enhanced safety level s^1. In other words, CV represents exactly the WTP for the safety improvement, introduced in Section 3.5.1.

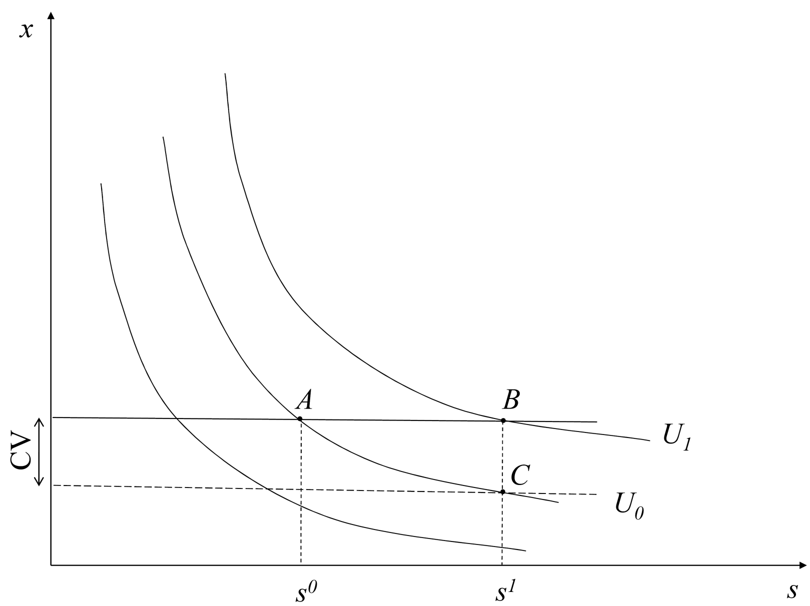


Figure 4.5: The compensating variation

This is easily retraced by means of Figure 4.5 that depicts the consumer's preferences in a single private good - public good safety space. As stated above, the increase in safety is assumed to be free of charge so that the consumer's budget line is horizontal. The status quo situation of the consumer is given by point A on an indifference curve corresponding to utility level U_0. After the improvement in the safety level due to the public project, the consumer is able to attain a point B of higher satisfaction allocated on a higher indifference curve corresponding to utility level U_1. Now, reducing the consumer's income by an amount equal to CV under the new safety conditions will bring her back to the initial indifference curve consuming the bundle C, so that she is equally well off as before the change. Similar argumentation reveals, that a reduction in the publicly provided safety level requires a monetary payment CV' received by the consumer to enable the attainment of the initial utility level U_0. In case of a reduction in the safety level the CV therefore represents the WTA.

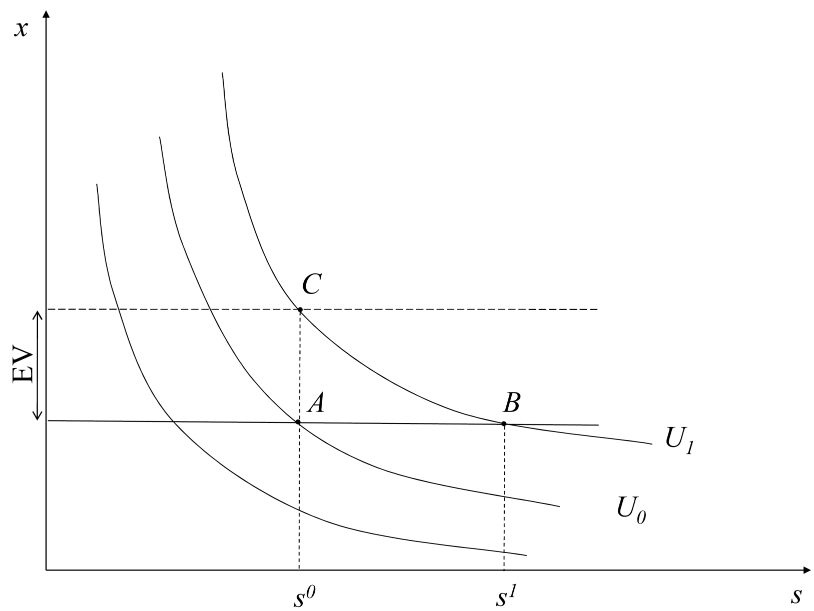


Figure 4.6: The equivalent variation

The Equivalent Variation (EV)

The EV represents the minimum amount of money that must be given to the consumer before an improvement in the publicly provided safety level to make her exactly as well off as she would have been after the change. Formally, the EV is implicitly defined by means of the following equation:

$$V^c(p, w, y^c + EV^c, s^0) = V^c(p, w, y^c, s^1) \tag{4.43}$$

Thus, the maximal achievable utility level of the consumer at status quo safety level s^0 and a by means of EV augmented income y^c is equal to the maximal achievable utility level under new safety conditions s^1. As a consequence, the consumer reaches the same higher indifference curve by receiving the EV and enjoying status quo safety level as she could have attained with the safety enhancing public project and status quo non labor income. In other words, the EV is exactly the WTA to forgo the project.

This is easily verified on basis of Figure 4.6. As before, the status quo condition is given by point A on an indifference corresponding to utility level U_0. After the implementation of the public project the consumer could reach point B on indifference curve corresponding to utility level U_1. Increasing the consumer's income by EV under the status quo safety conditions would enable her to reach point C with utility level U_1 also without the project, making her as well off as she would have been after the change. Similarly, in case of a safety decrease, the EV corresponds to the WTP of the consumer to prevent the project from being realized. Table 4.5 summarizes the relation between CV, EV and WTP and WTA for positive and adverse safety changes.

Welfare change measure	Safety increase (Utility increase)	Safety decrease (Utility decrease)
Compensating variation	WTP	WTA
Equivalent variation	WTA	WTP

Table 4.5: Welfare change measures

The WTP/WTA Discrepancy

Traditionally, economists have been quite indifferent about which welfare measure to employ in economic valuation: WTP and WTA have both been considered to be acceptable. By large, the literature has focused on WTP. It has been confirmed by numerous empirical studies however, that the WTA tends to be significantly higher than the WTP in many applications [102, 272]. This discrepancy is shown in Table 4.6 for diverse kinds of goods.

Both measures rely on the assumption of substitutability in preferences, but each adopts different reference points. The WTP reference point is the absence of the safety improvement, while the WTA reference point is the presence of the improvement. Therefore, WTP and WTA need not be exactly equal for an equal size improvement of the publicly provided safety level. Obviously, the WTP is constrained by consumer income, whereas there is no upper limit on what the individual would require to forgo the improvement (WTA). In addition, it has been observed that people place higher values on things they own than on things they do not, a phenomenon commonly known as the endowment effect [246].

These differences still would not matter if property rights were always clearly defined [200]. WTP in the context of a potential improvement is clearly linked to the right for status quo. It presumes that the individual has no right to the safety improvement but does have a right to the original level of s and must pay for the enhancement. Similarly, if the situation is one of losing the status quo, then WTA for that loss is the appropriate measure. It presumes that the individual has a right to the new, higher level of safety and must be compensated if the new level is not attained.

Type of good	WTA/WTP ratio	Standard error
Public or non market	10.4	2.5
Health and safety	10.1	2.3
Private	2.9	0.3
Lotteries	2.1	0.2
Timing	1.9	0.2
All goods	7.2	0.9

Table 4.6: WTA/WTP for different kinds of goods [Horowitz and McConnell [118]]

Based on this interpretation of the two measures, some economists have argued that the choice between them is basically an ethical one [46]. As policy decisions in the context of public risk reduction project appraisal deal with improvements rather than degradation in the publicly provided safety level, there is strong favor for the assumption that the WTP constitutes the appropriate measure for employment. For marginal safety changes however, the discussion about which measure to choose becomes dispensable from a theoretical point of view, as the two measures coincide. For practical purposes instead, it might be relevant also for marginal changes.

4.3.2 Impact of Marginal Safety Changes on Consumer Welfare

So far the welfare change measures have only been defined implicitly by means of the indirect utility function V^c. In this paragraph explicit formulations for the consumer's WTP for safety are derived. Under the assumption that the risk reduction project has only marginal effects on the total publicly provided safety level and potentially other exogenous variables, the partial and total derivatives of the indirect utility function may be taken into account to estimate the resulting change in consumer welfare.

Ceteris Paribus Safety Changes

It is investigated firstly, how a small ceteris paribus change in the publicly provided safety level s affects consumer utility (per unit of safety). This is done by constructing the partial derivative of the indirect utility function V^c with respect to safety:

$$\frac{\partial V^c(p,w,y^c,s)}{\partial s} = \frac{\partial U^c(x^c(p,w,y^c,s), l^c(p,w,y^c,s), s)}{\partial s} \tag{4.44}$$

$$= \frac{\partial U^c}{\partial x^c}\frac{\partial x^c}{\partial s} + \frac{\partial U^c}{\partial l^c}\frac{\partial l^c}{\partial s} + \frac{\partial U^c}{\partial s} \tag{4.45}$$

$$= \lambda^c p\frac{\partial x^c}{\partial s} - \lambda^c w\frac{\partial l^c}{\partial s} + \frac{\partial U^c}{\partial s} \tag{4.46}$$

In this chain of equations, expression (4.46) has been obtained from equation (4.45) by making use of the first order optimality conditions $\frac{\partial U^c}{\partial x^c} = \lambda^c p$ and $\frac{\partial U^c}{\partial l^c} = -\lambda^c w$ of the utility maximization problem stated in (4.13) and (4.14). As discussed above, the first two terms in equation (4.46) imply that the optimal levels of consumption and labor supply may be affected by a change in the safety level s through their marginal utilities as

$$\frac{\partial^2 U^c}{\partial x^c \partial s} \neq 0 \qquad \text{and} \qquad \frac{\partial^2 U^c}{\partial l^c \partial s} \neq 0 \tag{4.47}$$

generally holds. But under the ceteris paribus assumption of constant prices p and w and income y^c, these indirect effects cancel out as can be demonstrated by means of the income constraint [96]. Under the optimal consumer behavior the following equation must hold for the income restriction:

$$y^c - px^c(p,w,y^c,s) + wl^c(p,w,y^c,s) = 0 \tag{4.48}$$

Differentiating the income constraint (4.48) with respect to safety and multiplying all terms by the marginal utility of income to convert them in utility changes leads to:

$$\lambda^c p\frac{\partial x^c}{\partial s} - \lambda^c w\frac{\partial l^c}{\partial s} = 0 \tag{4.49}$$

Now, reinserting condition (4.49) in equation (4.46) yields:

$$\frac{\partial V^c}{\partial s} = \frac{\partial U^c}{\partial s} \tag{4.50}$$

Thus, the influence of a ceteris paribus safety change on optimal utility is equal to the marginal utility of safety and all second order effects on consumption and labor supply are negligible. This result is of great importance in microeconomic theory in general and is commonly known as envelope theorem, which is formally derived in Appendix C. For the constrained utility maximization problem it states that the partial derivatives of the indirect utility function with respect to the exogenous parameters equal the respective derivation of the Lagrange function Λ^c evaluated at the optimum:

$$\frac{\partial V^c}{\partial z} = \frac{\partial \Lambda^c}{\partial z}, \qquad z \in \{p, w, y^c, s\} \tag{4.51}$$

As safety does not enter the budget constraint by construction, for the above considered ceteris paribus safety change (4.51) vanishes to (4.50). The envelope theorem is of great importance for further argumentation.

To sum up, the total change in optimal utility for a marginal safety change ds is given by:

$$\frac{\partial V^c}{\partial s} ds = \frac{\partial U^c}{\partial s} ds \tag{4.52}$$

Now, by dividing equation (4.52) by the marginal utility of income $\frac{\partial V^c}{\partial y^c}$, the change in utility is converted to monetary value, leading to consumer c's marginal WTP for safety:

$$WTP^c = \frac{\frac{\partial V^c}{\partial s}}{\frac{\partial V^c}{\partial y^c}} ds = \frac{\frac{\partial V^c}{\partial s}}{\lambda^c} ds \tag{4.53}$$

The identity $\frac{\partial V^c}{\partial y^c} = \lambda^c$ may be verified by means of the envelope theorem result (4.51). It is easily seen that a subtraction of WTP^c from the consumer's income neutralizes the utility increase of the safety improvement, so that the consumer is as well off with the safety change and a lower income than without the change and initial income. For any fraction of the NCHS that the consumer has to expend for the project that is below the consumer's WTP, an increase utility would result and the project would be worthwhile for the individual. Accordingly, it can be validated, that an addition of WTP^c to consumer income makes her as well off without the safety improvement as she could have been with the change and initial income.

Consequently, for a marginal ceteris paribus safety change the WTP^c given in formula (4.53) is identical to the above introduced measures CV^c and EV^c and these two measures coincide. This is due to the fact that the marginal utility of income remains constant throughout the change. Therefore, equation (4.53) is to be interpreted as the consumer's pricing rule for small ceteris paribus safety changes. Furthermore, it has to be emphasized that the WTP formula is invariant under monotone transformations of the ordinal utility function because the change in V^c is offset by the change in the marginal utility of income resulting from the transformation [105].

Complex Safety Changes

If the ceteris paribus assumption of the safety change is relaxed and thus, prices p, wages w and income y^c are allowed to vary in response to the safety change ds, the realm of measuring individual safety changes in a general equilibrium setting is reached. At individual level however, the changes in prices, wages and income must be considered as exogenously given, as outlined above. They are treated endogenously when cost benefit rules in general equilibrium based on the representative consumer model are derived in Section 4.4. Now, the individual WTP measure for such a complex safety induced welfare change must fulfill the following implicit condition

$$V^c(p^1, w^1, y^{c1} - WTP^c, s^1) = V^c(p^0, w^0, y^{c0}, s^0) \tag{4.54}$$

where superscripts 0 refer to the status quo state before the implementation of the project and superscripts 1 denote the post project situation. For reasons of simplicity the characterization of the WTP by means of equation (4.54) has relied on the CV measure. In order to arrive at an explicit form for the WTP measure, the indirect utility function V^c needs to be totally differentiated to firstly obtain a functional form for the consumer's utility change

$$\begin{aligned} dV^c &= \sum_{i=1}^{n} \frac{\partial V^c}{\partial p_i} dp_i + \sum_{j=1}^{k} \frac{\partial V^c}{\partial w_j} dw_j + \frac{\partial V^c}{\partial y^c} dy^c + \frac{\partial V^c}{\partial s} ds && (4.55) \\ &= -\lambda^c \sum_{i=1}^{n} x_i^c dp_i + \lambda^c \sum_{j=1}^{k} l_j^c dw_j + \lambda^c dy^c + \frac{\partial V^c}{\partial s} ds && (4.56) \\ &= -\lambda^c (x^c dp - l^c dw - dy^c) + \frac{\partial V^c}{\partial s} ds && (4.57) \end{aligned}$$

where equation (4.56) has been derived from equation (4.55) by applying the envelope theorem and equation (4.57) corresponds to (4.56) in vector notation. Equation (4.57) captures the change in consumer utility due to the safety induced changes in the exogenous variables. Dividing equation (4.57) by the marginal utility of income λ^c converts the utility change in monetary units and thus, yields the generalized WTP formula of the consumer for complex safety induced changes:

$$WTP^c = -x^c dp + l^c dw + dy^c + \frac{\frac{\partial V^c}{\partial s}}{\frac{\partial V^c}{\partial y^c}} ds \tag{4.58}$$

Equation (4.58) may be interpreted as the individual consumer's cost benefit pricing rule for a marginal safety change in a general equilibrium analysis. The direct benefits to the consumer are given by $\frac{\partial V^c}{\partial s} / \frac{\partial V^c}{\partial y^c} ds$, and the project has some spillover effects on a variety of markets, where it causes adjustments in consumer and producer behavior. Therefore, the previously given equilibrium prices p and wages w are no longer stable and adjust to new values, as discussed above. These price changes in turn have an effect on the individual consumer surpluses on the affected markets, which are captured by the first two terms of equation (4.58). The third term represents the change in the consumer's non labor income that results from the safety change. As was true for the small ceteris paribus safety change also here the WTP^c measure is interpretable either in terms of CV or EV, since the two measures coincide. This holds, because the marginal utility of income remains constant along a marginal change in the exogenous variables.

If prices and wages should vary in non marginal amounts in response to the marginal safety change however, the problem of path dependency is faced: when relying on the Marshallian demand curves to analyze consumer surplus changes on the affected markets, the sequential order in which the prices are changed generally has an impact on the WTP measure and can cause inconsistencies.

These inconsistencies occur whenever the cross price derivatives of the demand functions are not symmetric. In this case, the welfare change measures must be based on Hicksian compensated demand curves to ensure a reliable result. The relation between Marshallian and Hicksian demands is provided in Appendix D. In a general equilibrium setting however, welfare changes that result from price changes net out on aggregate level, as the effects of price changes occur with reversed signs both on consumer and on producer side, as demonstrated below. Therefore, the issue of path dependency is not further explored in this thesis. For a detailed discussion on this issue it is referred to [139].

4.3.3 Producer Welfare Change Measures

In this section it is investigated how marginal safety changes affect producer welfare. As stated above, the ultimate goal of the producers is to maximize profits. Therefore, profit is the obvious candidate for a measure of producer welfare. On the producer side there is no path dependency problem involved in the measurement of welfare changes on markets where non marginal price adjustments occur and the derived measures can be given the interpretation of both CV and EV since they coincide. Producers simply maximize the difference between revenues and cost under the given safety conditions. In analogy to the indirect utility function of the consumer, the profit function of the producer constitutes a valuable tool to analyze safety induced welfare changes:

$$\Pi^f(p,w,r,s) = pF^f(L^f(p,w,r,s), K^f(p,w,r,s), s) - wL^f(p,w,r,s) - rK^f(p,w,r,s) \tag{4.59}$$

As illustrated above, the publicly provided safety level enters the production function and therefore affects the amount of output the producer is able to produce from a given amount of inputs. According to equation (4.59) profits are equal to revenues minus variable and fixed cost. In general, the fixed cost rK^f are independent of the level of production the producer decides to produce in the short term. For reasons of simplicity therefore, the fixed cost is treated as a constant henceforth and the dependence of the functions x and L on the fixed stock of capital is suppressed. Accordingly, the following form for the profit function is obtained:

$$\Pi^f(p,w,s) = pF^f(L^f(p,w,s), s) - wL^f(p,w,s) - r\overline{K}^f \tag{4.60}$$

Now, if the publicly provided safety level is increased from s^0 to s^1 ceteris paribus, the following change in the firm's profit results:

$$\Delta\Pi^f = \Pi^f(p,w,s^1) - \Pi^f(p,w,s^0) \tag{4.61}$$

Straightforwardly, the change in profit is measured directly in monetary terms, so that equation (4.61) already represents an adequate measure for the producer's welfare change. Furthermore, provided that capital is fixed, it becomes obvious that the influence of capital nets out in calculating the profit change. Therefore, the changes in profits are uniquely determined by the changes in revenues minus changes in variable cost, which is equal to the firm's producer surplus or quasi rent. As a consequence, in the firms short term production decisions it is focused on producer surplus or quasi rents rather than directly on profits. The term quasi rent is used because on the one hand it is a rent on the fixed factors of production and on the other hand it may not persist for a long time, since the firm is able to adjust capital stock long term [129].

4.3.4 Impact of Marginal Safety Changes on Producer Welfare

As the profits have been identified to be the proper tool to evaluate producer welfare changes, it is now proceeded with analyzing the impacts of both ceteris paribus and more complex safety changes on producer welfare.

Ceteris Paribus Safety Changes

In order to analyze the effects of a marginal safety change on producer welfare, it is assumed firstly, that the safety change does not impact prices and wages on other markets. The effect of such a ceteris paribus safety change on producer welfare can be assessed by differentiating the profit function with respect to safety:

$$\begin{aligned} \frac{\partial \Pi^f}{\partial s} &= p(\frac{\partial F^f}{\partial L^f}\frac{\partial L^f}{\partial s} + \frac{\partial F^f}{\partial s}) - w\frac{\partial L^f}{\partial s} && (4.62) \\ &= (p\frac{\partial F^f}{\partial L^f} - w)\frac{\partial L^f}{\partial s} + p\frac{\partial F^f}{\partial s} && (4.63) \\ &= p\frac{\partial F^f}{\partial s} && (4.64) \end{aligned}$$

Here, equation (4.64) has been obtained from equation (4.63) by making use of the first order conditions of profit optimization (4.23). As a result, the impact of the safety change on the firm's profits are given by the value of the marginal product of safety $p\frac{\partial F^f}{\partial s}$ and all second order safety effects may legitimately be neglected. This is a result of the envelope theorem for unconstrained optimization problems, that is provided in Appendix C.

The producer's WTP for the marginal ceteris paribus safety increase is then easily calculated

$$WTP^f = p\frac{\partial F^f}{\partial s}ds \tag{4.65}$$

which simply corresponds to the additional profits or producer surplus the producer obtains from the enhanced safety level. If the amount WTP^f was transferred to the producer under status quo safety level, she would realize exactly the same profits as after the project. Alternatively, taking away this amount of money from the producer after the safety change would enable her to realize the same profits as in status quo condition and she would be equally well off. Therefore, the producer welfare measure (4.65) can be interpreted in terms of CV and EV and the latter two always coincide since there are no income effects involved.

Complex Safety Changes

In order to complete the analysis on producer welfare, an expression to measure producer welfare changes for complex safety changes is derived. Here, the ceteris paribus assumption is relaxed and prices and wages are permitted to vary in response to the safety improvement. Taking the total differential of the profit function yields

$$d\Pi^f = \sum_{i=1}^{n} \frac{\partial \Pi^f}{\partial p_i} dp_i + \sum_{j=1}^{k} \frac{\partial \Pi^f}{\partial w_j} dw_j + \frac{\partial \Pi^f}{\partial s} ds \tag{4.66}$$

$$= x^f dp - L^f dw + p \frac{\partial F^f}{\partial s} ds \tag{4.67}$$

where equation (4.67) has been obtained from (4.66) by applying the envelope theorem. As the profits are already given in monetary units, equation (4.67) already constitutes the appropriate expression for the producer's WTP under complex safety changes. Investigating equation (4.67) more closely, it is seen that the total change in producer welfare is given by the producer surplus changes on the product and labor markets due to the modified prices and wages resulting from the safety change, augmented by some additional producer surplus originating from the higher production capacity under the new safety conditions.

4.3.5 Impact of Marginal Safety Changes on Social Welfare

Once the above described procedure to measure consumer welfare changes has been carried out for each consumer individually, the single consumer's WTPs have to be aggregated on social level to finally arrive at a consumers' SWTP for the marginal safety increase under investigation. Here, the aggregation is performed by summing up the individual WTPs over all consumers:

$$SWTP^C = \sum_{c=1}^{C} WTP^c \tag{4.68}$$

This result holds because of the public good properties of safety: it is enjoyed by all and the consumption of safety of one person does not reduce the availability of safety to another. So all individuals pay collectively for a unit increase in safety and the aggregate marginal value of safety is the sum of all individual marginal values.

The aggregation of the single producer's WTP on social level to obtain the producers' SWTP is performed analogously by summing up the individual producer's WTPs:

$$SWTP^F = \sum_{f=1}^{F} WTP^f \tag{4.69}$$

The total welfare change to society is given by the sum of the changes in consumer and producer welfare and is calculated by summing up the respective WTPs of all economic agents that comprise society:

$$SWTP = SWTP^C + SWTP^F \tag{4.70}$$

This value corresponds to the social price for safety in the economy. Should the SWTP be higher than the NCHS of the pure safety problem, the project is approved to be efficient in social sense. Consequently, the Kaldor-Hicks test is passed and a potential Pareto improvement is realizable, as discussed in Chapter 3. In a consecutive step it then has to be investigated, if the project really results in a welfare increase to society by means of a social welfare function that includes a value judgment about distributional desirability.

This is usually done by attaching distribution weights to the individual WTPs to elevate the relative influence of comparatively weak individuals in the project decision process. The justification to avoid an assignment of distribution weights, that is commonly made by economists, is the assumption that the welfare distribution in society is already on an optimal level, so that the marginal social utility of income is equal across all individuals [131]. An alternative assumption that could be placed is that the government smoothens possibly occurring inequities by a progressive taxation of the project cost. If the marginal social utility of income is equal across individuals, the following relation holds

$$SWTP - NCHS > 0 \Leftrightarrow \Delta W > 0 \tag{4.71}$$

i.e. efficiency in safety is equivalent to a social welfare increase in society.

Up to now, it has been demonstrated, how consumers and producers WTP for changes in the publicly provided safety level can be assessed and aggregated on social level to obtain a social price for safety. All previous derivations were based on the individualistic concept, i.e. each single individual's valuation has been included in the derivation of a social price for the safety change, in line with the individualistic postulate of welfare economics. In real project appraisal however, such an approach would hardly be possible considering the great heterogeneity of individual preferences involved, which are generally unknown, and even less at a reasonable cost. At the individual level there is a wealth of information available that is reflected in the price building process of particular goods. Abstaining from market distortions, the price of single goods processes all that information and constitutes a valuable indicator for true social scarcity values, that contain each participating individual's WTP contribution.

In the attempt to price non market goods on population level however, this wealth of information does not exist. In the future, with advancements in information processing and computer technology and in a perhaps more ideal system of public decision making, such information could become accessible for public planning. But in contemporary policy analysis planning on population level often entails the use of representative agent models that rely on a few characteristics only [51]. The proper question to be considered in this context is whether aggregate validity is improvable by better handling the characteristics of the representative agent. The construction of a representative agent model is subject of the following section and serves as theoretical basis for the LQI in Chapter 5.

4.4 Evaluating Welfare Changes in a General Equilibrium Setting

Actual economies consist of millions of consumers and producers that are potentially affected by the public risk reduction project. Therefore, in theory, the individual cost benefit rules for consumers and producers that are derived above need to be carried out for each economic agent individually to finally arrive at the SWTP for the considered safety change provided in (4.70). Beyond doubt, such a detailed analysis would be very demanding and costly and requires precise information about the preferences of single individuals and firms, which is not easily obtained. As a consequence, public policy analysis is often being performed by relying on representative agent models to investigate changes in a general equilibrium setting or alternatively, it is restricted to partial equilibrium analysis on single affected markets ceteris paribus.

In order to keep with general equilibrium analysis, it is conventional in welfare economics to proceed as if the consumers and producers could be aggregated in one single or representative agent each

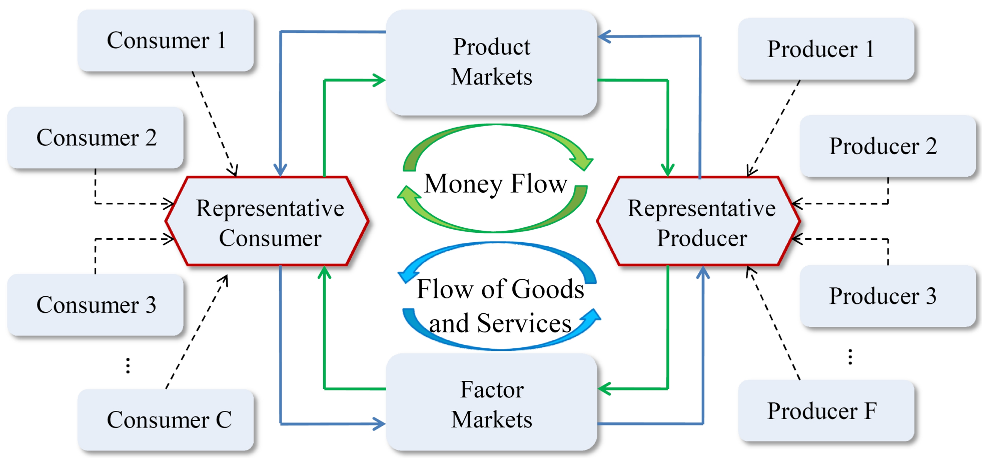


Figure 4.7: The representative agent model

[31], as illustrated in Figure 4.7. Then, to simplify the analysis it is assumed that all agents of the same type are identical and show similar behavior or alternatively, that agents differ, but act in such a way that the sum of their choices is mathematically equivalent to the decision of one representative individual or firm, respectively. The whole analysis is based on average values. Especially in the evaluation of public goods in a general equilibrium setting where each subject is affected by the same quantity changes, the representative agent model is widely applied [31]. The advantage of this procedure is that it allows for the use of aggregate market data in evaluating welfare changes and to eschew the issue of aggregating welfare measures over individuals. Instead it is only focused on the allocative efficiency effects of a welfare change [31].

Samuelson [226] points out that there are two circumstances where the representative agent model may legitimately be used to represent many person economies for normative purposes:

1. The government is continually redistributing income optimally by a set of lump sum transfers.

2. Preferences are restricted to be identical and homothetic for all individuals.

Either of these two restrictions is to be looked upon as being quite stringent, but the approach may serve well as an approximation under limited information to derive a first statement about project efficiency and thus constitutes a useful analytical device.

4.4.1 The Representative Agent Model

At this point is proceeded as if there was just one single consumer and one single firm in the model economy and either one or both of the above stated prerequisites for the representative agent model are assumed to be fulfilled. As before, the ultimate goal of the representative consumer is to maximize her utility subject to a given budget constraint and the firm's objective is to maximize profits. Therefore, all CBA rules derived above hold also in the representative agent case. But one further simplification is possible: because the firm is ultimately owned by the consumer, all profits must accrue directly to the consumer as part of her income. This implies that the two problems of utility and profit maximization are linked through the consumer's income and a social welfare analysis can be carried out by focusing only on the representative consumer's utility changes. For this purpose it is sufficient to explicitly account for profit income in the consumer's indirect utility function and thus, to split up non labor income y in its components K, Π and τ, so that changes in the firms profits become visible and are translated into utility changes of the consumer

$$V(p, w, K + \Pi - \tau, s) := \max_{x^d, L^s} \left\{ U(x^d, L^s, s) : K + \Pi + wL^s - \tau = px^d \right\} \tag{4.72}$$

where the superscripts c have been omitted and the new introduced superscripts d and s refer to supply and demand, respectively. In the following it is assumed, that the consumer considers profit income as being independent of her own actions. From the point of view of the whole economy however, price and safety changes will affect consumer income through their influence on profits. Accordingly, changes in consumer income are explicitly accounted for in the model. Furthermore, by differentiating the indirect utility function (4.72) with respect to p and w and setting the resulting expressions equal to zero it is easily seen, that social welfare is maximized at the set of prices where supply equals demand on all markets, so that a general equilibrium is obtained.

It must be emphasized however, that one price needs to be fixed, since general equilibrium models are only able to determine relative prices, as outlined above. At this point it is assumed, that a general equilibrium has been reached under status quo safety conditions before the implementation of the public risk reduction project. Thus, the general equilibrium under initial safety level constitutes the starting point of the welfare analysis.

4.4.2 Pricing Marginal Safety Changes in General Equilibrium

Relying on the results of Sections 4.3.1 and 4.3.3 it is now studied, how a change in the publicly provided safety level is priced in a representative agent model and rules for application in CBA are derived. As before, it is only investigated how marginal safety changes influence the model economy and it is not distinguished between ceteris paribus and more complex safety changes, as the latter contains the former as a special case. An increase in the publicly provided safety level, however small, will potentially cause adjustments in prices and wages on distinct markets within the economy. As stated above, it is not considered how the public risk reduction project is financed and therefore the tax variable τ is kept constant throughout the analysis. Also the capital variable K is assumed to be constant as it is only focused on the derivation of a SWTP for a marginal safety change short term. Furthermore, as the considered economy consists only of one representative consumer and the firm's profits are included as part of consumer income, the WTP of the representative consumer already constitutes the SWTP. Under these simplifications the following implicit formulation for the SWTP for the safety change is obtained:

$$V(p^1, w^1, K + \Pi^1 - \tau - SWTP, s^1) = V(p^0, w^0, K + \Pi^0 - \tau, s^0) \tag{4.73}$$

where superscript 0 refers to the situation in status quo and superscript 1 stands for the conditions after project implementation. In order to arrive at an explicit formulation for the SWTP for safety, firstly the indirect utility function has to be totally differentiated to make utility changes visible:

$$\begin{aligned} dV &= \sum_{i=1}^{n} \frac{\partial V}{\partial p_i} dp_i + \sum_{j=1}^{k} \frac{\partial V}{\partial w_j} dw_j + \frac{\partial V}{\partial \Pi} d\Pi + \frac{\partial V}{\partial s} ds && (4.74) \\ &= -\lambda \sum_{i=1}^{n} x_i^d dp_i + \lambda \sum_{j=1}^{k} L_j^s dw_j + \lambda d\Pi + \frac{\partial V}{\partial s} ds && (4.75) \\ &= -\lambda (x^d dp - L^s dw - d\Pi) + \frac{\partial V}{\partial s} ds && (4.76) \end{aligned}$$

Here, the terms corresponding to the constant variables τ and K have been omitted and the derivation has been carried out by making use of the envelope theorem. Furthermore, all derivatives have been evaluated with respect to status quo equilibrium before the project is implemented. Dividing (4.76) by the marginal utility of income converts the change in the representative consumer's utility to monetary value, yielding the SWTP for the safety increase:

$$SWTP = -x^d dp + L^s dw + d\Pi + \frac{\frac{\partial V}{\partial s}}{\frac{\partial V}{\partial y}} ds \tag{4.77}$$

Now, by making use of the formula for the producer's profit change (4.67), which under the new notation becomes

$$d\Pi = x^s dp - L^d dw + p \frac{\partial F}{\partial s} ds \tag{4.78}$$

the firm's profit changes can be included in formula (4.77), so that the following cost benefit rule for the marginal safety change in general equilibrium is derived:

$$SWTP = (x^s - x^d) dp + (L^s - L^d) dw + p \frac{\partial F}{\partial s} ds + \frac{\frac{\partial V}{\partial s}}{\frac{\partial V}{\partial y}} ds \tag{4.79}$$

It is important to note that if under perfect competition the prices and wages adjust in response to the safety improvement in order to restore a new general equilibrium by equating supply and demand on each market under the new safety conditions, the first two terms in the general equilibrium SWTP formula (4.79) vanish. This implies that even if the project affects all relative prices in the economy, the price changes can be neglected in the project evaluation. The reason for this is that positive effects of price changes on the consumer side are exactly offset by the corresponding negative effects on producer side and vice versa. They do not contribute to an increase in efficiency if no distortions have been present in the economy before [105]. Thus, under perfect competition, equation (4.79) can further be simplified to:

$$SWTP = p \frac{\partial F}{\partial s} ds + \frac{\frac{\partial V}{\partial s}}{\frac{\partial V}{\partial y}} ds \tag{4.80}$$

Equation (4.80) impressively highlights that the value of the safety improvement to society is equal to the sum of the representative consumer's direct valuation of the increased safety level and its direct impact on the representative firm's profits, which in turn result in a higher consumer income. Price effects can completely be neglected under perfect competition in a general equilibrium setting.

Before closing this section, it has to be noted that the general equilibrium safety pricing rule (4.80) is perfectly in line with a result obtained by Jones-Lee [133] in his influential book on the value of life. In this work he states, that a reduction in the social fatality rate will have three primary effects on society: firstly, it will reduce the social resource cost borne by society due to damage of physical capital (e.g. vehicles, roads etc.). Secondly, it will lead to a net contribution or increase of social output. The third (and almost certainly most important) effect of a reduction in the fatality rate is the direct effect on personal wellbeing.

Reconsidering equation (4.80) and keeping in mind that it has been focused on the pure safety problem throughout the analysis, it becomes clear that all three effects are being accounted for. The reduction in the social resource cost has been considered in the construction of the pure safety problem, while the first term of (4.80) captures the increase in social output and the second term accounts for the net effect of increased personal wellbeing of the representative consumer. Therefore, equation (4.80) finds good academic support.

4.5 Model Extensions

The above presented model to derive a social price for increased public safety levels has been very general so far, so that extensions in several directions are thinkable. Possible add-ons are presented in the following, that help to make the model more applicable for practical purposes.

4.5.1 Private Defensive Expenditures

In the previous sections it has been outlined, that the safety level that is enjoyed by each individual of society is the result of a twofold process. There is the basic safety level that is provided by the government that applies to all individuals simultaneously and there is a superstructure of safety that allows for individual variability. The latter is mainly influenced by the choices for certain safety related consumption goods and the selection of a particular type of labor with a certain risk exposure. Consequently, the individual has a certain influence on her personal safety status. This interaction between private and public safety standards may be incorporated in the above described model by introducing a so called safety production function

$$s^c = f(s, x, l) \tag{4.81}$$

that endogenizes the safety level for each individual. Personal safety s^c accordingly, is the result of the interplay between the publicly provided safety level s and the choices that are made in favor of certain safety related consumption goods x and labor supply l. Consequently, the production function describes, how different combinations of safety related goods affect total personal safety. Substituting this function for the individual safety level in the utility functions of the consumers would properly account for the twofold safety structure and allow for individual variability in safety. The general approach to derive a SWTP for safety however, would remain unaffected by this modification. More details on this issue are provided in [131].

4.5.2 Intertemporal Extension

The safety pricing models considered thus far have been discussed without any assumption about time. Therefore, they constitute so called atemporal or one period models. As public risk reduction projects are usually designed for comparatively long time periods, the model has to be extended to account for these intertemporal aspects. Furthermore, even a onetime increase in the publicly provided safety level leads to an increase in survival probability not only in the considered period, but also in all consecutive periods of the consumers' remaining lives. This holds, because all survival probabilities in consecutive years are conditional on the survival probability today, as further discussed in Chapter 5. These dynamic aspects certainly have to be accounted for in deriving a social price for safety.

Another important feature of intertemporal models in contrast to one period models is that the consumer is not assumed to spend all her income in the considered period but can partly abstain from current consumption and save for future periods. These savings augmented by an interest payment in turn enable the consumer to consume more in the future. Thus, the consumer is able to distribute her consumption expenditures arbitrarily over her lifetime by means of a properly defined interest rate r, which generally yields additional utility. Consequently, in an intertemporal context, the problem of utility maximization turns into the maximization of lifetime utility. The lifetime utility model can be formulated either in discrete and continuous time.

In the discrete case, the above introduced models may also be given a finite time dynamic interpretation. For this purpose, the commodity and labor vectors as well as the price and wage vectors need to be extended in their dimension: the vector components then do not simply represent different goods, but each good of the same kind purchased at different instants in time is treated as a distinct good that has its own (future) price. Accordingly, the public safety variable must be substituted by a safety vector that contains the safety levels for each time period as its components. Finally, the income variable has to be replaced by the present value of lifetime income. If the discrete intertemporal model is considered over $t = 1, ..., T$ periods, by introducing the following additional notation

$$\begin{aligned}
x &= (x_{11}, ..., x_{1T}, x_{21}, ..., x_{2T}, ..., ..., ...x_{n1}, ..., x_{nT}) \\
l &= (l_{11}, ..., l_{1T}, l_{21}, ..., l_{2T}, ..., ..., ...l_{k1}, ..., l_{kT}) \\
p &= (p_{11}, ..., p_{1T}, p_{21}, ..., p_{2T}, ..., ..., ...p_{n1}, ..., p_{nT}) \\
w &= (w_{11}, ..., w_{1T}, w_{21}, ..., w_{2T}, ..., ..., ...w_{k1}, ..., w_{kT}) \\
s &= (s_1, ..., s_T) \\
y &= \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}}
\end{aligned} \tag{4.82}$$

the above presented models in vector notation are applicable also in an intertemporal world. Such an approach however requires that agents have perfect foresight with respect to prices and each decision must be made in presence of presumed future development [93]. It has to be emphasized at this point, that the trade-off between income and survival probability/time in a one period model converts into a trade-off between wealth and life expectancy in a lifetime model. This and further intertemporal aspects are elaborated in detail in Chapter 5.

4.5.3 Risk Extension

Before introducing the above sketched safety pricing models it has been outlined, that there are generally two interpretations of the safety variable s: firstly, it may be interpreted in terms of annual survival probability and secondly, it permits a measurement in terms of expected annual survival time, measured in years, as the considered time horizon has been outlined to be one year. In absolute terms the two measures straightforwardly coincide. For the purpose of modeling safety in terms of a public good and analyzing its influence on consumer and producer behavior in a general equilibrium setting, the interpretation in terms of time is better suited as it is more tractable in welfare analysis and allows for straightforward comprehension.

In a great part of WTP literature however, safety s is being modeled in terms of survival probability (or mortality risk) and thus, the underlying model explicitly accounts for risk. Accordingly, the latter is based on the von Neumann Morgenstern expected utility theory [184]. In these models it is often assumed that utility depends only on consumption and that there are only two states of the world, namely survival A or death D, so that the following function is to be maximized by the individual

$$E[u(x)] = s \cdot u_A(x) + (1 - s) \cdot u_D(x) \tag{4.83}$$

where E denotes the expected value operator. According to the von Neumann Morgenstern expected utility hypothesis [31], the utility function u must be based on a cardinal utility concept, while the expected utility $E[u]$ is of ordinal nature. An individual WTP for safety is then derived by analyzing the trade-off between safety and consumption:

$$\frac{dx}{ds} = \frac{u_A(x) - u_D(x)}{su'_A(x) + (1 - s)u'_D(x)} \tag{4.84}$$

Many authors [52] further hypothesize, that the utility in case of death may be assigned a zero value, so that u_D vanishes from expressions (4.83) and (4.84). Under this assumption, the consumer utility function that has been employed in the models above can also be given the following interpretation

$$U(x, l, s) = E(u(x, l)) = u(x, l) \cdot s \tag{4.85}$$

so that an implicit inclusion of risk becomes possible. As outlined above however, it must be assumed that the subutility function $u(x, l)$ in expression (4.85) is cardinal, whereas the total utility function $U(x, l, s)$ is ordinal as before. The results obtained for the individual WTP then coincide with (4.84), by setting $u_D = 0$ and letting x act as the numeraire good.

4.6 Discussion

In this chapter, the impact of safety on a market economy has been studied and approaches have been presented that allow for the determination of a social price for safety in a general equilibrium framework. At the outset, an extensive literature review on existing approaches for deriving a WTP for safety has been carried out and exemplary VSL values originating from distinctive studies have been documented.

The valuation approaches have been subdivided into revealed and stated preferences approaches. VSL values from labor market studies resulted in the highest VSL estimations in the order of US$

7.8 million, whereas consumer market studies and stated preference approaches generally led to significantly lower values in the range of US$ 5 million. Throughout all studies, a comparatively wide range of values could be observed. Several reasons might explain this result. Among these are possible market imperfections, a difference between perceived and observed risk reduction efficiency and several other factors that are mainly attributed to individual characteristics and the context in which the studies have been conducted.

On a population level it might not be admissible to apply values that have been extracted in different contexts and from different sub populations involved in the market transactions or surveys. A desirable goal for public policy analysis would be in contrast to use standard VSL values in project evaluations or the identification of a functional relationship that enables the construction of context specific VSL values. For this reason, meta analyses have been presented that subsume a great diversity of VSL studies with the objective to statistically extract reasonable VSLs on social level. Even the reviewed meta studies could not contribute to significantly more homogeneity in values however, so that no rule of thumb could be obtained. In order to assess a reliable and stable VSL on social level, it is necessary to analyze and price safety induced changes on a great diversity of markets and populations.

To approach this task, a model has been developed that clearly reveals the far reaching influence of safety in a market economy. In this sense, it has firstly been observed, that the safety level that is enjoyed by each individual of society is the result of a twofold process: there is a basic infrastructure that applies to all and is publicly provided and there is some kind of superstructure that allows for individual variability. With respect to the latter, the market perspective is of particular importance as private safety precaution is mainly bought on the market when choices are made in favor of certain safety related goods. The focus of study has been placed on the publicly provided safety level however, as this is impacted by public risk reduction interventions. Good reason has been presented to interpret the public safety level as a public good, that simultaneously affects all consumers and producers in the model economy to equal amounts.

It has further been outlined that safety as a public good impacts both consumers' and producers' economic behavior in their ambitions to maximize utility and profits, respectively. Consequently, a safety change generally leads to shifts in demand and supply curves on a great diversity of markets and has therefore an influence on the stability of both single market equilibria as well as the general equilibrium of the model economy. By this cognition it has been demonstrated, that safety improvements allow for an interpretation in terms of a social resource increase, that in turn leads to a shift in the general equilibrium which is attributed to an increase in efficiency. Consequently, the change in social welfare in the move from status quo equilibrium to post project equilibrium represents the social price for safety, if converted to the money scale.

In a next step, consumer and producer welfare measures have been reviewed and individual safety pricing rules for marginal safety changes have been derived. The total WTP on social level has been obtained by summing the individual WTPs over all economic agents of society. The limitations of this approach have clearly been outlined as no distributional value judgment has been included in the aggregation, which is justifiable under restrictive conditions only. Furthermore, it has been emphasized that the presented individualistic approach goes perfectly in line with the postulates of welfare economics, but is hardly applicable in the context of public non market goods that potentially affect all members of society because of the stringent information requirements.

Based on this critic, a representative agent model has been introduced that departs from the individualistic concept with the ambition to make the safety pricing assessment more tractable from a practical point of view. This however comes at the price of a violation of the welfare economic postulate in general, as it is theoretically justifiable under restrictive assumptions only. The representative agent model merges the consumers' and producers' problems by including the firm's profits as a part of consumer income, which in turn allows for the endogenous treatment of the consumer's non labor income in general equilibrium.

From the obtained SWTP safety pricing formula it became clear, that price changes do not impact the social safety price as they net out in the move from one equilibrium to another. As it is only focused on efficiency aspects of the safety change in the representative agent model, this is intuitive, as prices have predominantly redistributive purposes. The final social price for safety has been shown to be composed of the consumer's direct valuation of the safety increase and the increases in total produced output that are attributed to the enhanced safety level.

Concluding the chapter, three possible model extensions have been presented that allow for the endogenous treatment of individual safety, the explicit treatment of risk and the expansion of the approaches to an intertemporal setting. Especially the latter two are of great importance in the following chapter.

The major essence that has been revealed in this chapter is that a certain discrepancy between the applicability of a model and its compliance with the individualistic welfare economic postulate is observable. While individual approaches are in perfect accordance with welfare economic theory, they are of limited use in the evaluation of non market goods on population level, such as public safety. Representative agent models in contrast deliver quite fast and sound results, but contradict with the welfare economic understanding of valuation in general. It might be concluded that theoretical economists need a far better understanding of the pressures that affect actual decisions, but those who make actual decisions perhaps also need a far better understanding of economics [200].

In the following chapter top down approaches for valuing safety are presented that refrain from the individualistic concept and attempt to extract VSL values directly on social level. In particular, the LQI is discussed, being itself a macroeconomic concept, whose microeconomic foundations lie in the representative agent model. Therefore, the results obtained above are fundamental for a comprehensive understanding of the LQI concept and its role in economic theory.

Chapter 5

Pricing Safety - Top Down Approaches and the Life Quality Index

So far, it has been demonstrated how a social willingness to pay (SWTP) for safety can be derived, following a bottom up approach: based on preferences towards safety, individual willingness to pay (WTP) measures for marginal safety improvements have been assessed and aggregated on social level. Now, it is refrained from the individualistic approach and top down strategies to directly obtain a SWTP for safety on aggregate level are presented. Here, the above introduced representative consumer model is to be looked upon as the transition model from bottom up to top down approaches. In particular, the Life Quality Index (LQI) concept is introduced and analyzed with respect to its economic foundations. The LQI represents a compound social indicator, taking the GDP per capita, life expectancy and the fraction of lifetime devoted to work as its life quality defining arguments. In the LQI context, safety changes are priced in terms of increased life expectancy. Several distinct approaches to derive the LQI exist in literature, leading to slightly different functional forms and showing particular conceptual shortcomings each. Addressing this issue, an innovative economically consistent derivation of the LQI is presented, that clearly reveals the underlying assumptions and economic reasoning. Here, it is departed from LQI literature in the sense the LQI is firstly derived in a one period general equilibrium model and then extended to account for intertemporal aspects. Beside these new economic insights, the major contribution of this chapter is to extend the LQI based safety pricing rule to include second order income effects and to present strategies for an improved calibration. In this way, eventually more extensive WTP and value of statistical life (VSL) measures are obtained.

5.1 Selected Macroeconomic Indicators

To create the basis for the top down safety pricing rules that are developed in the following, several selected macroeconomic indicators, namely the GDP (per capita), the life expectancy and the life working fraction are briefly introduced in this section. These measures are of particular importance in the sequel as they are employed in the formulation of the LQI, which is formally introduced in Section 5.3 below.

Figure 5.1 provides an overview of these indicators for selected economies and displays how this information is comprised in the LQI. In particular, in the chart's plane region the allocation of different countries with respect to their life expectancy at birth and their GDP per capita is depicted.

The total GDP on an economy level in contrast, is reflected in the surface size of the countries' pies, while their height mirrors the values the LQI assigns to them.

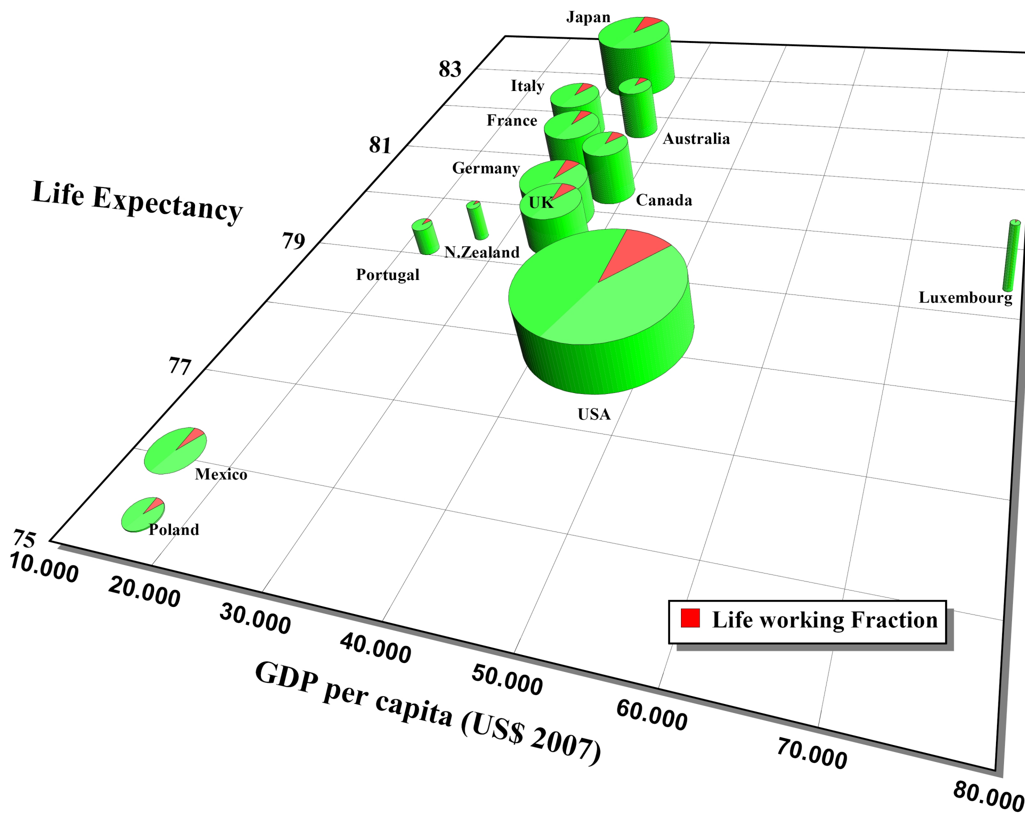


Figure 5.1: The LQI and its components - country overview

In addition, each country's pie shows, how the average individual in the respective economies splits her total lifetime into working time and leisure time, which are represented by the red and green regions on the pies' surfaces, respectively. It is seen from the height of the pies that Luxembourg and Japan followed by Canada are the countries with the highest life quality, while Portugal, Poland and Mexico compose the lower end of the life quality ranking, according to the LQI philosophy.

5.1.1 The Gross Domestic Product (GDP)

The gross domestic product (GDP) is a common indicator of national income and constitutes a measure for the total output of a country's economy. It represents the monetary value of all final goods and services produced within a country's borders in a given year. In this respect, only value added activities of the production process are included and double counting is carefully avoided by excluding intermediate goods and transfer payments. The GDP can therefore be considered as a surrogate measure for wealth related aspects of life quality or standards of living [217]. It has become the standard measure for economic performance and income of a nation, as used for instance by the United Nations [250].

As the national economy on a macro level constitutes a circular flow of goods and services (flow of product) and of money (flow of cost) between the productive and the consumer sector, the GDP is measurable in two distinct ways which are conceptually identical. The two flows of goods and money between consumers and producers have already been introduced in Chapter 4 when the simplified economy model was discussed. On a macro level however, it is refrained from considering single product or factor markets in isolation but they are analyzed on an aggregate or national level. The expenditure method now measures the flow of money at the point where it flows into the productive sector, whereas the income method calculates the GDP at the receiving end of the consumer sector [198].

Approaches to Measure GDP

The first approach to measure the GDP is the so called **expenditure method** and it is the most common approach to quantifying the GDP. Seen from this perspective, the GDP constitutes the value of total expenditures for all final goods and services and is thus equal to aggregate demand:

$$GDP = X + I + G + (EX - IM) \tag{5.1}$$

In this equation X stands for consumption, I for investment, G for government spending and $(EX - IM)$ represents the trade balance, i.e. the exports minus the imports. The consumption component represents all expenditures of households for clothing, food, heating and so on and accounts for around 60% of total GDP in most developed countries. The investment fraction is that part of total GDP which is spent for the purchase or replacement of productive assets such as plants, equipment, machinery and infrastructure which are not consumed but are used for future production. The investment part constitutes usually around 20% of total GDP. Government spending comprises mainly public expenditures on health care, education, policy and defense and accounts for around 20% of total GDP. The trade balance $EX - IM$ in contrast, represents in most cases only a minor part of total GDP [217].

Expenditure Method	**Income Method**
private consumption	wages
+ government spending	+ interests and profits
+ investments	+ indirect taxes
+ exports	- trans-border income transfers
- imports	+ depreciation
= GDP	= GDP

Table 5.1: National accounting

The second way to calculate the GDP is the **income method**. Here, the GDP represents the total of earnings or income generated in the production process. In particular, this encompasses compensation of labor W in terms of wages and benefits and of capital in terms of economic rents R, interests I and profits Π.

$$GDP = W + R + I + \Pi + \text{ Statistical Adjustments} \tag{5.2}$$

Labor comprises all human contribution to the output and capital is the stock of all productive assets, such as land, machines, housing and so on.

In summary, the expenditure and the income method lead to the same absolute value of the GDP and represent only different points at which the flow of money is measured. This is due to the fact that all economic transactions involve both a buyer and a seller and therefore every dollar that is expended on one side arrives in terms of income at the other side [158].

GDP Production

A macroeconomic production function describes the functional relationship how factors of production, in particular labor L and capital K, are transformed into the total output of an economy, i.e. the GDP. The GDP production function may be interpreted as the macroeconomic counterpart of the firms' production functions that have been introduced in the last chapter [197]. One of the most important and widely used production functions has been developed by Cobb and Douglas [50] in 1927, based on the observation that the relative contributions of labor and capital in the GDP production remained quasi constant over a comparatively long period of time [158]. This implies that GDP is produced with constant relative factor inputs. The Cobb Douglas production function has been extracted to have the following functional form:

$$Y = F(K, L) = AK^{\alpha}L^{\beta} \qquad \text{with } 0 < \alpha, \beta < 1 \tag{5.3}$$

The labor input L in the macroeconomic production function is measurable in different ways. On the one hand, it might be thought of as corresponding to the total hours of employment, while on the other hand it might be interpreted as the total number of employees in the economy. The capital stock K corresponds to the quantity of machines (or more explicitly, equipment and structures) used in production, and it is typically measured in terms of the value of these assets. There are multiple ways in thinking of capital and equally many ways of specifying how capital comes into existence. Since the objective at this point is to start out with a simple workable model, the simplifying assumption is made that capital is the same as the final good of the economy [2]. The factor A in the Cobb Douglas production function denotes a technology factor that describes the total factor productivity and α and β are the output elasticities of capital and labor, respectively. It is easily seen that when the two exponents α and β sum up to one, the production function exhibits constant returns to scale, indicating that an increase in both capital and labor to a certain amount leads to exactly the same increase in output.

Given the functional form of the Cobb Douglas production function (5.3), it can be investigated how total output changes due to a variation in factor input. Calculating the partial derivatives of the production function yields the marginal product of capital and labor, respectively:

$$\begin{aligned} MPK &= \frac{\partial Y}{\partial K} = \alpha A K^{\alpha-1} L^{\beta} = \alpha \frac{Y}{K} \\ MPL &= \frac{\partial Y}{\partial L} = \beta A K^{\alpha} L^{\beta-1} = \beta \frac{Y}{L} \end{aligned} \tag{5.4}$$

From these marginal product equations it follows that an increase in labor input reduces the marginal product of labor, meaning that each additional unit of labor leads to lesser and lesser increases in total output. At the same time an increase in labor input causes the marginal product of capital to rise, making it more profitable to employ additional capital in the production process. This eventually leads to the observed constancy in the relative factor inputs.

Furthermore, if the two factors of production are always paid their marginal products, the total capital income and total labor income amount to $MPK \cdot K$ and $MPL \cdot L$, respectively. Then, it is easily verified from (5.4) that

$$\begin{aligned} MPK \cdot K &= \alpha \cdot Y \\ MPL \cdot L &= \beta \cdot Y \end{aligned} \tag{5.5}$$

indicating that total GDP splits into αY and βY, and α and β represent the share of rents on capital and wages in total GDP. Therefore, the factor shares of total GDP are only determined by the parameters α and β and are independent of the total amount of capital and labor employed in the production process as well as of the technology factor A [158]. Furthermore, it becomes clear that if the factors are always paid their marginal products under constant returns to scale production, the total GDP will be exhausted without any surplus or deficit and profits are zero [198]. Figure 5.2 provides an exemplary overview of wage shares in the GDPs of different countries. It is seen that in most developed economies the values cluster around 50-60%.

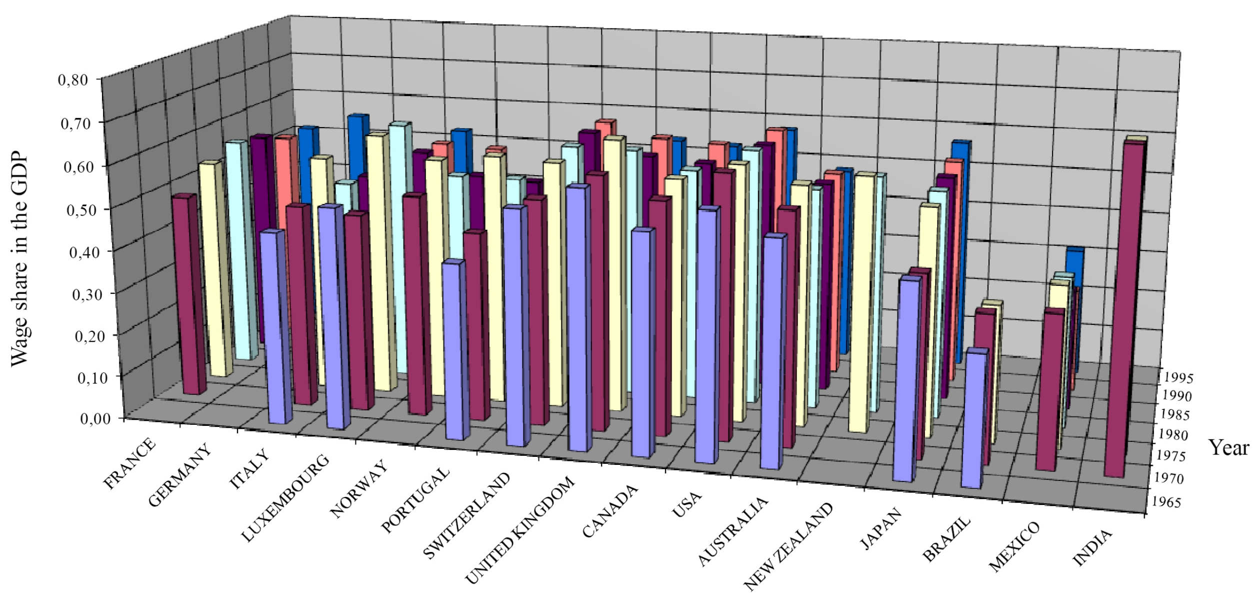


Figure 5.2: Wage shares in GDP [data source: Extended PennWorld Tables [159]]

The GDP represents one of the major indicators for the economic situation and development of a nation. However, there is a large body of literature on the shortcomings of using GDP as an indicator for social welfare and standard of living. Major criticism addresses that non-market activities such as housework and volunteer services as well as impacts of production on the environment are not accounted for. In this sense, a country might achieve high GDP growth rates by exploiting natural resources or polluting the environment as the sustainability of economic development is not an issue in national accounting. Moreover, income distribution is not considered implying that a high GDP might go in line with large disparities in income between the rich and the poor. Alternative indicators like e.g. the Index of Sustainable Economic Welfare (ISEW) or the Human Development Index (HDI) have been developed to address these deficiencies. Using these indices, an increase in GDP may not automatically imply an increase in these alternative indicators [165].

Potential GDP tends to change over time due to three key supply-side events [245]:

1. Changes in the labor force [quantity and quality];
2. Changes in the stock of productive capital [quantity and quality];
3. Changes in "total factor productivity" resulting from technological improvements in the efficiency with which labor and capital inputs are transformed into output.

Measuring changes in the quantity of labor and capital inputs, and to some extent in the quality of labor and capital inputs, is possible in principle. Measuring changes in total factor productivity on the contrary is more problematic. Therefore, many researchers attempt to measure changes in total factor productivity, or "productivity change" for short, as a residual change in actual measured real GDP after changes in the quantity and quality of labor and capital inputs have been accounted for.

5.1.2 The Life Expectancy

According to Pandey and Nathwani [197] the employment of life expectancy as an indicator for human development in the LQI rests on three main motives: the intrinsic value of longevity, its value in helping people pursue various goals and its close relation with characteristics such as safety, good health and medical care. In particular with respect to safety, the life expectancy is a valuable indicator as safety improvements reduce the probability of death of people at various ages and can be quantified intertemporally by means of increases in life expectancy. Moreover, data on life expectancy are concise, reliable and widely available in terms of life table statistics. Figure 5.3 displays the development of the life expectancy at birth in the period from 1960 to 2005 for selected countries. A clearly increasing trend throughout all selected countries is observable, being more emphasized in developing countries. In the developed world, the life expectancies nowadays cluster in the range of 79-83 years, with Japan taking the lead with a value of 82.9 years.

The close relation between mortality, survival probability and life expectancy at various ages is briefly discussed in the following, by reviewing the basics about life table calculations: let $F_D(t)$ denote the cumulative probability function of not having survived until age t and $f_D(t)$ the corresponding probability density function of living exactly for t years. Then, the age specific mortality or death rate is characterized by the following equation:

$$\mu(t) := \frac{f_D(t)}{1 - F_D(t)} \tag{5.6}$$

Accordingly, the age dependent mortality rate equals the probability of dying at age t under the condition that age t has been reached before. More illustratively, the rate is obtained by analyzing how many people of a given cohort have reached age t and did not survive until the end of the interval $t + 1$. Dividing the second amount by the first yields the age specific mortality rate.

By means of the distribution F_D or alternatively by taking the age specific mortality rate μ into account, the survival probability is easily calculated:

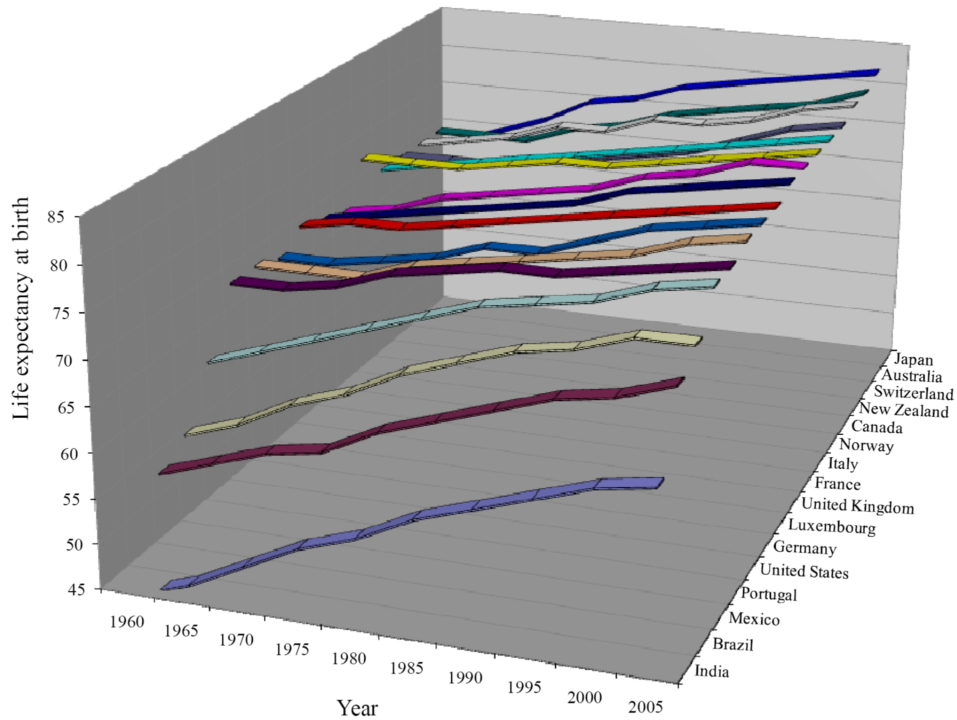


Figure 5.3: Life expectancy at birth [data source: WDI [262]]

$$S(t) := 1 - F_D(t) = \exp\left[-\int_0^t \mu(\tau)d\tau\right] \tag{5.7}$$

Straightforwardly, the survivor function expresses the probability of survival up to at least age t. Now, by integrating the survival probability from 0 to a maximum attainable age t_{max}, the life expectancy at birth is obtained:

$$e(0) := \int_0^{t_{max}} S(t)dt = \int_0^{t_{max}} \exp\left[-\int_0^t \mu(\tau)d\tau\right] dt \tag{5.8}$$

Thus, the life expectancy at birth is calculated by summing up the survival probabilities for infinitesimal small time intervals from 0 to the maximum attainable age t_{max}. In publicly available life tables, these time intervals are usually normalized to one year so that the life expectancy at birth is just the sum of annual expected lifetimes. Another important concept that will be of interest in the following is the life expectancy at age x, that reflects the expected remaining lifetime of a person that has already attained age x:

$$e(x) := \int_x^{t_{max}} \frac{S(t)}{S(x)}dt = \int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)d\tau\right] dt \tag{5.9}$$

The above introduced concepts constitute the basis for applying the LQI results in an intertemporal setting and are repeatedly used in this context.

5.1.3 The Life Working Fraction

The third component that is contained in the LQI constitutes the life working fraction a. The life working fraction is defined as the fraction of total lifetime the average individual of society spends in economic production, i.e. in income generating work. The inclusion of this variable in the LQI is based on the idea that not only a long and healthy life with a high level of income contributes to high living standards but also the time available to enjoy life. This appears to be a reasonable assumption considered that most work is dull, boring, troublesome and sometimes dangerous [214].

It can also be verified based on an historical argument. In the 1870ies the average annual time spent working was approximately 2900 hours, in 1950 still 2000 hours and in present times only around 1600 in developed economies [217]. Figure 5.4 provides an overview of the development of the annual working hours per employee over time for selected OECD countries. At the same time life expectancy increased from 45 to around 80 years due to advancements in safety, health care and nutrition and the GDP per capita rose more than tenfold due to higher productivity [155]. Consequently, beside higher consumption and life expectancy a significant increase in leisure time contributed to the great progress in the development of living standards. In addition, it has been argued repeatedly, that leisure time is the ultimate source of life quality, provided that economical and health conditions are sound [271].

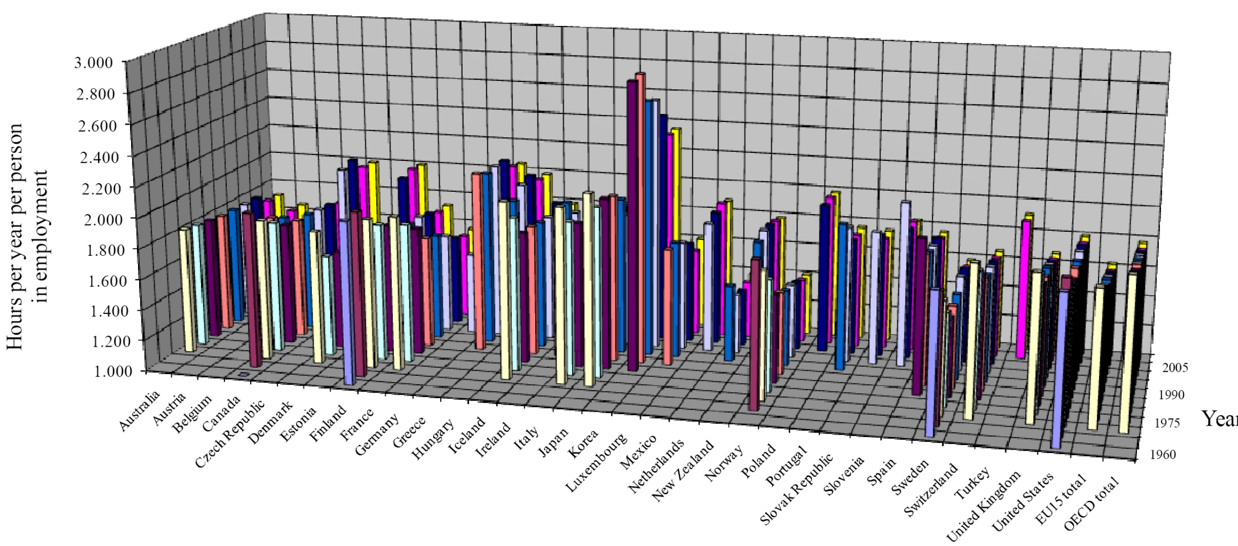


Figure 5.4: Annual working hours per person in employment [data source: OECD [188]]

In the LQI literature a continuous discussion about how to calibrate the parameter a is observable [62, 181, 215]. Central discussion points include the question whether to calibrate a relative to a 24-hour day or to subtract the necessary sleep time, whether to count commuting time to the work place or housework as economic activity and whether to relate a to the entire life span or solely to the effective work period excluding early childhood and retirement age. As several of these issues contain subjective judgments (e.g. on how much sleep is necessary) and therefore a high impact for controversy and would hardly be obtainable from empirical data, it is proposed to define a relative to a 24 hour day and the entire life span, counting effective work only. Moreover, as becomes clear later, the definition of the life working fraction has to be in accordance with the measurement of the labor force in the GDP production function. Counting housework and commuting time might therefore cause inconsistencies.

Now, in order to measure the life working time for the average individual of society, principally two distinct approaches can be followed. Firstly, according to [181] the following formula can be employed:

$$a := \frac{\text{life working period}}{e(0)} \cdot \frac{\text{annual working hours per employee}}{24 \cdot 365} \tag{5.10}$$

The second approach in contrast, is based on a snapshot of the current economy and enables to obtain data on a by considering statistics of the current economy:

$$a' := \frac{\text{number of employees}}{\text{total population}} \cdot \frac{\text{annual working hours per employee}}{24 \cdot 365} \tag{5.11}$$

The two approaches will lead to identical results only under special conditions, e.g. if the ratio between life working period and the life expectancy at birth equals the fraction of total number of employees to total population. Conditions under which this is fulfilled are discussed in [151]. As will become clear later, the second equation is considered as being more adequate in the LQI calibration and furthermore, data are easier obtainable.

5.2 VSL Estimations based on Macroeconomic Indicators

In contrary to the approaches presented in the previous chapter, the valuation of saved lives may also be based on macroeconomic indicators. The two traditional methods for the valuation of life are variants of the human capital approach or livelihood measure. Here, the value of life is assessed in proportion to a person's contribution to the production of the economy. The first method looks at the economy in terms of national income. The value of life then represents the discounted value of future earnings over the expected remaining lifetime. The second variant of the human capital approach is quite similar to the first except for the fact that all consumption expenditures that people make over their lifetime are subtracted from future earnings. The assumption behind the latter is that society loses earnings less consumption due to the premature death of one of its individuals [35].

For the first approach values around US$ 500,000 were obtained in a study of Forester et al. [84] in 1984, whereas Van Manen and Vrijing [253] came to an estimate of 500.000 Euros in a more recent study performed in 1996. For the second variant of the human capital approach straightforwardly lower values must be obtained. In this respect, Forester et al. [84] arrived at a value of US$ 369,000 using a 30% consumption rate derived from budget studies perfomed by the US department of labor.

It becomes obvious that the human capital method treats a life just as an asset of productive capital and estimates the production that is lost from premature death. Or as Acton [3] puts it:

> "The normative viewpoint which apparently motivates this approach is either that people are properly thought of as the chattel of the state, and the loss of a life has a cost to the state comparable to the cost of a slave's death to his owner; or the proper objective of public policy is to maximize gross national product."

Therefore, the human capital approach lacks satisfactory normative justification as it does not take individual preferences into account and consequently, strongly contradicts with the individualistic principle of welfare economics [131]. Freeman [88] emphasizes this point even more drastically by stating:

> "This measure is the antithesis of the individualistic premise of conventional welfare economics"

If one accepts the view that production is not an end in itself for people, but rather a necessary intermediate step which allows people to enjoy the outcome of production, then the human capital approach is clearly inappropriate. Increases in safety and life expectancy help to ensure the continuation of an individual's ability to enjoy the pleasures of her life and the continuation of the pleasure which her family and friends derive from their relationship with her, and it is the value of prolonging this enjoyment which should be assessed in measuring the benefit of public programs that affect safety [3].

The assessment of VSL values based on the WTP measures in contrast, reflects the whole range of cost associated with premature death: loss of production (as in the human capital approach), suffering, losses imposed on other family members and society, and all complex attributes associated with human life. These WTP estimates are consequently much higher, on average, than those derived from the human capital approach [68]. For all these reasons the human capital approach is full of methodological and moral problems and is not further considered in this thesis. For a comprehensive discussion on the assessment of the human capital approach it is referred to Schelling [229], whereas Gegax [91] provides a sound overview about critics and historical development of the approach.

What is therefore needed in top down pricing strategies is a method that also takes the "quality of life" of the average individual into account. This is done by means of utility functions that are indicators of the level of satisfaction the average citizen derives from living under particular circumstances. In literature dealing with this aspects [149, 154, 187, 234], economists attempt to rank countries with respect to the quality of life and take this as a maxim for deciding on public policy issues. The quality of life in this context is often defined as a weighted average of specific country statistics or macroeconomic indicators. These include for example income levels per capita, life expectancy, the literacy level, infant mortality rates and so on.

It is without question that these variables may be very important for the utility levels of single individuals, the individual utility levels themselves however are not measured by these statistics. A directly imposed problem then is how to weight the variables against each other. Will quality of life improve when infant mortality rates are lowered by 1% and the income is decreased by 1% as a result of the project cost? Or might this 1% decrease in income could have been better invested to elevate the literacy level of the population? It is clear that when it is not attempted to evaluate these changes on an indivudual basis by means of personal preferences, the weights chosen in the design of the index become the decisive criterion. The problem thus of how to weigh these different variables into a composite quality of life index is a critical issue that is not surprisingly the main source of disputes among researchers [254].

The LQI may be seen as an approach that tackles this task by consistently combining the three macroeconomic indicators GDP, life expectancy and life working fraction into a single index [137]. One of the major contributions of the LQI constitutes the fact that it derives a justifiable rule for the exchange of lifetime and income [136] and can thus conveniently be applied to derive a SWTP for reductions in mortality risk.

5.3 The Life Quality Method

The LQI is a social indicator that has been introduced by Nathwani et al. [181] and is particularly designed to support risk management decisions affecting human safety in social interest. Although the LQI has evolved in many different forms over time, it constantly represents a function of the GDP per capita y as an indicator for personal income, the life expectancy e, and the fraction of lifetime devoted to work a. Based on a chain of economic assumptions and mathematical derivations, the LQI has been extracted to have the following functional form in its more recent versions [193, 198] that are based on the utility concept:

$$LQI = y^q(1-a)e, \qquad \text{where } q = \frac{1}{\beta}\frac{a}{1-a} \tag{5.12}$$

Assuming that a public risk reduction intervention results in a reduction of the mean disposable income and a prolongation of (work free) life expectancy for the average individual, the LQI may be used to judge on the social desirability of the project, summarized in the net benefit criterion.

Throughout its persistence the LQI has been continuously subject to controversial discussions concerning its theoretical foundations and the calibration of the inherent parameters. This resulted in the development of several distinct versions of the index differing in their functional forms as well as in the way of application in accordance with economic theory. In this respect, Rackwitz [212, 215] remarkably expanded the LQI framework and applied it for the first time to derive optimal safety levels for civil engineering infrastructures. In Maes et al. [156] the LQI was employed to optimize the life cycle cost of structures and it has further been applied in cost benefit analyses (CBA) of Canadian air quality standards [194] and nuclear safety design practices [195]. Rackwitz [215] also performed an extensive study of economic cross country data to present an empirical validation of the index. Finally, other interesting contributions in the LQI research field that deserve special attention were put forward by Ditlevson et al. in a series of articles [64, 65, 66]. By focusing the analysis on the relation between work time, leisure time and productivity, Ditlevson converted the conventional LQI to pure units of time, which he in turn named the life quality time allocation index.

In the following, the LQI is analyzed with respect to its economic foundations. Firstly, the traditional LQI derivation approach to arrive at formula (5.12) is presented and its conceptual shortcomings are discussed. In a second step, new time and equilibrium consistent LQI derivations are developed and more extensive acceptability criteria derived. It will become clear, that these are in accordance with the economic theory surrounding the SWTP for safety assembled throughout the preceding chapter. In this way, the implicit assumptions behind the index will become more transparent, as these resulted in some controversial discussion in literature. In this sense, it was put into question whether the LQI represented a normative or an empirical concept [63].

5.3.1 The Traditional LQI Derivation Approach

The original derivation of the LQI presented in [181] was based on the assumption that life quality in a country may be defined as the product of a special income and a particular leisure function. By means of some elegant mathematical derivations based on a differential equation of the LQI and an employment of the labor leisure trade-off, the LQI was deduced in its initial form. It is important to emphasize that the entire derivation was carried out without making use of utility functions. The single steps of the derivation as well as the LQI in its initial form may be reviewed in Appendix E.

Putting into effect in later articles [193] that the initial LQI derivation was less intuitive and did not provide sufficient insight about the assumptions and economic implications inherent in the LQI formula, alternative derivations closely related to utility theory and decision making have been developed with the aim to promote communication to a wider audience. The utility based reformulation of the LQI began in [196] and has been extended in [193, 198], where a more realistic relation between GDP and work time based on the Cobb Douglas production function was introduced. The final two publications are interpreted as the current state of the art and lead to the LQI formula (5.12) that is currently most often applied. Therefore, only the latter formulation constitutes the subject of study in the following. The LQI derivation that is presented subsequently closely follows the schematic steps in [193, 198] and is given little additional interpretation in order to accentuate the followed approach.

Starting point for the LQI derivation is the assumption that a person's enjoyment of life or utility in an economic sense arises from a continuous stream of resources available for consumption over the entire remaining life. Therefore, income required to support consumption and the time to enjoy life are two determinants of life quality. Accordingly, the potential lifetime utility of a person can be interpreted in terms of income utility over the remaining life. Then, a general ordinal utility function of the form

$$L_Q = y^q(1-a)e \tag{5.13}$$

is defined in order to serve this purpose.[1] By assuming that y is a constant over the remaining lifetime, the derivation proceeds by calibrating the index with respect to the parameter q.

The actual calibration of the LQI is carried out by formulating the utility function (5.13) in terms of labor time and then invoking the "labor leisure trade-off", which is broadly considered as the central LQI innovation in LQI literature [151, 217]. This is done by taking the Cobb Douglas GDP production function introduced in (5.3) into account to relate income to (leisure) time. For this purpose, the Cobb Douglas production function has to firstly be transformed in per capita units by dividing it by the total population size N

$$F(\frac{K}{N}, \frac{L}{N}) = A(\frac{K}{N})^{\alpha}(\frac{L}{N})^{\beta} = Ak^{\alpha}a^{\beta} = f(k,l) = y \tag{5.14}$$

to obtain the average income per person under the assumption of constant returns to scale, i.e. $\alpha + \beta = 1$. In equation (5.14) lower case letters denote per capita values and the life working

[1] Note that the LQI derivation starts directly from taking income y as a utility defining parameter as opposed to consumption, which is the conventional approach in economics and also the basis of argumentation in LQI literature before concretely deriving the index.

fraction a comes into play to represent average labor supply in one year. In a next step, equation (5.14) is supplemented in equation (5.13) to obtain:

$$L_Q = (f(k,l))^q (1-a)e = [Ak^\alpha]^q (a^\beta)^q (1-a)e \tag{5.15}$$

Under the assumption of constant per capita capital $\overline{k}$ and technological knowledge A it is easily verified, that the utility function L_Q is now solely defined in terms of labor time, represented by the life working fraction a. Accordingly, under the hypothesis that capital per person and the factor A are independent of the work time fraction a, the leisure labor trade-off is employed, meaning that equation (5.15) is optimized with respect to a. Setting the derivative of L_Q with respect to a equal to zero gives the first order condition that is then taken into account to obtain the previously unknown parameter q:

$$\frac{dL_Q}{da} = 0 \qquad \Rightarrow \qquad q = \frac{1}{\beta}\frac{a}{1-a} \tag{5.16}$$

Inserting this value for q in equation (5.13) eventually yields the final formulation of the LQI as provided in (5.12). The parameter q represents the elasticity of utility with respect to consumption and serves as a measure of trade-off between the utility derived from longevity and consumption [196]. This becomes clear in the application of the net benefit criterion, which is discussed next.

5.3.2 The Net Benefit Criterion

In order to judge on the efficiency of public risk reduction interventions on basis of the LQI, the net benefit criterion [196] is often applied in LQI literature. The net benefit criterion can be interpreted as the general acceptability condition for investments into safety. To obtain this condition, it is assumed that the project under investigation leads to a marginal increase in (leisure) lifetime de due to the enhanced safety levels at constant a and has costs that result in a reduction of income dy equal to the net cost into human safety (NCHS) per capita. To analyze how small changes in life expectancy and income affect the LQI, it is totally differentiated:

$$dLQI = \frac{\partial LQI}{\partial y}dy + \frac{\partial LQI}{\partial e}de \tag{5.17}$$

Now, for the investment to be approved in social interest it is required that the project induced changes in the LQI variables result in an increase in total life quality, i.e. $dLQI \geq 0$. Calculating equation (5.17) concretely for LQI formula (5.12) yields the net benefit criterion for risk reduction in social interest:

$$\frac{dy}{y} + \frac{1}{q}\frac{de}{e} \geq 0 \tag{5.18}$$

Accordingly, a project is considered as socially beneficial, if the relative change in income is compensated by the relative increase in life expectancy weighted by the inverse of the elasticity parameter q, so that the total change in life quality is not negative for the average individual, as illustrated graphically in Figure 5.5. The WTP of the average individual of society for the safety induced change in life expectancy is now easily obtained by setting equation (5.18) equal to zero and thus, determining the threshold change in income that causes total life quality to stay at its initial level:

$$WTP = \frac{\frac{\partial LQI}{\partial e}}{\frac{\partial LQI}{\partial y}} de = \frac{y}{qe} de \tag{5.19}$$

As a consequence, the WTP based on the LQI is equivalent to the compensating variation (CV) introduced in Chapter 4. It represents exactly the amount of money that can be taken away from the average individual of society after a safety induced change in life expectancy, so that she is as well off as before the change. If the WTP is greater than the NCHS of the risk reduction project per capita, the project is approved in social sense. Straightforwardly, the net benefit criterion and the WTP approach always produce similar results and are just two distinct ways of illustration.

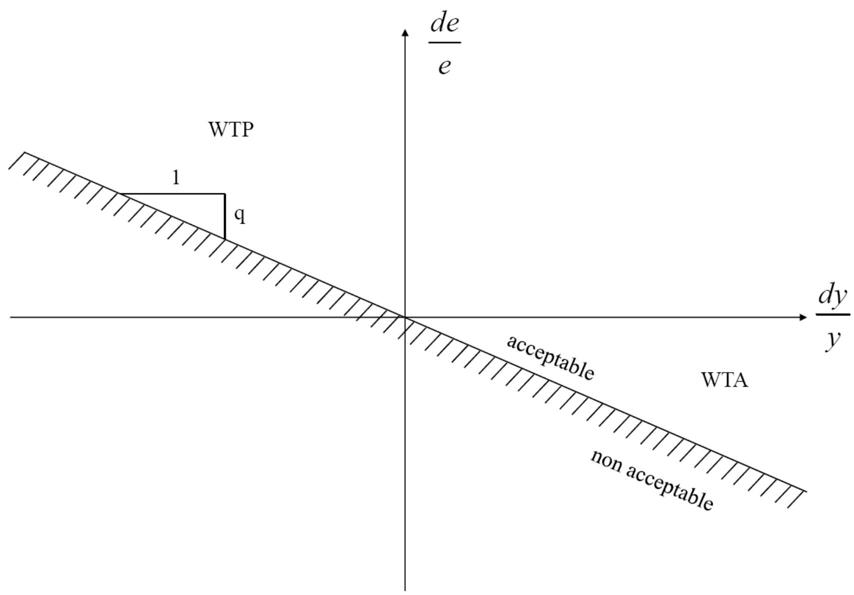


Figure 5.5: The net benefit criterion [Nathwani et al. [181]]

To finally obtain a SWTP for the risk reduction project on aggregate levels, the WTP of the average individual has to be simply multiplied by total population size N:

$$SWTP = WTP \cdot N \tag{5.20}$$

At this point it is important to emphasize, that the WTP formulae (5.19) and (5.20) only hold for a ceteris paribus safety change, leaving all other variables, i.e. income y, constant (see Section 4.3.2). What is not considered however is that, leaving the cost of the project aside for the moment, the pure safety change causes also income to increase. This follows by construction of the life quality model and can be retraced as follows:

The life working fraction parameter a has been optimally determined in the LQI derivation and is assumed to be constant over lifetime. Therefore, an increase in total lifetime de decomposes in an increase in leisure time $(1-a) \cdot de$ and **also** in an increase in labor time $a \cdot de$, proportions being determined by the parameter a. The increase in leisure time is directly valued in the LQI utility function and converted into monetary units by division through the marginal utility of income, as shown in equation (5.19). The increase in labor time however, is not being accounted for. An increase in labor time leads to an increase in total production according to the relation specified in the Cobb Douglas production function that is part of the model. These increases in per capita income in turn bring additional benefits to the individual, which are not valued by the net benefit criterion. Therefore, the concept values only the direct effects of risk reduction in terms of increases in (leisure) longevity. As a consequence, the reversed critics that apply for the human capital approach hold for the LQI: it values additional lifetime only by the pleasure of longevity without taking the productive value of time in the GDP production into account.

5.3.3 Critics

As discussed above, the presented LQI derivation represents the current state of the art and has been developed as a mean to improve the economic foundations of the LQI to promote the concept to a wider audience [193]. Equipped with the economic theory about utility and profit maximization and WTP measurement assembled throughout the preceding chapter however, several questions remain unanswered. In particular, the following aspects are conspicuous in the LQI approach:

- The LQI represents the utility function of a representative consumer that is maximized with respect to the life working fraction. In the maximization process however, the constrained utility maximization problem is never stated explicitly and budget constraints do not appear. Furthermore, the relation between consumption and income (GDP per capita) remains obscure.
- The optimized value for q is inserted in the function L_Q to obtain the LQI, implying that the LQI constitutes a value function. The distinction between direct and indirect utility is not made though.
- The LQI derivation is not time consistent. It contains the annual income, but employs total life expectancy as the other utility defining component. Therefore, it is not obvious at first glance if the LQI methodology relies on an atemporal or dynamical model.[2]

The first two points, i.e. the omittance of clear budget constraints and the non distinction between direct and indirect utility, are closely related and eventually lead to the fact that changes in income due to improved safety levels are not being accounted for in the net benefit criterion. Furthermore, all three objections collectively make it difficult to verify that the LQI concept actually is of general equilibrium nature. In fact, there is no publication within LQI research that is known to the author, where the general equilibrium foundation of the LQI is explicitly stated.

The following sections are designed to approach the above critics. Firstly, the LQI is derived in a static one period general equilibrium setting by clearly stating constrained utility and profit maximization problems and eventually presenting an improved net benefit criterion that allows far reaching interpretations. Here, the economic reasoning is closely related to the representative agent model discussed in Section 4.4, which can be looked upon as the LQI's microfoundation. In contrast to Chapter 4 however, the perspective here is to be integrated in macroeconomic theory. In a second step then, intertemporal aspects are included in the LQI methodology, so that eventually a WTP for safety and a VSL estimate are obtained.

[2] In numerous articles [194, 196, 215, 214, 217] the LQI is explicitly considered in dynamic life cycle consumption models. Here however, the derivation of the LQI is carried out as above, while intertemporal aspects are only added in application (see Appendix E.2).

5.4 New Equilibrium Consistent LQI Derivation in a One Period General Equilibrium Model

It is assumed that the economy consists of N identical consumers that maximize their utility with respect to their budget constraints and only one firm that maximizes its profits subject to the whole economy's output production function, which is of the Cobb Douglas type with constant returns to scale. Furthermore, there are only three markets available in the economy, namely the labor market, the capital market and the goods market, where only composite goods are traded. The influence of the government in terms of taxes is omitted in the model and the economy is closed, i.e. there is no trade with foreign countries. The focus of attention is the short term perspective, implying that the consumer is not able to shift consumption throughout different periods and the firm cannot adjust its capital structure. Therefore, capital is assumed to be constant and the capital market is in equilibrium. As stated in Section 4.2.6 the general equilibrium model is only suitable to define relative prices, so the price for consumption is set arbitrarily equal to one, implying that the composite consumption good acts as a numeraire.

5.4.1 The Representative Consumer

In the above sketched model it can be concentrated on one single representative consumer that freely decides how much work l to supply on the labor market and how much to consume x on the product market. Furthermore, she is endowed with a certain amount of time s, fixed capital $\overline{k}$ and receives profits π as the partial owner of the firm, so that her income is determined with the choice of labor supply. The income y imposes a restriction on her ambition to consume goods and leisure time to maximize her utility. Here, it is important to outline that total time available to the representative consumer is modeled stochastically, equivalent to her total expected survival time in the considered period of one year

$$\begin{aligned} s &:= (365 \cdot 24) \cdot (1-\mu) \quad [\text{ hours }] \\ &:= 1 \cdot (1-\mu) \quad [\text{ years }] \end{aligned} \tag{5.21}$$

where μ denotes the crude mortality rate affecting all consumers uniformly, as they are identical. In a one year time span the expected annual survival time s, measured in years, and the annual survival probability coincide in absolute values and are therefore used interchangeably henceforth. Therefore, the variable s corresponds to the publicly provided safety level in Chapter 4. The total expected annual time s can freely be distributed among labor l and leisure time $\widehat{l}$. The representative consumer takes wages w, rent on capital r and the survival probability s as fixed and exogenous, i.e. outside her control.

Now, the life quality assumption comes firstly into play. It is hypothesized that the consumer derives utility L_Q from the consumption of the composite good x and leisure time $\widehat{l}$, which is at risk and dependent on her expected annual survival time s. The relation between expected labor time $E[l]$, expected leisure time $E[\widehat{l}]$ and expected annual survival time s is modeled conveniently by taking the annual work supply fraction a_s into account:

$$\begin{aligned} E[\widehat{l}] &= s(1 - a_s) \\ E[l] &= sa_s \\ s &= E[\widehat{l}] + E[l] \end{aligned} \tag{5.22}$$

By these conditions and the fact that the annual survival time s is exogenously given, the utility maximization with respect to leisure time can be reduced to maximizing the utility with respect to a_s, as with the work supply fraction a_s labor and leisure time are uniquely determined. Therefore, it is focused on a_s rather than $\widehat{l}$ or l in the following.

According to the life quality approach, the utility function of the representative consumer is assumed to have the following mathematical representation

$$L_Q(x, a_s, s) = E\left[u(x, \widehat{l})\right] = x^q(1 - a_s)s \tag{5.23}$$

where E denotes the expected value operator, accounting for the fact that the enjoyment of utility is uncertain and depending on survival s. The utility function L_Q is closely related to equation (5.13) the LQI derivation departed from, except for the fact that income y is replaced by consumption x and life expectancy e by the expected annual survival time s to make the utility function time consistent. Furthermore, it becomes obvious that the utility function L_Q is a von Neumann Morgenstern [184] expected utility function, that assigns zero utility to the state of non survival, as discussed above. As a consequence, L_Q may be interpreted in terms of ordinal utility, while the subutility function $u(x, \widehat{l})$ must be of cardinal nature in order to be applied in an expected utility framework according to the von Neumann Morgenstern expected utility hypothesis [31].

A restriction to the consumer's utility maximization problem in terms of a budget constraint is now derived. The consumer earns her income y by selling her full annual time endowment s on the labor market, by the rents on fixed capital $\overline{k}$ and by obtaining profits π as a partial owner of the firm. Therefore, total income y amounts to:

$$y = ws + r\overline{k} + \pi \tag{5.24}$$

She spends her income y entirely by consuming goods x on the product market and leisure time $\widehat{l}$, so that total expenditures are given by:

$$y = x + wE[\widehat{l}] \tag{5.25}$$

Here, it has to be noted that the consumer first sells her entire time endowment on the labor market and then repurchases a fraction of this total time in terms of leisure consumption. This approach might seem a little peculiar but is common practice in general equilibrium theory and labor economics when modeling leisure time as an utility argument [243]. Furthermore, by this way of modeling it becomes clear, that the wage rate w in general equilibrium is not only the price obtained for selling labor on the labor market but also the value of leisure in terms of opportunity cost [256]: every hour that the representative consumer spends in leisure activities could have been supplied on the labor market, increasing her income by the respective wage rate. Therefore, whenever making a decision of how much labor to supply and how much leisure to consume, the consumer faces this labor leisure trade-off, which is a central concept of labor economics as discussed for instance in [42, 122].

Subtracting equation (5.25) from equation (5.24) and making use of relation (5.22) yields the final budget constraint the representative consumer faces in the utility maximization process

$$\begin{aligned} x &= wE[l] + r\overline{k} + \pi \\ &= wsa_s + r\overline{k} + \pi \end{aligned} \tag{5.26}$$

which is again formulated in terms of labor time that is at risk. In summary, the representative consumer's life quality maximization problem may be stated as follows:

$$\begin{aligned} \max_{x,a_s} L_Q(x, a_s, s) &= x^q(1-a_s)s \\ \text{s.t. } x &= wsa_s + r\overline{k} + \pi \end{aligned} \tag{5.27}$$

The first order conditions to this constrained maximization problem are easily obtained by means of Lagrange multipliers

$$\frac{\partial \Lambda}{\partial x} = qx^{q-1}(1-a_s)s + \lambda = 0 \tag{5.28}$$

$$\frac{\partial \Lambda}{\partial a_s} = -x^q s - \lambda w s = 0 \tag{5.29}$$

$$\frac{\partial \Lambda}{\partial \lambda} = x - wsa_s - r\overline{k} - \pi = 0 \tag{5.30}$$

where λ represents the Lagrange multiplier and thus, the marginal utility of income. Dividing equation (5.29) by equation (5.28) and rearranging terms yields the marginal rate of substitution (MRS) between the work supply fraction a_s and consumption x:

$$MRS_{a_s - x} = \frac{x}{q(1-a_s)} = sw \tag{5.31}$$

This result highlights the fact that the representative consumer's decision to supply labor on the labor market is influenced both by the persisting wage rate w as well as the annual survival probability s. At higher annual survival probabilities each infinitesimal increase in the annual life working fraction brings more absolute additional labor time on the market than if the survival probability was at a comparatively lower level. Dividing equation (5.31) by the annual survival time s yields

$$MRS_{\widehat{l} - x} = \frac{x}{q(1-a_s)s} = w \tag{5.32}$$

the MRS between leisure time and consumption, showing that at the optimum, the representative consumer trades leisure for consumption at the (real) market wage rate w. In principle, it could be proceeded with solving the life quality maximization problem, but at this point this would produce only unnecessary clutter. After switching to the producer side it will become clear that condition (5.32) is already sufficient to derive the LQI in a general equilibrium setting.

5.4.2 The Representative Firm

When switching to the producer side the steps to be employed are more easy and straightforward than those on the consumer side. Here, it is possible to rely to large extents on the previously obtained results. The only difference to the derivations in Chapter 4.2.5 is that the production function takes the Cobb Douglas form and that the labor component L has to be modeled stochastically. By assumption, there is only one firm that decides how much output Y to produce and how much labor L to hire. In analogy to the consumer side, the decision how much labor to employ may be characterized by a parameter a_d, which represents the fraction of annual time the firm wishes to demand from the average individual. It takes the capital stock $\overline{K}$ as fixed and follows the profit maximization strategy under exogenous wages w, interest rate r and survival probability of the consumers s. The ambition to maximize profits is limited by the production technology captured by a Cobb Douglas production function given through:

$$Y = F(\overline{K}, E[L]) = F(\overline{K}, NE[l]) = F(\overline{K}, Nsa_d) = A\overline{K}^{\alpha}(Nsa_d)^{\beta} \tag{5.33}$$

Here, it is important to outline that the labor component in the production function refers to labor in terms of time, i.e. total hours of employment, and not in terms of number of people employed. As a consequence, an increase in the survival probability s under constant a_d leads to the fact that more labor time enters the production function, which in turn causes total output Y in the economy to rise. Furthermore, it becomes clear under the placed assumptions of constant capital and technology, that the total output of the economy is uniquely determined by the firm's labor demand decision, represented by the parameter a_d.

Dividing equation (5.33) by total population size N and assuming constant returns to scale, the per capita production function is obtained as follows:

$$y = f(\overline{k}, E[l]) = f(\overline{k}, sa_d) = A\overline{k}^{\alpha}(sa_d)^{\beta} \tag{5.34}$$

By means of this per capita production function, the per capita profit is defined as:

$$\pi = \pi(a_d) = f(\overline{k}, sa_d) - wsa_d - r\overline{k} \tag{5.35}$$

Maximizing the per capita profit with respect to a_d yields the following first order condition:

$$\frac{\partial \pi}{\partial a_d} = \beta A\overline{k}^{\alpha}(sa_d)^{\beta-1}s - sw = \beta\frac{y}{sa_d}s - sw = 0 \tag{5.36}$$

Rearranging terms leads to

$$\beta\frac{y}{a_d} = sw \tag{5.37}$$

which shows that also the firm's decision to demand the labor fraction a_d from the representative consumers depends both on the wage rate w and the annual survival time s. This is due to the fact that at higher annual survival times an incremental increase in labor demand a_d brings more absolute labor time in the production process than at comparatively lower survival times. Dividing equation (5.37) by the annual survival time s yields

$$MPL = \beta\frac{y}{sa_d} = w \tag{5.38}$$

which shows that in optimum, the firm employs labor until the MPL is equal to the (real) wage rate. In the given model, the MPL also corresponds to the marginal rate of transformation (MRT) between labor and consumption. Furthermore, as labor employment constitutes the only decision variable of the producer, equation (5.38) already characterizes the firm's optimal decision. Eventually, if labor and capital are always paid their marginal products, at the optimum a zero profit results from the constant returns to scale property of the production function.

5.4.3 General Equilibrium

It has been demonstrated so far that the consumer's optimal decision must satisfy equation (5.32), whereas the producer's optimal decision is characterized by equation (5.38). Now, under perfect competition the wage rate w would adjust to a value w^* at which labor supply a_s equals labor demand a_d for the given survival probability s and the utility and profit maximizing ambitions become accordable. Thus, at the optimal wage rate w^* the MRS of leisure for consumption equals the MRT between labor and consumption, as illustrated previously. Furthermore, as the capital market is in equilibrium by assumption and the optimal wage rate w^* causes also the labor market to clear, it follows from Walras' Law, that the goods market must be in equilibrium as well. As a consequence, a general equilibrium solution is obtained and total output is consumed, so that $x = y$ holds.

Rather than adjusting the wage rate to its optimal value, it has to be kept in mind that the parameter q of the life quality function L_Q has not yet been determined. To calibrate q, the life quality approach therefore implicitly assumes that the considered economy is currently in an equilibrium state where the observed wage rate already constitutes the optimal one and thus $a_s = a_d = a$ holds. Under this premise, the parameter q is determined by equalizing the MRS (5.32) and the MRT (5.38):

$$\frac{x}{q(1-a)s} = \beta \frac{y}{sa} \tag{5.39}$$

By simply rearranging terms and keeping in mind that total output is consumed $x = y$, the parameter q is obtained:

$$q = \frac{1}{\beta} \frac{a}{1-a} \tag{5.40}$$

Inserting this value for q in the utility function L_Q gives the LQI as in equation 5.12, with e substituted by the annual survival time s.

The above shown LQI derivation is being referred to as decentralized equilibrium derivation as the consumer's and firm's problems have been considered separately to highlight the assumptions behind the approach. Now, it is also possible in a general equilibrium setting to integrate the consumer's and producer's problem into the so called Social Planner's Problem (SPP) and determine the parameter q in this framework. By means of the latter it is more convenient to analyze, how changes in the survival probability due to enhanced safety standards affect the total model economy. An important feature of the SPP is that it operates without prices, as price changes do not contribute to welfare increases in a competitive general equilibrium setting, as demonstrated in Section 4.4.2.

5.4.4 Social Planner's Problem to Maximize Life Quality

In macroeconomic general equilibrium models the SPP is well-known and widely applied [2]. Here, the representative consumer's utility maximization problem and the firm's profit maximizing problem are unified in one single constrained optimization problem, being representative for the whole economy. Accordingly, it is often being referred to as centralized allocation problem. In this model, a fictitious and benevolent social planner is introduced, that maximizes consumer utility by choosing consumption and labor on their behalf subject to the aggregate resource constraint, given by the production function. Among all possible solutions the social planner searches for the best possible allocation, which is Pareto optimal. Furthermore, he does not follow prices, but he understands opportunity cost [266]. Starting again from the direct utility function L_Q under the new notation, the SPP for the life quality maximization is formulated as:

$$\begin{aligned} &\max_{x,a} L_Q(x,a,s) = x^q(1-a)s \\ &\qquad \text{s.t. } x = f(\bar{k}, sa) = A\bar{k}^{\alpha}(sa)^{\beta} = y \end{aligned} \tag{5.41}$$

As the economy as a whole is considered in the SPP, the previously introduced distinction between a_s and a_d becomes superfluous so that solely a is employed in the formulation of (5.41). The solution to this problem gives the best feasible bundle of consumption and leisure that is attainable in the model economy and is therefore Pareto optimal. The SPP may now be solved either by directly imposing the constraint through the insertion of the production function in the objective function, as done by the LQI derivation approach in Section 5.3.1, or by Lagrange optimization. As the use of Lagrange multipliers more evidently shows that the solution to the SPP is absolutely identical to the decentralized equilibrium solution obtained in the last section, it is briefly discussed subsequently. The Lagrange function to (5.41) is constructed to be

$$\Lambda = L_Q + \lambda(f(\bar{k}, sa) - x) \tag{5.42}$$

so that the following first order conditions are obtained:

$$\frac{\partial \Lambda}{\partial x} = \frac{\partial L_Q}{\partial x} - \lambda \qquad = qx^{q-1}(1-a)s - \lambda = 0 \tag{5.43}$$

$$\frac{\partial \Lambda}{\partial a} = \frac{\partial L_Q}{\partial a} + \lambda\frac{\partial f}{\partial a} \quad = -x^q s + \lambda\beta\frac{y}{sa}s = 0 \tag{5.44}$$

$$\frac{\partial \Lambda}{\partial \lambda} = x - A\bar{k}^{\alpha}(sa)^{\beta} = 0 \tag{5.45}$$

Dividing equation (5.44) by equation (5.43) yields

$$\frac{x}{q(1-a)} = \beta\frac{y}{a} \tag{5.46}$$

which is identical to the general equilibrium solution (5.39) after dividing it by s. Finally, by making use of the constraint to the social planner's problem, which enforces that $x = y$, equation (5.46) can be solved for q to obtain the form of the LQI as stated in (5.12), with e replaced by its annual counterpart s.

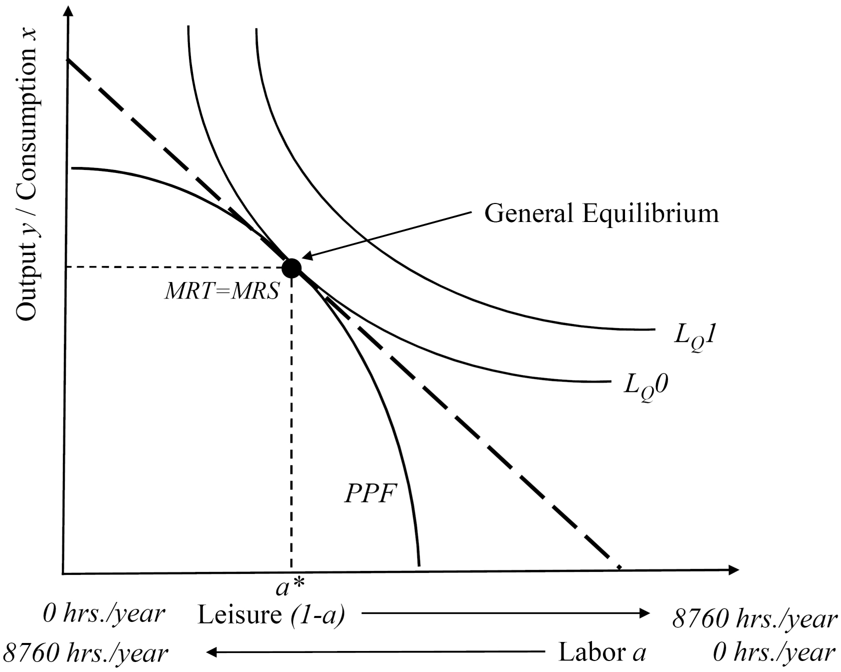


Figure 5.6: Solution to the SPP

To sum up, the LQI is the utility function of a representative consumer that is calibrated with respect to the general equilibrium. Either a decentralized or a centralized approach can be followed to obtain the LQI in its final form. In the LQI derivation known in literature, the function L_Q constitutes the objective function of a SPP that is restricted by the economy's resource constraint, represented by the GDP production function. The problem is solved by directly imposing the constraint. The solution of the problem yields the optimal trade-off between labor and leisure a^*, by equalizing the MRS of leisure for consumption to the MRT between labor and consumption, as visualized in Figure 5.6. Accordingly, the general equilibrium solution is allocated at the point where the economy's production possibilities frontier (PPF)[3] is just tangent to the highest reachable indifference curve of the representative consumer's utility function L_Q. Consequently, as the LQI represents the maximized value of L_Q

$$LQI = LQI(y, s) := \max_{x,a} \left\{ L_Q(x, a, s) : x = f(\bar{k}, sa) = y \right\} \tag{5.47}$$

it constitutes a value function in analogy to the indirect utility functions introduced in Chapter 4.

5.5 New Annualized Net Benefit Criterion

With the annualized version of the LQI at hand and equipped with the new insights on the economic foundations of the LQI, the net benefit criterion is reconsidered in this section. The LQI's constrained maximization problem and in particular the restrictions to the problem have clearly been stated. Furthermore, it has been emphasized that the LQI constitutes a value function that yields the optimal L_Q for a given income and safety level. Therefore, it can be conveniently applied to perform

[3]Represents a graph depicting the maximum amount of output that can be produced out of a given amount of input, given society's production technology.

comparative statics and analyze, how life quality changes when the safety level is elevated and the annual survival probability increased due to the implementation of a public risk reduction project.

The WTP for a marginal increase in survival probability from s_0 to s_1 formulated terms of the CV measure introduced in Section 4.3.1 must satisfy:

$$LQI(y - WTP, s_1) = LQI(y, s_0) \tag{5.48}$$

To arrive at an explicit formulation for the WTP, it is proceeded as in Chapter 4 and the LQI is totally differentiated:

$$dLQI = \frac{\partial LQI}{\partial y}dy + \frac{\partial LQI}{\partial s}ds \tag{5.49}$$

Now, by keeping in mind that the consumer's income $y = f(\bar{k}, sa)$ is in particular a function of the expected annual survival time s, the following expression for the income change in response to the safety change is obtained:

$$dy = \frac{\partial f(\bar{k}, sa)}{\partial s}ds \tag{5.50}$$

Inserting equation (5.50) in equation (5.49) yields

$$\begin{aligned} dLQI &= \left(\frac{\partial LQI}{\partial y} \cdot \frac{\partial f(\bar{k}, sa)}{\partial s} + \frac{\partial LQI}{\partial s}\right)ds & (5.51) \\ &= (qy^{q-1}(1-a)s \cdot \beta\frac{y}{sa}a + y^q(1-a))ds & (5.52) \end{aligned}$$

which represents the change in life quality utility resulting from an incremental increase in the publicly provided safety level ds. Dividing this expression by the marginal utility of income $\lambda = \frac{\partial LQI}{\partial y}$ to convert the change in utility to monetary units yields:

$$\frac{dLQI}{\lambda} = \left(\frac{\partial f}{\partial s} + \frac{\frac{\partial LQI}{\partial s}}{\frac{\partial LQI}{\partial y}}\right)ds \tag{5.53}$$

This amount corresponds to the monetized change in LQI utility resulting from the considered safety change ds. If the representative consumer's income is reduced by this amount after the safety change, she maintains exactly her initial level of life quality. Consequently, expression (5.53) already represents the consumer's WTP for the safety change ds. Note that WTP formula (5.53) coincides with the safety pricing rule of the representative agent model discussed in Section 4.4. Solving equation (5.53) concretely gives:

$$WTP = (\beta\frac{y}{sa}a + \frac{y}{qs})ds \tag{5.54}$$

Now, by keeping in mind that $q = \frac{1}{\beta}\frac{a}{1-a}$ and by definition $E[l] = sa$ represents the expected annual labor time of the representative consumer, equation (5.54) can slightly be rearranged to yield a surprising economic interpretation:

$$\begin{aligned} WTP &= (\beta\frac{y}{sa}a + \frac{y}{s}\beta\frac{1-a}{a})ds && (5.55) \\ &= (\beta\frac{y}{E[l]}a + \beta\frac{y}{E[l]}(1-a))ds && (5.56) \\ &= (MPL \cdot a + MPL \cdot (1-a))ds && (5.57) \\ &= MPL \cdot ds && (5.58) \end{aligned}$$

The WTP for the marginal safety change obtained from the annualized LQI equals the total increase in expected annual survival time ds valued at the MPL. The representative consumer directly benefits from the increase in expected leisure time $(1-a) \cdot ds$, which is valued at its opportunity cost equal to the MPL. In addition, as a represents a constant that has already been optimized, the increased expected annual survival time causes also consumer income to increase. This is due to the fact that at constant a an increase in total expected survival time leads to an increased expected labor input $a \cdot ds$ in production. This in turn causes total output and thus, GDP to increase by the MPL times the increased labor time. To sum up, the increase in safety leads to an increase in total expected time endowments of the individuals of society which is valued at the MPL.

Accordingly it becomes clear in particular, that the LQI values increases in safety in terms of increased expected survival time. As the WTP formula given in equation (5.55) has been derived in an one period model however, it values only the extension in survival time that accrues in the considered period: if m lives are saved in society due to the implementation of a public risk reduction project, the mortality rate is decreased by m mortality risk reduction units $m \cdot \frac{1}{N}$, so that each individual enjoys a time gain of

$$\begin{aligned} ds &= s_1 - s_0 = 8760(1-\mu_1) - 8760(1-\mu_0) \\ &= 8760(1 - \frac{M-m}{N}) - 8760(1 - \frac{M}{N}) = 8760 \cdot \frac{m}{N} \qquad [\frac{\text{hours}}{\text{year}}] \end{aligned} \qquad (5.59)$$

in the considered period. As above, M refers to the total number of expected fatalities in the considered time horizon of one year prior to the implementation of the risk reduction project and 8760 is the total number of hours in one year. Therefore, the WTP for the current time gain of each (identical) individual of society can be reformulated to be

$$WTP = (\frac{\partial f}{\partial s} + \frac{\frac{\partial LQI}{\partial s}}{\frac{\partial LQI}{\partial y}}) \cdot 8760 \cdot \frac{m}{N} \qquad (5.60)$$

leading to the following expression for the SWTP:

$$SWTP = WTP \cdot N = (\frac{\partial f}{\partial s} + \frac{\frac{\partial LQI}{\partial s}}{\frac{\partial LQI}{\partial y}}) \cdot 8760 \cdot m \qquad (5.61)$$

Now, by either dividing the individual WTP formula (5.60) by the size of the mortality risk reduction $d\mu$ or the SWTP formula (5.61) by the total number of lives saved $dM = m$, the social value of one statistical life year (VSLY) is obtained:

$$VSLY = \frac{WTP}{d\mu} = \frac{SWTP}{m} = \left(\frac{\partial f}{\partial s} + \frac{\frac{\partial LQI}{\partial s}}{\frac{\partial LQI}{\partial y}}\right) \cdot 8760 \tag{5.62}$$

Table 5.2 provides a collection of VSLYs for selected economies. The VSLY estimates cluster in the range US$ 150,000-250,000 for industrialized western economies with the outliers Luxembourg that shows the highest value of more than US$ 0.5 million and Mexico that is allocated at the lower end of the estimates with a VSLY of only US$ 50,000. Furthermore, it is clearly seen that beside the GDP, the most influential parameters on the VSLY of a country are the life working fraction a and the parameter β that all together determine the labor productivity in a country and thus the MPL.

Table 5.2: VSLY estimates from the annualized LQI

Country	**GDP**[1]	β[2]	**a**[3]	**s**[4]	**VSLY**[5]
France	32,686	0.52	7.7	0.991	220,375
Germany	34,391	0.53	8.1	0.989	227,475
Italy	30,381	0.47	8.7	0.989	165,827
Luxembourg	79,793	0.51	7.3	0.992	555,375
Poland	15,989	0.45	10.2	0.990	71,697
Portugal	22,815	0.45	10.4	0.989	100,737
UK	35,669	0.53	9.6	0.990	199,612
Canada	38,500	0.52	10.6	0.992	191,863
USA	45,489	0.57	10.6	0.992	248,863
Australia	37,565	0.49	9.7	0.993	190,070
New Zealand	27,431	0.55	10.4	0.993	146,356
Japan	33,626	0.57	10.6	0.991	180,956
Mexico	14,004	0.31	8.6	0.995	50,346

[1] (US$ 2007), from OECD [188]
[2] from PennWorldTables [110, 159]
[3] [%], from OECD [188] and WDI [262], after formula (5.11)
[4] from CIA World Factbook [49], after formula (5.21)
[5] after formula (5.62)

Now, in order to take into account the full benefits in terms of increased survival time that result from a reduction in crude mortality, it has to be emphasized that a reduction in crude mortality today has an impact not only on the survival time in the considered time span of one year, but affects all consecutive survival probabilities over the representative consumer's remaining life. This is due to the fact that the survival probabilities $S(t)$ in all future years t are conditional on survival today, as becomes clear from formula (5.7). Consequently, in order to arrive at a VSL measure on social level, these increases in future survival times have to be assessed and included in the valuation. This is subject of the next section.

5.6 Intertemporal Extensions

The safety pricing rule derived from the LQI constitutes a measure to place a value on increased survival time. Whereas in the one period model, the increase in annual survival time is based on the change in the crude mortality rate and equal across all (identical) individuals, in the lifetime evaluation further refinements are necessary in order to measure the total time gain over the average individual's life. These refinements address in particular the fact that people of different ages have different remaining life expectancies, so that a change in the mortality rate today will impact the gain in total lifetime to different extents depending on age. Moreover are individuals of different ages exposed to age specific mortality rates $\mu(x)$ rather than to a uniform crude mortality rate μ, which represents the average of all age specific mortality rates.

5.6.1 Mortality Risk Reduction Regimes

In order to quantify the total time gains that individuals of society enjoy from a change in the mortality rate, it is first necessary to specify the so called mortality risk reduction regime [218], which defines the way changes in crude mortality distribute across the age specific mortality rates $\mu(x)$. There are generally two standard cases how this may be accomplished: the proportional or relative risk reduction regime and the uniform or absolute mortality risk reduction regime. When considering changes in age specific mortality due to the implementation of some specific risk reduction measure, the general all cause level of mortality a subgroup of the population is exposed to at age x is commonly being referred to as background mortality $\mu(x)$ [151].

The relative mortality risk reduction regime is based on the assumption that a change in crude mortality $d\mu = \delta\mu$ impacts individuals of different age classes proportional to their respective background mortality

$$\mu_1(x) = \mu_0(x)(1+\delta) \qquad \text{for all } x \tag{5.63}$$

where subscript 0 refers to the background mortality before the implementation of the risk reduction measure and 1 describes the respective value in post project state. This criterion implies that people who are subjected to a higher background risk are more affected by the mortality change δ. As background mortality is in tendency increasing with age, this mortality risk reduction scheme assigns the major benefits of risk reduction to people of advanced ages. It might describe reality well in case of disease prevention and the reduction of air pollution, but is generally considered to be incompatible with the equality principle of modern societies [215], which dictates that a distinction between different age groups is not feasible in the derivation of a VSL.

By applying the uniform mortality risk reduction regime, it is assumed that an infinitesimal change in crude mortality $d\mu = \Delta$ distributes equally as a constant throughout all ages. Therefore, the mortality change has an impact on every member of society regardless of her age and background mortality so that

$$\mu_1(x) = \mu_0(x) + \Delta \qquad \text{for all } x \tag{5.64}$$

is obtained for the change in age specific mortality, where subscripts are defined as above. This mortality risk reduction regime is of particular relevance for technical applications, such as structural reliability, flood protection, earthquake resistant design and disaster risk reduction interventions in general [214]. If for instance a structure collapses, every occupant inside faces more or less the same risk of dying, independent of actual background mortality, age and sex. Certainly, the mobility of affected persons, which is generally higher at low and medium ages, will also influence the risk of dying in a disaster in reality, but the regime is nevertheless assumed to hold in good approximation. Furthermore, it is implicitly hypothesized, that the subgroup of affected people is distributed representatively, in accordance with the age distribution of the entire society. Also this assumption might be criticized, but the democratic equality principle prohibits a distinction between different age groups and enforces a valuation on basis of average values [215]. For all those reasons, the absolute mortality risk reduction regime is adopted henceforth.

5.6.2 Converting Changes in Mortality to Changes in Life Expectancy

After the mortality risk reduction regime has been specified, it is now analyzed, how changes in mortality risk translate into changes in total lifetime for the average individual of society. It has been outlined above that a change in mortality risk $\mu(x)$ today has an impact on all future survival probabilities $S(t), t > x$ the individual faces over her entire remaining life, as they all are conditional on survival today. If a person of age x is considered, the survival probability to survive until at least period t is conditional on survival to age x and can be expressed as

$$S(t|x) = \frac{S(t)}{S(x)} = \frac{1}{S(x)} \exp\left[-\int_0^t \mu(\tau)d\tau\right] = \exp\left[-\int_x^t \mu(\tau)d\tau\right] \tag{5.65}$$

making the dependency of the survival probabilities on $\mu(x)$ obvious. Now, as it is aimed at assessing all changes in future survival probabilities due to the mortality change today, the remaining life expectancy at age x is the proper measure to be employed as it subsumes all future survival probabilities over the individual's remaining life

$$e(x,\mu) := \int_x^{t_{max}} S(t|x)dt = \int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)d\tau\right] dt \tag{5.66}$$

where t_{max} represents some maximum attainable age taken from life table records and is in the range of 110 years. By formula (5.66) the relationship between mortality rates, survival probabilities and remaining life expectancy at an arbitrary age x is clarified. In particular, it is easily verified that a change in mortality today has a larger impact the lower the age of the considered individual, as there are more remaining increased survival probability terms to be summed until the maximum attainable age is reached, which in turn results in a larger gain in total life expectancy. Now, in order to make these gains in life expectancies time consistent and to enable a valuation on basis of the VSLY measure introduced above, the additional time increments that occur in the future have to be discounted back to the time of valuation. This can conveniently be done on basis of the so called discounted life expectancy $e_d(x)$[4]

$$\begin{aligned} e_d(x,\mu,r) &:= \int_x^{t_{max}} S(t|x) \exp\left[-\int_x^t r(\tau - x)d\tau\right] dt \\ &= \int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau) + r(\tau - x)d\tau\right] dt \\ &= \int_x^{t_{max}} S_d(t|x)dt \end{aligned} \tag{5.67}$$

[4]Note that in equation (5.67) the mortality rate $\mu(\tau)$ is dependent on the considered individual's age, while the discount factor is age independent and refers to the time the considered individual aged x will be τ years old, so that the factor $r(\tau - x)$ is to be applied to discount the survival probabilities.

where r denotes a suitably chosen discount rate that is further discussed below. In essence, the discounted life expectancy e_d at age x is a measure of the individual's total remaining disposable time adjusted for the individual's time preferences and economic growth. It allows for an interpretation in terms of the net present value (NPV) of total remaining lifetime and takes on values of around 22-24 years if evaluated for $x = 0$ at a discount rate of 4% following the typical age structure of industrialized countries. Now, the discounted life expectancy is taken into account to value the total time gain an individual of age x derives from a reduction in mortality risk today. If the vector

$$\mu_0 = (\mu(x), \mu(x+1), ..., \mu(t_{max} - 1)) \tag{5.68}$$

denotes the collection of age specific mortality rates prior to the risk reduction intervention and

$$\mu_1 = (\mu(x) + \Delta, \mu(x+1), ..., \mu(t_{max} - 1)) \tag{5.69}$$

represents the vector of age specific mortality rates the considered x-year old individual faces after project implementation over her remaining life, the total gain in discounted life expectancy at age x amounts to:

$$de_d(x, \Delta, r) = e_d(x, \mu_1, r) - e_d(x, \mu_0, r) \tag{5.70}$$

This difference represents the present value of the total expected time gain the individual derives from an immediate mortality risk reduction over her remaining life at age x and is measured in years. As a consequence, the WTP of an individual at age x for an instant mortality risk reduction $d\mu = \Delta$ can be valued by means of the VSLY measure derived above:

$$WTP(x) = VSLY \cdot de_d(x, \Delta, r) \tag{5.71}$$

As the total population consists of individuals of all age classes, the age specific WTP measures are to be averaged over the age distribution of the total population in order to arrive at a measure that represents the WTP of the average individual of society. For a constant population growth n over the considered time horizon, individuals are distributed over different age classes by the function

$$h(x, n) := \frac{\exp\left[-nx\right] S(x)}{\int_0^{t_max} \exp\left[-nx\right] S(x) dx} \tag{5.72}$$

so that the average individual's WTP for the considered reduction in mortality risk becomes:

$$WTP = \int_0^{t_max} WTP(x) h(x, n) dx \tag{5.73}$$

The SWTP is then easily calculated by multiplying the age averaged WTP by total population size N:

$$SWTP = WTP \cdot N \tag{5.74}$$

Eventually, the VSL comprises all the above information and standardizes the WTP measures to the value of saving one statistical life in society:

$$VSL = \frac{WTP}{d\mu} = \frac{SWTP}{dM} \tag{5.75}$$

As the above described procedure to derive the VSL from the age specific WTP values constitutes a rather time consuming and elaborate procedure, a possible simplification based on a linear approximation is introduced before closing this section. Reconsidering formula (5.71) it becomes obvious that the only variable factor in the age specific WTP is given by the gain in discounted remaining lifetime $de_d(x, \Delta, r)$, which is in particular depending on the age of the considered individual. Therefore, in the derivation of a VSL on social level it is possible to abstract from age specific individual considerations by introducing the concept of age averaged discounted life expectancy

$$\begin{aligned}\widetilde{e}_d := E_h\left[e_d(x,\mu,r)\right] &= \int_0^{t_{max}} e_d(x,\mu,r)h(x,n)dx = \\ &= \int_0^{t_{max}}\int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)+r(\tau-x)d\tau\right] dth(x,n)dx\end{aligned} \tag{5.76}$$

which constitutes the population average of all age dependent discounted life expectancies $e_d(x)$ over all existing age classes. It is strongly dependent on the age distribution of the considered population as well as the discount factor chosen to determine the discounted life expectancies. Values of around $\widetilde{e}_d = 16 - 18$ years are obtained for typical demographies of industrialized countries, when applying a discount rate of 4%. By means of the age averaged discounted life expectancy $\widetilde{e}_d$ changes in discounted life expectancy due to the mortality change under investigation can directly be assessed on social level. In this context, Rackwitz [214, 215, 217] proposes a linearization of the relationship between the change in mortality risk $d\mu = \Delta$ across all age classes and the change in age averaged discounted life expectancy $d\widetilde{e}_d$ by means of the first term of a McLaurin series:

$$\begin{aligned}E_h\left[\frac{de_d(x,\Delta,r)}{e_d(x,\mu,r)}\right] &\approx \int_0^{t_{max}} \frac{-\frac{d}{d\Delta}e_d(x,\Delta,r)|_{\Delta=0}\cdot\Delta}{e_d(x,\mu,r)}h(x,n)dx \\ &= \int_0^{t_{max}} \frac{-\frac{d}{d\Delta}\int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)+\Delta+r(\tau-x)d\tau\right] dt|_{\Delta=0}\cdot\Delta}{\int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)+r(\tau-x)d\tau\right] dt}h(x,n)dx \\ &= -\int_0^{t_{max}} \frac{\int_x^{t_{max}}(t-x)\exp\left[-\int_x^t \mu(\tau)+r(\tau-x)d\tau\right] dt}{\int_x^{t_{max}} \exp\left[-\int_x^t \mu(\tau)+r(\tau-x)d\tau\right] dt}h(x,n)dx\cdot\Delta \\ &= -C_\Delta\cdot\Delta\end{aligned} \tag{5.77}$$

Clearly, such an approximation is only permissible for reasonably small changes in mortality risk, which constitute the subject of study throughout this thesis. It has to be noted however, that in the derivation of approximation (5.77) a simultaneous change throughout all age specific mortality rates has been considered, so that an additional division by $e_d(x, \mu, r)$ is necessary to obtain the correct relation between Δ and $d\widetilde{e}_d$. Furthermore, it is easily verified that the factor C_Δ obtained in (5.77) is a constant that is specific for the demography of the considered country and represents a measure for the shape of the survival curve $S(x)$ [212]. For the typical age structure of industrialized countries and a discount factor of 4% it takes values in the range of 15-18 and equals roughly the total age averaged discounted life expectancy $\widetilde{e}_d$. The convenience of approximation (5.77) is to be seen in the fact that once the demographic factor C_Δ has been calculated, it can repeatedly be applied in WTP and VSL calculations as the demographic structure of society is not expected to change significantly in the short to medium term perspective.

Finally, the age independent WTP of the average individual of society for the considered uniform mortality risk reduction change $d\mu = \Delta$ is easily obtained

$$WTP = VSLY \cdot d\widetilde{e}_d = VSLY \cdot C_\Delta \cdot \Delta \tag{5.78}$$

which can in turn straightforwardly be employed to calculate the VSL in the considered society:

$$VSL = \frac{WTP}{d\mu} = VSLY \cdot C_\Delta \tag{5.79}$$

The so obtained VSL calculation formula is in line with the VSL estimation approach suggested in [94]. Table 5.3 eventually provides an overview of VSL estimates for the above chosen economies.

Country	**GDP**[1]	e^2	**a**[3]	γ^4	**r**[5]	**n**[6]	$C_\Delta{}^7$	**VSL**[8]
France	32,686	80.9	7.7	1.9	4.6	0.55	16.2	3,575,293
Germany	34,391	79.9	8.1	1.9	4.6	-0.05	15.6	3,556,438
Italy	30,381	81.6	8.7	2.0	4.6	-0.05	15.7	2,610,904
Luxembourg	79,793	79.4	7.3	2.3	4.9	1.17	15.9	8,809,751
Poland	15,989	75.4	10.2	1.6	4.2	-0.05	16.0	1,149,663
Portugal	22,815	79.1	10.4	2.1	4.6	0.28	15.9	1,602,856
UK	35,669	79.5	9.6	1.3	4.0	0.28	17.0	3,385,692
Canada	38,500	80.4	10.6	2.0	4.5	0.82	16.5	3,162,347
USA	45,489	78.2	10.6	1.8	4.4	0.98	16.7	4,156,087
Australia	37,565	81.7	9.7	2.0	4.6	1.2	16.8	3,198,041
New Zealand	27,431	79.4	10.4	1.2	3.9	0.94	17.7	2,594,928
Japan	33,626	82.9	10.6	2.7	5.1	-0.19	14.8	2,676,402
Mexico	14,004	76.06	8.6	1.4	4.0	1.13	17.9	901,193

Table 5.3: VSL estimates from the annualized LQI

[1] (US$ 2007), from OECD [188]
[2] taken from period lifetables from the Human Mortality Database [267]
[3] [%], from OECD [188] and WDI [262]
[4] [%], from Maddison [155] and extended Penn Worldtables [159]
[5] [%], after formula (5.84) below, with ρ=3% and $\epsilon = 1 - q$
[6] [%], from CIA Worldfactbook [49]
[7] after formula (5.77)
[8] (US$ 2007), after formula (5.79)

It is clearly seen from Table 5.3 that the VSL estimates derived from the annualized LQI are in general significantly lower than those obtained from the microeconomic approaches introduced in Chapter 4. The mean VSL obtained from the LQI across all selected countries is US$ 3,183,046 with a standard deviation of US$ 1,951,709, while the mean value throughout all microeconomic studies clustered in the range of US$ 5-6 million. If it is focused only on those countries instead, for which microeconomic data have been collected in Chapter 4, e.g U.S., Canada and Western

Europe, the mean VSL from the LQI amounts to US$ 4,179,502, which is still at the lower end of the microeconomic studies, but in the same order of magnitude however. The reason for these differences might be seen in the fact that in most microeconomic studies, respondents are directly faced with the risk of death and asked to state the amount of money they would require as compensation for changes in the risk level. Apart from the broadly observed difficulty to judge on small changes in mortality risk correctly, people not only fear the event of death per se, but also the dreadful process of dying and are potentially willing to pay large amounts of money to avoid such events. In the LQI methodology in contrast, the trade-off between risk and money is determined in a less emotional context [151]. By observing the daily choices that people make to allocate their time between work and leisure activities, a social value of time is derived, which is eventually transferred to evaluate highly emotional decisions, involving the prospect of death.

5.6.3 Relation to Conventional LQI Approach

The above described procedure to determine a VSL based on the LQI methodology differs significantly from the conventional approaches presented in literature. Here, the LQI has been derived firstly in a one period general equilibrium model, where the life expectancy term has been replaced by the annual survival probability to make the derivation time consistent. Based on this approach an annualized version of the net benefit criterion has been obtained that could be employed to derive a VSLY, which constitutes a standardized measure of the average individual's WTP for increased annual survival time and may be seen as the annual counterpart of the VSL.

By the insight that the LQI based acceptability criterion values increased safety in terms of time, the safety pricing rule obtained in the one period model has been applied to value changes in total lifetime that accrue to the average individual as a result of an instant reduction in mortality risk. By assessing the total time gains over the individual's life and discounting them back to the decision point, the VSLY could be employed to derive a VSL on social level. As a consequence, the LQI derivation procedure and the valuation of intertemporal safety effects have been split.

This is to be seen in sharp contrast to conventional LQI approaches to estimate the VSL. Here, the LQI is firstly derived as illustrated above to calibrate the parameter q and then somehow integrated in a lifetime utility model to account for intertemporal aspects, as shown in detail in [217] and summarized in Appendix E.2. In the lifetime utility approach however, it is started from a general unspecified power utility function that contains consumption as its sole argument which is then optimized to maximize lifetime utility:

$$\max_{c(t)} L(x) = \int_x^{t_max} u(c(t)) S_d(t|x) dt \tag{5.80}$$

The relation of this utility function $u(c(t))$ to life quality utility remains unclear though. After a constant consumption $c(t) = c$ over lifetime has been declared to be the optimal consumption path over lifetime, the time invariant utility function is then taken out of the integral to obtain:

$$L(x) = u(c) e_d(x) \tag{5.81}$$

Finally, the parameter q of the LQI derivation is substituted for the so far unspecified exponent of the general power utility function $u(c)$, consumption c is replaced by income y and the function $L(x)$ is age averaged over the age distribution of society $h(x, n)$ to obtain the societal LQI (SLQI):

$$SLQI = u(c)\widetilde{e}_d = y^q \widetilde{e}_d \tag{5.82}$$

The latter formula (5.82) is then taken into account to determine the WTP for safety after equation (5.19):

$$WTP = \frac{y}{q} \cdot E_h \left[\frac{de_d}{e_d} \right] = \frac{y}{q} \cdot C_\Delta \cdot \Delta \tag{5.83}$$

which closely resembles the WTP equation (5.78) derived above. Essentially, the two formulae are closely related, the crucial difference is to be seen in the fact that formula (5.78) contains the additional second order income effect in the VSLY measure, which is neglected in (5.83), as discussed above. As a consequence, the newly derived WTP equation (5.78) yields estimates that are in the range of 10% higher than those obtained by the conventional formula (5.83) for the same size of risk reduction Δ.

As observed for the one period case, also in the above sketched lifetime utility model, budget constraints are not explicitly considered in the optimization process. It is simply stated that under perfect market conditions, a constant consumption rate can be shown to be the optimal path, as demonstrated by Shepard and Zeckhauser [235, 236]. This however holds only if the rate of time preferences with which utilities are discounted equals the employed interest rate and if perfect and fair life insurance is available [127]. The assumption of fair life insurance in turn makes it necessary to include the survival probabilities in the budget constraint, so that it follows from the dynamic envelope theorem that changes in the budget constraint would have to be accounted for as well in evaluating increases in safety, as shown for instance in [6, 41, 132, 224]. These additional changes in the optimal consumption path are not considered in the intertemporal WTP criterion (5.83).

Furthermore, it remains questionable if the parameter q that has been calibrated with respect to the general equilibrium in a one period model may simply be substituted to define the elasticity of an instantaneous utility function that is employed in a lifecycle model. In this respect, it has to be emphasized that while the utility exponent q defines the trade-off between income and safety in a one period model and allows for an interpretation in terms of a risk aversion parameter, in an intertemporal model it determines the intertemporal rate of substitution between consumption today and consumption in the future [224]. This issue certainly requires additional research to be resolved.

As the LQI was originally meant to represent "a snapshot of the current economy" [197], the above illustrated methodology to derive the LQI in a one period model and then to discount additional lifetime back to the time of evaluation seems to go much better in line with this kind of thought than to consider the LQI in an intertemporal lifecycle model of consumption without really accounting for intertemporal aspects. In this way, rather complex issues of dynamic optimization for determining the optimal consumption program and capital accumulation over time are avoidable and at the same time an economically consistent derivation and application assured.

5.7 Calibration

In LQI literature it is continuously discussed how to calibrate the parameters inherent in the LQI formula as well as the social discount rate to account for time preferences in order to guarantee an economically consistent application. As the new derivation approach presented above has shed some new light on the LQI concept, several controversial aspects are readopted in the following and given some new interpretation.

5.7.1 GDP

When the net benefit criterion is applied to derive a SWTP for safety changes it is frequently discussed whether to employ the full GDP per capita in the formula or just that part of the GDP that is available for consumption, which amounts to roughly 60% of total GDP. Authors [151, 214, 217] who recommend using only 60% of the GDP base their arguments on the fact that only the private consumption fraction of the GDP is actually available to be spent for risk reduction activities, as the other parts like investment and government spending are necessary to maintain the functionality of the economy and are therefore to be excluded from the consideration. From the author's point of view in contrast, the full GDP should be taken into account for two main reasons. Firstly and most importantly, as demonstrated in the derivation of the modified net benefit criterion, it has become clear that the LQI values increased safety in terms of time, which is priced at the MPL. If only 60% of the GDP was taken into account in the WTP formula (5.54), the productivity of labor would be underestimated which in turn would result in a biased estimate of the MPL and therefore of the money value of time. Secondly, in special application to disaster risk reduction interventions it can hardly be stated in general who takes over the cost of the project eventually. This is to be determined case specifically and therefore, potentially all parts of the GDP might be affected by the project.

5.7.2 Life Working Fraction

In the estimation of the life working fraction a it is often hypothesized whether to count housework or commuting time to the workplace as labor time. Beside the fact that it would be rather difficult to obtain reliable and objective estimates for these time spans on aggregate level, it has to be emphasized, that the estimate of the parameter a must be in accordance with the labor values that enter the Cobb Douglas production function. As commuting time and housework are not included in national accounting, they do not count as GDP producing activities and are therefore to be excluded from the valuation. Furthermore, by keeping in mind that the above derived procedure to value safety is based on "a snapshot of the current economy", formula (5.11) is clearly advantageous over formula (5.10) to determine the parameter a.

5.7.3 Life Expectancy

Lastly, there is the question whether to employ life expectancy at birth, age averaged life expectancy or age averaged discounted life expectancy in the formulation of the LQI. Here, it has to be distinguished if the LQI is seen as a social indicator that is taken into account to describe the socio economic situation of a country or if it is interpreted in terms of a utility function of the representative consumer. In the first case life expectancy at birth or age averaged life expectancy might be advantageous to give a straightforward and easily interpretable insight in a society's health and safety conditions.

In the second case in contrast, discounting and age averaging becomes a necessity. Here the time gains are to be seen as an economic good, that bring utility to the consumer. Only if they are reliably assessed in the periods they occur, discounted back to the point of valuation and age averaged to represent the time gain for the average individual, a consistent WTP measure for safety can be established.

5.7.4 Social Discounting

Discounting is a fundamental issue in economics and the choice for a certain discount factor has a main influence on the outcome of project decisions. Public investments into disaster risk reduction must be discounted as any other investments that are made by private firms. Confronting public with private investments however, a crucial distinction has to be made. Private investors would certainly refuse to invest in (risky) projects with a yield on capital below the market interest rate, as they could otherwise purchase fixed government bonds and receive approximately the market interest rate without taking any risk. So the market interest rate serves as a baseline in private sector decision making. Public investments in contrast are usually made with a considerably larger time horizon where both present and future generations are concerned. Employing such a comparatively high discount factor could therefore be problematic as it implicitly attaches little weight to future outcomes. Considering the fact that preventative risk reduction measures are often undertaken with design periods well above 50-years, aspects of sustainable development and intergenerational justice seem very relevant in this context.

Public interest rates vary greatly among countries and rates in the range of 2-8% can be observed [217], indicating that there is no general consensus on this issue as the rates are often set according to quite different rationals. While for instance Weinstein and Statson [263] are convinced that life saving measures should be based on the same discount factor as any other cost and equal the market interest rate, others [39, 230] do not employ any dicounting at all, primarily on ethical reasons when nuclear waste disposal, global warming or the extensive use of natural ressources are concerned. These two opinions are rather two polar cases, rational discounting presumably lies somewhere in between.

The public interest rate is strongly related to real economic growth, where it is important to take per capita values because of the counteracting effects of population and economic growth [217]. Among others, there are two important macroeconomic models that are widely applied and employable to determine the optimal discount rate for stable economic growth: the Ramsey Neoclassical Growth Model [220] and the so called Overlapping Generations Model, devised by Allais [8] in 1947 and popularized by Samuelson in 1958 [227]. According to the Ramsey neoclassical approach, the market interest rate for optimal stable economic growth under perfect market conditions is given through

$$r = \rho + \epsilon\gamma \tag{5.84}$$

where ρ denotes the rate of pure time preferences, γ represents the consumption (income) growth rate per capita and ϵ is the elasticity of marginal utility of consumption. The subjective element in this equation is the rate ρ, which describes the phenomenon that people value things that occur in the future less than if they occurred today, due to human impatience, myopia, the lack of telescopic capacity and so on. While the determination of ρ often results in controversy, its existence has widely been observed in economic literature [209]. The growth rate per capita γ, on the other hand, is often being determined from observed long term averages for developed economies. Comparing the GDP per capita in the time from 1870-1992 gives $\gamma = 1.9\%$ for industrialized western economies such as Western Europe, USA, Canada, Japan; $\gamma = 1.4\%$ for Eastern Europe and $\gamma = 0.9\%$ for Africa [155]. The elasticity of marginal utility of consumption ϵ stems from the concrete formulation of the utility function employed in the Ramsey model, which is often hypothesized to be of the constant proportional risk aversion type. The employment of this class of utility functions entails that the effect of increased wealth upon marginal utility is expressed by a constant elasticity value. It is commonly assumed that $\epsilon = 1$. For the total value of r Nordhaus [186] obtains 0.05%, with $\rho = 0.03$, $\gamma = 0.2$ and $\epsilon = 1$, which seems to be a quite reasonable range where many other sustainable development studies fall into [215].

In economic literature the adequacy of the Ramsey model has been seriously put into question. The critics mainly address the issue that there is a conflict between the preferences of persons currently alive $\rho > 0$ and the needs of sustainability and intergenerational equity. Often, the above mentioned Overlapping Generations Model is advocated instead. The main reasoning behind this approach is that interests of living generations should be discounted at the full discount rate provided in equation (5.84), while for all yet unborn generations only the fraction $\epsilon\gamma$ should be employed: pure time preferences ρ are an attribute of the living generation, while yet unborn individuals are unable to express their corresponding demands. With respect to the latter, society should act as a trustee [151]. For a detailed discussion on the Overlapping Generations Model it is referred to [23, 211].

5.8 Discussion

This chapter dealt with top down approaches for valuing increases in the publicly provided safety level. Refraining from the individualistic concept it has been shown, how macroeconomic indicators may be employed to value reductions in mortality risk. In particular, the human capital approach has been reviewed, that bases the VSL estimation on the concept of forgone earnings that are lost in case of premature deaths of single individuals. As in the human capital approach human life is just seen as a factor of production and the life quality that people derive from the pleasure of living is not accounted for, it lacks satisfactory normative justification and is full of moral and conceptual problems. Therefore, it is commonly rejected and has not been further elaborated.

Addressing the critics of the human capital approach, the LQI has been introduced as a concept that combines several macroeconomic indicators in a composite index, with the intention to measure life quality on aggregate scales and improving the normative justification of estimating VSLs in a top down fashion. In particular, the LQI combines the GDP per capita, the life expectancy and the life working fraction into one single index that is supposed to reflect the most important human concerns by measures that are widely available.

Throughout its evolution over time, the LQI has been continuously subject to controversial discussions regarding its derivation and role in economic theory. This resulted in a great variety of different functional forms and ways of calibrating the index for practical application. The latest LQI derivation based on the utility concept has been the subject of study. By reviewing the contemporary derivation approach, several critics have been placed: most important has been the omittance of budget constraints and the time inconsistency of defining a utility function over annual income but total life expectancy. As a consequence of the latter critic it is not clear, if the LQI concept is to be integrated in static or dynamic models. Furthermore, the general equilibrium nature of the LQI approach has never been explicitly stated in literature.

Addressing these critics, a new equilibrium and time consistent derivation of the LQI has been presented in a one period general equilibrium model. To highlight the theory surrounding the index, the new derivation has firstly been carried out by means of a decentralized derivation approach, i.e. by considering representative consumers and firms separately. In a second step, the index has been derived in a centralized economy, based on the SPP. It turned out that the conventional LQI derivation constitutes a particular solution approach to the SPP.

Having obtained the solution to the SPP, it has been demonstrated that the LQI constitutes a value function that is calibrated with respect to the general equilibrium of the model economy. Furthermore, it has been shown that the conventional approach to derive a SWTP by means of the net benefit criterion neglects second order income effects. Increases in the public safety level lead to increases in total annual survival time for the average individual, that decompose in an increase in leisure but also in an increase in labor time, at a fixed and optimized annual working fraction. Only increases in leisure time are valued by the net benefit criterion, while the additional benefits in terms of higher production and thus higher income due to increased labor input are not considered.

The reason for this neglect is the omittance of budget constraints. Extending the net benefit criterion to include these effects, a surprising result has been disclosed: the LQI, if applied time consistently, values additional total time due to increased safety levels at the MPL, which corresponds to the wage rate in general equilibrium under perfect competition. The conventional LQI pricing strategy in contrast, values only increases in leisure time at the MPL. As the considered time period has been restricted to one year, the standardization of the WTP measure to one mortality risk reduction unit yielded the VSLY.

Based on the observation that instant reductions in mortality risk do not only cause annual survival time to increase, but affect all survival probabilities over the average individual's remaining life, the approach has been extended to account for these intertemporal effects. By evaluating the total increases in lifetime that result from an increase in safety today and discounting them back to the decision point, firstly age dependent WTP measures have been derived. By averaging these over the population's age distribution, a SWTP for safety and a VSL estimate could be obtained.

Additionally, it has been shown, how the VSL calculation procedure can be simplified by linearly approximating the relation between reductions in mortality risk and total increases in age averaged discounted life expectancy. The VSL values obtained from the LQI turned out to be in the range of US$ 4 million for countries considered in the studies of Chapter 4. Thus, the VSL estimates based on the LQI are significantly lower than the microeconomic estimates, but are in the same order of magnitude though.

Eventually, the relation of the presented approach to the conventional LQI methodology has been outlined and recommendations for calibrating the LQI have been provided. The central innovation of the presented approach lies in the fact that the derivation procedure of the index and the accountancy for intertemporal aspects have been split, while they are inconsistently mixed in the conventional method. The finally obtained WTP and VSL formulae resembled each other, with the difference that second order income effects are included in the new LQI pricing formula while they are neglected in the conventional one. This yielded increases in the VSL of around 10%, depending on the actual value of the life working fraction. In addition, it has been shown to be adivisable to employ the full GDP instead of only the 60% fraction that is available for consumption in order to obtain a correct social value of time. In summary, the innovations and new insigts have lead to an increase well above 50% in the LQI based VSL estimates, which is seen as a valuable improvement, considering the fact that the latter have been and still are significantly lower than the estimates from microeconomic studies.

Chapter 6

Uncertainty - The Real Option Approach

The previously presented strategies to determine the WTP for safety have been carried out under the assumption of perfect information. Perfect information is a basic assumption of perfect competition that may not approximate many real world situations [139]. In practical applications of social CBA, it is hardly possible to determine cost and benefits with certainty, but uncertainty can be reduced by gathering information. Any investment into risk reduction that is made now, which commits resources or generates cost that cannot be recovered subsequently, constitutes a so called irreversible investment. In the realm of irreversible investments that are made under uncertainty it may pay to delay the investment to some future point in time to reduce the risk of misinvestment. The value of information that is gained from that delay is the option value or quasi option value of the project. This chapter explains how this option value arises and presents strategies how it may be priced in the context of public risk reduction investments based on a discrete option pricing model. In this way, profound decision rules whether to invest immediately, to postpone the investment or to reject the project are derived. The main ideas behind the presented methodology have been worked out in [147] and further been extended in [202], which serves as a baseline for the subsequent presentation.

6.1 Option Pricing

6.1.1 Options in Finance

The term "call option" or often labeled just "call" describes a financial contract between two parties. The buyer or holder of the option purchases the right, but not the obligation to buy a predefined quantity of a commodity or particular financial instrument, the underlying S, from the seller of the option at a certain time, the expiration date T, at a certain price, the so called strike price P. In opposition to the buyer, the seller is obliged to sell the underlying in case the buyer decides to make use of her right, i.e. to exercise the option. Therefore, the buyer has to expend a cost of acquiring the option in order to enter in the contract. Whether the buyer will finally exercise the option depends on the price development of the underlying. Thus, options are often being referred to as contingent derivatives. Is the market price S_T of the underlying at expiration time T below the strike price P, the owner of the call option will prefer to purchase the underlying on the market instead of exercising the option. In this case the possession of the option is not beneficial to the owner. Should the market price of the underlying S_T move up above the strike price P at expiration time T in contrast, the option is said to be "in the money" and the owner will receive a payoff $S_T - P$ from exercising the option. At this point it is important to emphasize that the option's payoff is conceptually different from the option's profit. The profit of the option is to be seen as the net gain resulting from exercising the option and is obtained by subtracting the option price C_T from it's payoff $S_T - P$. Moreover, it is important to realize that the owner of the option sometimes will exercise the option and make a loss overall. This case occurs when the notation of the underlying at expiration T exceeds the strike price P only by an amount that is smaller than the option price C_T. Nevertheless is this behavior advantageous for the holder to limit overall losses as the price for the option has already been expended initially. Therefore, in general, call options should always be exercised at the expiration date if the stock price is higher than the strike price. To summarize, the payoff C_T of a call option is depending on the market price of the underlying S_T at expiration time T, leading to the following relationship:

$$C_T = max\{S_T - P, 0\} \tag{6.1}$$

From this equation it becomes clear, that the owner of a call option is able to fully profit from the positive development of the underlying while she is at the same time safeguarded against potential losses. This asymmetric payoff stream ensures that the option value can never become negative. As depicted in Figure 6.1, the payoff streams may be visualized on basis of so called "payoff functions", that display an option's payoff as a function of the underlying's price.

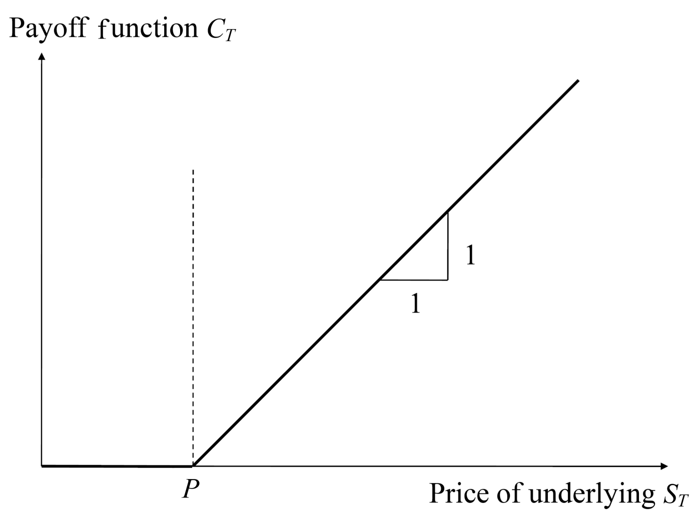


Figure 6.1: A call's payoff function in dependence of the underlying's market price

So far the concept of call options has been illustrated on basis of European call options, which allow the holder to exercise the option only on expiration date T itself. American call options in contrast, may be exercised at any time before and at maturity. In this case T constitutes only the latest possible exercise date. This flexibility in the exercise time directly raises the question to the owner under which circumstances it is profitable to exercise the option prior to maturity. In order to find the optimal time to exercise the option, she has to constantly weigh the payoffs obtained from an immediate exercise and the corresponding loss of the purchasing right against the probable price evolution of the underlying as well as potentially due dividend payments.

Within option pricing theory it is generally assumed that the price evolution of an option is merely a function of the price development of the underlying and of time. The most important factors that have an influence on the option price are consequently:

- The current price (spot price) of the underlying S_t
- The volatility of the underlying $\overline{\sigma}$
- The exercise price P
- The time to maturity T
- The interest rate r
- Due dividend payments D

From equation (6.1) it becomes clear that the option value is positively correlated with the price of the underlying S_t and furthermore increases with decreasing strike price P. Whereas for European call options no general statement for the influence of the time to maturity T on the option value can be derived, for American call options a longer time to maturity causes an augmentation in option value. This is due to the fact that the right to exercise the option is valid for a longer time period which in turn increases the probability that the spot price S_t will surpass the strike price P.

The volatility $\overline{\sigma}$ of the underlying is an important measure for the uncertainty in its price development over time. With increasing volatility the probability that the stock price fluctuates in both directions increases, which generally has a positive influence on the option value. This follows from the asymmetric payoff structure of the option, i.e. the owner takes the full benefits from a positive development of the underlying while she is safe against adverse evolutions and can lose maximally the price she expended for the option.

The risk free interest rate r affects the price of an option in a less clear cut way. A higher risk free interest rate implies that investors generally demand higher expected returns from the stock. In addition, the present value of any future cash flow received by the holder of the option decreases. The combined effect on the option price is generally positive[1] [120].

Dividend payments D normally cause a reduction in the option value because the owner of the option does not profit from dividend payments, while they are at the same time anticipated in the price of the underlying. Table 6.1 summarizes the influences on the option value schematically.

6.1.2 The Binomial Model of Cox Ross and Rubinstein

After the basics about options as well as the influencing factors on the option price have been provided in the last section, it is now discussed how the option price may be determined explicitly on basis of an option pricing model. The subsequently introduced option pricing model of Cox, Ross and Rubinstein [57] is built on the principle of arbitrage free pricing, that requires a short explanation.

In finance, the term "arbitrage" characterizes a market transaction that involves no negative cash flows at any temporal state and a positive cash flow in at least one state. In simpler terms, arbitrage corresponds to a risk free profit [37]. In the simplest case, arbitrage can be realized by buying an

[1] Here it is assumed that the interest rate changes while all other variables, such as the price of the underlying, remain constant. In practice, when interest rates rise, stock prices tend to fall. Therefore, the overall effect of an interest rate increase and the accompanied stock price decrease remains ambiguous.

Parameter	**European call option**	**American call option**
Price of underlying	+	+
Strike price	−	−
Time to maturity	?	+
Volatility	+	+
Interest rate	+	+
Dividends	−	−

Table 6.1: Influences on the option price

asset at a low price on one market and reselling it simultaneously on another market for a higher price. Thus, the risk free profit is obtained by taking advantage of a price differential between two or more markets. In a perfect capital market in equilibrium state[2], that is in particular free of information- and transaction cost, the absence of arbitrage directly follows [86]. The absence of arbitrage is the first necessary condition of arbitrage free pricing and directly implies the so called "law of one price" that may be formulated as follows [36]:

If two financial positions A and B that lead to the same payoffs $z^s_{A,t} = z^s_{B,t}$ at every future point in time t under all envisioned potential future situations s, they need to be traded at the same price if the capital market is in equilibrium.

The second necessary condition for arbitrage free pricing is the assumption of a complete capital market. A complete capital market is given, when there are as many distinctive financial positions with linear independent payoff vectors as there are possible future outcomes [53]. Under this premise, every possible cash flow may perfectly be replicated by constructing an adequate replicating portfolio. Is the composition of the replicating portfolio as well as the market prices of all contained positions known, the law of one price is employable to determine the price of the replicated cash flow.

Under the presumption of perfect and complete capital markets in equilibrium, the Binomial Model of Cox, Ross and Rubinstein is now introduced. It belongs to the class of discrete option pricing models, implying that the value of the option and that of the underlying are only permitted to change at discrete points in time. Moreover are the stochastic variables, that are considered in the model, restricted to adopt particular predefined values only. Thus, it follows that the underlying, that is assumed to be a stock in the following, can only take two values in the respective next period when departing from the current quoting S_0:

$$S_1^u = S_0 \cdot (1+u) \qquad \text{or} \qquad S_1^d = S_0 \cdot (1+d) \tag{6.2}$$

In this respect, $u > 0$ generally describes the case of an upturn in the stock quoting ("up"), whereas $-1 < d < 0$ represents a downturn in the stock price ("down"). If ϕ denotes the probability of an upturn in the stock price, $1 - \phi$ must capture the probability of a downturn as there are only two possible value developments of the stock price in the change from one period to another. For a

[2] Implies the absence of taxes and transaction cost, information efficiency, perfect competition and rational behavior of all market participants [206].

European call option on the stock S with strike price P and execution time T, the exercise payoff in case of an upturn or downturn respectively is given through:

$$C_1^u = \max\{S_1^u - P, 0\} = \max\{S_0(1+u) - P, 0\} \tag{6.3}$$
$$C_1^d = \max\left\{S_1^d - P, 0\right\} = \max\{S_0(1+d) - P, 0\} \tag{6.4}$$

Because there are only two possible outcomes in $t = 1$, the option's payoffs are reproducible by means of two positions with linear independent cash flows. This is done by constructing a replicating portfolio out of Δ stock shares S and investing B units in a bond at the risk free interest rate r:

$$\Delta \cdot S_0 \cdot (1+u) + B \cdot (1+r) = C_1^u \tag{6.5}$$
$$\Delta \cdot S_0 \cdot (1+d) + B \cdot (1+r) = C_1^d \tag{6.6}$$

Subtracting the second equation from the first allows for an elimination of B:

$$\Delta \cdot S_0 \cdot (u-d) = C_1^u - C_1^d \tag{6.7}$$
$$\Rightarrow \quad \Delta = \frac{C_1^u - C_1^d}{S_0 \cdot (u-d)} \qquad \Rightarrow \quad B = \frac{C_1^d \cdot (1+u) - C_1^u \cdot (1+d)}{(u-d)\cdot(1+r)} \tag{6.8}$$

By construction the replicating portfolio now leads to the same cash flows as the call option under all possible circumstances, so that the law of one price implies that the value of the portfolio in $t = 0$ must be equal to the price of the call option C_0 . The latter is then easily calculated by taking Δ and B into account:

$$C_0 = \Delta \cdot S_0 + B = \frac{1}{1+r} \cdot (C_1^u \cdot \frac{r-d}{u-d} - C_1^d \cdot \frac{u-r}{u-d}) \tag{6.9}$$

Denoting $\frac{r-d}{u-d} =: p$, this option price formula simplifies to:

$$C_0 = \frac{1}{1+r} \cdot (C_1^u \cdot p - C_1^d \cdot (1-p)) \tag{6.10}$$

This expression resembles an expected value, that is being discounted by the risk free interest rate r. If $p \in (0,1)$ was the probability that the stock price moved upwards, equation (6.10) would correspond to the option pricing formula of a risk neutral investor. In fact, p must be identical to ϕ in a risk neutral world. This is due to the fact that a risk neutral investor evaluates the stock only on basis of its expected value, implying that the expected return on the stock must be equal to the risk free interest rate r, so that the identity $p = \phi$ must hold:

$$E(\frac{S_1 - S_0}{S_0}) = \phi \cdot u + (1-\phi) \cdot d =: r \quad \Rightarrow \qquad \phi = \frac{r-d}{u-d} = p \tag{6.11}$$

This equation illustrates an important general principle in option pricing theory known as risk neutral valuation. The principle states, that it is permissible to assume that the world is risk neutral when pricing an option. The price that is obtained is correct not just in a risk neutral world but in the real world under arbitrary risk preferences as well. This can be verified by emphasizing that equation (6.10) has been derived only by relying on arbitrage free pricing, without employing a risk neutrality requirement. Under non risk neutral preferences, p constitutes the artificial probability of an upturn, which is to be clearly distinguished from the real upturn probability ϕ. Remarkable is at this point, that the true probabilities ϕ and $1-\phi$ for the two possible states of stock price development are not being accounted for in the option pricing formula. The reason for this is, that the option price is not valued in absolute terms but in terms of the underlying stock price. Consequently, as the expectations of the market participants with respect to the stock price development are already included in the current stock quoting, it is not necessary to consider them again when valuing the option in terms of the stock price [120].

The above described one period Binomial Model can now be generalized to the n period case. This intention in mind it is assumed that the parameters u and d characterizing the stock price development remain constant in each of the considered periods. In this way it is assured that an up move followed by a down move in two consecutive periods lead to the same stock price as if first a down move and then an up move had occurred. The stock price development is best illustrated by means of a binomial tree that visualizes the stock price evolution over time, as depicted in Figure 6.2. Hereby, each node of the binomial tree represents a possible stock price resulting from different combinations of up- and downturns in the single periods.

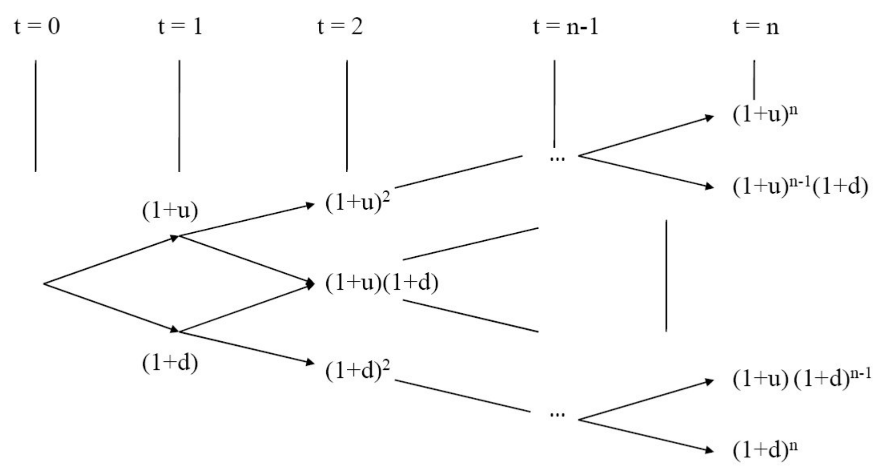


Figure 6.2: The Binomial Model of the underlying's price development for n periods

The artificial risk neutral probability p of an up move in a single period is to be calculated absolutely analogously to the one period case:

$$p = \frac{r-d}{u-d} \in (0,1) \tag{6.12}$$

By means of (6.12) and simple combinatorics it is now possible to calculate the probability that the stock price in period n will be equal to $S_0 \cdot (1+u)^k \cdot (1+d)^{n-k}$ in a risk neutral world, after k up and $n-k$ down movements have taken place:

$$\frac{n!}{k!(n-k)!} \cdot p^k \cdot (1-p)^{n-k} = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \tag{6.13}$$

The remaining task is to determine the option price for each node of the binomial tree. After k up and $n-k$ down moves the following expression holds for the option price after n periods:

$$C_T^{k,n-k} = max\left\{S_0 \cdot (1+u)^k \cdot (1+d)^{n-k} - P, 0\right\} \tag{6.14}$$

Now, following the rationale of risk free pricing, this option value at time T is taken into account to determine the current option price C_0 at $t = 0$ by discounting the expected payoffs that have been determined by means of the risk free probabilities p:

$$C_0 = \frac{1}{(1+r)^n} \cdot \sum_{k=1}^{n} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \cdot C_T^{k,n-k} \tag{6.15}$$

In analogy to the single period case it can be shown that the option pricing formula (6.15) is also valid in real world applications with arbitrary risk attitudes other than risk neutrality. This is achieved by relying on the law of one price and constructing an adequate replicating portfolio. Here it has to be emphasized, that the replicating portfolio is held for the duration of one period only. After this, the composition of the portfolio needs to be modified to precisely duplicate the development of the option price in the next period. In the absence of transaction cost, which may be taken for granted due to the presumption of a perfect capital market, the portfolio adjustment in each period is self financed, meaning that in each period only changes in the composition of stock shares and risk free bond investment take place, without causing any cost [57].

The above listed prerequisites of the Binomial Model are limiting the applicability of the option pricing formula (6.15) in practice. Although the model is extendable in several directions, this often goes in line with complex computational requirements. But the simplified and time discrete modeling may also be looked upon as one of the model's advantages. Through simple backtracking even more complex problems of option pricing can be solved. Examples that are of particular interest in the following are the inclusion of dividend payments on the underlying or the pricing of an American call option that may be exercised at any time before expiration.

For this purpose, firstly a tree similar to that of Figure 6.2 of possible future states is constructed, based on which the option price at expiration time T is easily obtained. Like before, by means of risk neutral pricing, the option prices at all nodes in period $n-1$ are then easily calculated. Through gradual repetition of this procedure the root of the tree may eventually be reached, delivering the current option price. In addition, it has to be decided at each node if it pays to exercise the option prior to expiration.

6.2 Public Risk Reduction Opportunities as Real Options

The opportunity to invest in a risk reduction intervention within preventive disaster management can be interpreted as an American call option on the project. Hereby, the public decision maker represents the owner of the option that has to decide whether to invest immediately or to postpone the investment decision to some future point in time when better information is available. Because she has the authorization to freely decide about the time when to initiate the project, she will instantly invest only if the measure is judged to be cost efficient with adequate reliability.

Should the expected value of the estimated benefits only slightly exceed the investment cost and is there significant uncertainty involved in the benefit estimation process, it might be advantageous to postpone the investment to a later point. In this manner, the additional disposable time may be used to perform a detailed analysis to improve the information basis about the hazard and the efficiency of the risk reduction project.

Now, the analogy between public risk reduction possibilities and American call options is further illustrated with the objective to finally obtain a discrete pricing formula for the specific real option based on the Binomial Model of Cox, Ross and Rubinstein.

6.2.1 The Option Analogy of Investment Opportunities

In the (real) option analogy the risk reduction project under investigation corresponds to the stock on which the option contract is being issued. The value of the underlying risk reduction project is equal to the monetized social benefits that are expected to result from the reduced disaster risk. In Chapter 2 it has been outlined that these benefits are given by the monetized reduced disaster losses of all four consequence categories. Following the notation of Chapter 2, the total benefits B^T of the risk reduction project sum up to:

$$B^T = B^{econ} + B^{hum} + B^{env} + B^{CSH} \tag{6.16}$$

Here, in particular the benefit component B^{hum} corresponds to the social WTP for safety derived in the last two chapters. The total value of the risk reduction project at time t is then given by the net present value (NPV) of all the benefits that occur after t:

$$S_t = \sum_{x=t}^{T} \frac{B_x^T}{(1+r)^{x-t}} \tag{6.17}$$

Here, the planning horizon may also be infinite, i.e. $T = \infty$. The benefit estimation must always be performed on basis of imperfect information and therefore represents just an approximation of the true value. The true value however, is hardly assessable due to the numerous aleatoric uncertainties involved. What may be done instead is to improve the knowledge about the hazard induced endangerment by collecting more information and thus, to reduce the epistemic uncertainty. Then, under an improved information basis, the estimated social benefits of the risk reduction intervention may have to be properly adjusted. Because the direction of the adjustment cannot be foreseen in advance and depends on the specific outcome of the study, the value of the risk reduction project S_t will have to be modeled by means of a stochastic process that best describes the possible evolution of the discounted expected benefits over time.

Smaller scatterings in the value of the risk reduction project might be imposed by slight changes in the basic parameters, such as market prices of certain assets or a change in interest rates. In addition, also newly acquired information about the expected effectiveness of the measure contribute to fluctuations in the value. This new information might be obtained from performing a more detailed study on the risk, from latest research cognitions or from a more exhaustive data collection on past events. For all these reasons, the estimated value of the project must be continuously updated to perfectly go in line with the actual level of information.

In the following, all these and other imaginable influences on the social evaluation of the benefits are termed "steady information gains". In the short term, these information gains generally have only a small influence on the value of the risk reduction project, in the long term they might accumulate to significant changes in value though. In the considered Binomial Model these local up- or down moves are incorporated by multiplying the social value of the risk reduction project S_t at any discrete time step by the factors $(1 + u)$ or $(1 + d)$, respectively.

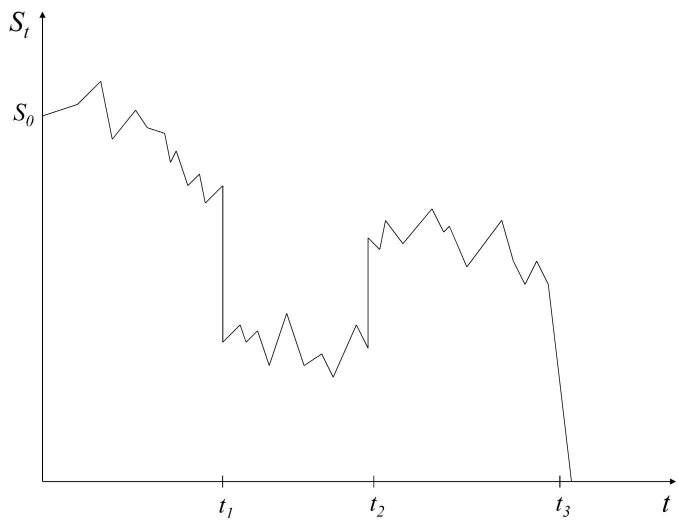


Figure 6.3: Value evolution of the risk reduction project over time

In sharp contrast to small value adjustments imposed by steady information gains is the information gain that results from the occurrence of a natural disaster and its immediate aftermaths. The occurrence of an extreme event might have a tremendous influence on the risk reduction project, causing an abrupt change in its value. Valuable data about the characteristics of the extreme event are collected, a large amount of information about the damages and losses is obtained and people's risk awareness instantly increases, generally leading to a higher WTP for protection.

All these impacts on the social value of the risk reduction project S_t are being referred to as "disaster imposed information gains" in the following. It is not possible to predict in which direction the social value of the (still not executed) project will change. It is thinkable that the natural event just highlights the necessity to treat the risk. On the other hand it might be possible, that the occurrence of the disaster and the newly acquired information show, that the risk reduction intervention would not have been effective anyway. Even a loss of the investment possibility is imaginable, for instance if a building that should have been rehabilitated against earthquake load has collapsed.

A possible evolution of the value of the public risk reduction project over time is depicted in Figure 6.3. At the beginning of the project evaluation, the project value is estimated to be S_0. Smaller value adjustments due to steady information gains are represented by the constant zigzagging movements between the three abrupt value changes at t_1, t_2 and t_3. The three jumps in the value development are due to disaster imposed information gains. At t_1 the value is sharply corrected downwards as it turned out that the value of the project has been overestimated in the first assessment. In t_2 in contrast, the value makes a significant jump upwards as the new disaster information showed that the measure could have prevented significant losses. In point t_3 the value of the project approaches zero as the disaster occurrence led to the destruction of the assets that should have been protected by the measure.

American call option (on a stock)	**Social real option (on a RR-project)**
Stock quotation S	Social value of RR-project
Strike price P	Investment cost
Expiration time T	Time for which RR-project is possible
Risk free interest rate r	Social discount rate
Dividends D	Forgone benefits from delaying the project

Table 6.2: The option analogy of risk reduction projects

The probability that a natural disaster occurs during a discrete time step is denoted as "disaster probability" Ψ in the following. It is further assumed that Ψ will remain constant over all considered time steps. The sudden jump in the change of the value of the social risk reduction project is integrated by means of a jump factor $1 + Y$. As neither the size nor the direction of change is predictable, Y represents a random variable. Therefore, a conditional probability function has to be defined, that describes the size of the jump in case of disaster occurrence. If it is hypothesized that all deterministic developments are anticipated in the current value of the risk reduction measure, the expected value change will be zero, so that $E_y(Y) = 0$ holds. Theoretically, all values $Y \geq 0$ are thinkable, even the total loss of the investment opportunity is feasible, leading to $Y = -1$ (multiplication of the project value by 0).

The value of the public risk reduction project is estimated under best available information and permits an interpretation in terms of the daily quotation S_t of the underlying. The life cycle cost of the risk reduction measure C^T, to be determined as discussed in Chapter 2, corresponds to the strike price P. The time for which the risk reduction investment opportunity persists is equivalent to the time to maturity T. This time may be finite, for instance for a building that has a limited service life, or the opportunity to invest never expires or at least not in a predictable time horizon. In the latter case $T \to \infty$ is fulfilled. Table 6.2 summarizes the option analogy of risk reduction opportunities.

Once a risk reduction project has been postponed, there is always the possibility that a natural disaster occurs in the meantime that could have been prevented or at least confined by an implementation in time. The losses that occur in this case are to be accounted for by means of lost profits in the model. This is being recognized by introducing a dividend payment D on the underlying that is discharged to its owner. For the holder of the option this dividend is lost. Problematically is hereby, that the dividend payment occurs at unknown times in the future as well as in not foreseeable amounts.

Such a stochastic dividend payment cannot be accounted for in established option pricing models: in the evaluation of options on stocks it is usually assumed that future dividend payments are announced in advance and are thus anticipated by the market. The risk reduction measure however, may only be evaluated by means of the expected reduced losses that occur statistically every year - if at all and when these reduced disaster losses actually occur is not predictable and can therefore not influence the investment decision.

The longer the implementation of the risk reduction investment is postponed, the higher is the probability that losses will occur, that could have been prevented by an execution of the measure in time. Therefore, it seems only logical to incorporate the forgone benefits in terms of forgone reduced losses on basis of an annual dividend payment D. The expectation towards the reduced disaster losses is changing over time, making a continuous adjustment of the project value necessary. Consequently, also the expectations towards the dividend payment are changing in the same way, so that it is modeled proportionally to the total value of the risk reduction measure. The size of the dividend payment relative to the absolute value of the project however, is assumed to be constant over time.

6.2.2 Prerequisites for Applying Real Option Theory

The above illustrated analogy between a financial call option and the possibility to invest in a public risk reduction project suggests to evaluate the project by means of financial option pricing models to account for the flexibility and uncertainty involved in the project decision. To proceed in this way, it has to be investigated firstly, if the prerequisites for applying real options theory are met. A detailed discussion on this issue is provided in [141]. Here, just the main points are discussed.

A necessary condition for arbitrage free pricing is that the cash flows of the real option's underlying, i.e. of the risk reduction project, are replicable by the cash flows of financial instruments that are traded on free markets. Because social risk reduction projects are not traded on markets, the real option on the project cannot be replicated simply by the underlying and a risk free bond, as it is done with options on stocks. Therefore, the existence of a traded asset having identical cash flows like the risk reduction project is often assumed. In reality however, it will hardly be possible to find such a "twin security" on the market. Also the alternative assumption of market completeness (see Section 6.1.2), which would in principle guarantee the replicability of an arbitrary cash flow, is at least questionable.

If real option pricing is understood as an extension of conventional investment theory in contrast, the real option can simply be replicated by a portfolio consisting of a risk free bond and the risk reduction project without the flexibility about the investment timing. The evaluation of the current value of the project, representing the current quote S_0, can then be carried out by relying on classical methods to evaluate investments, whereas the flexibility is priced by means of option pricing. As neither the real option nor the risk reduction project are traded on the market, the discussion of arbitrage free pricing is a pure theoretical issue. Nevertheless, the replicability assumption is justifiable at least in the sense that it is rather common in "inflexible" investment appraisal. Within the latter it is just attempted to guess the price that a non traded project would have on the free market [141]. It has to be emphasized however, that the evaluation of the option price on non traded investment projects on basis of arbitrage free pricing is to be seen just as an approximation.

Another important assumption for the applicability of real option theory is the full or at least partial irreversibility of the investment cost. The irreversibility of public risk reduction projects has been discussed in Chapter 2.6. If the investment cost was fully reversible, the evaluation of the flexibility to postpone the investment would become superfluous and a project appraisal by means of conventional investment theory would be sufficient.

Additionally, it is necessary that the investment option is exclusive for the owner, meaning that its value is not influenced by decisions of competitors. In case of public risk reduction projects this condition is straightforwardly fulfilled as they are not carried out under competition.

Finally, interactions between several distinctive investment projects as well as between single steps to be undertaken within one particular project have to be excluded (no inter- and intra-project interactions). If this prerequisite is violated, the option pricing approach has to be extended to evaluate several interdependent real options. In principle this would be possible, but comes at the cost of increased complexity.

In the following, the stated prerequisites for applying real option theory are assumed to be fulfilled. Only the first assumption about the replicability of the project's cash flows by traded assets has to be considered as being critical. Accordingly, the application of pricing models that have been derived under free market conditions in the context of public project appraisal may be challenged. But then also the application of several models of conventional investment theory, such as the discounting of a project's cash flows on basis of a risk adjusted interest rate, has to be questioned as it relies on similar or at least related assumptions.

6.3 Discrete Pricing Model for the Special Real Option

After the option analogy of public disaster risk reduction opportunities has been discussed, this section aims at deriving a concrete price for the flexibility inherent in the project decision. If a discrete random process is considered to be appropriate to describe the value development of the public risk reduction project, the impact of steady information gains can be modeled by relying on the Binomial Model of Cox, Ross and Rubinstein (see Section 6.1.2). The jumps in the project's value evolution due to disaster imposed information gains are accounted for by means of a special discrete approach developed by Amin [9].

6.3.1 Derivation of a Discrete Pricing Formula

In order to derive a discrete pricing model for the real option it is started with the construction of a portfolio consisting of the real option C_t itself, Δ parts of the investment project S_t without flexibility and an investment B in a risk free bond with interest rate r. Under the premise that the portfolio is self financed, the portfolio value at construction time t is zero:

$$V_t = \Delta \cdot S_t + B + C_t = 0 \tag{6.18}$$

As discussed in Section 6.2.1, the expected forgone risk reduction benefits are modeled by a constant dividend payment on the underlying that is paid in any period, also without the actual occurrence of a natural disaster. The size of the dividend payment is assumed to be proportional to the project value S_t, that is determined on basis of the best available information in the respective period, and will be included by means of a dividend rate D. Furthermore, it is hypothesized that the value of the risk reduction project due to newly available information only changes at the end of each period. The total amount of dividend payment at $t+1$ is consequently calculated by multiplying the dividend rate D with the project value of the previous period S_t. Therefore, the value of the total portfolio under both possible developments without disaster occurrence is obtained by:

$$V_{t+1}^u = \Delta \cdot S_t \cdot (1+u+D) + B \cdot (1+r) + C_{t+1}^u \tag{6.19}$$

$$V_{t+1}^d = \Delta \cdot S_t \cdot (1+d+D) + B \cdot (1+r) + C_{t+1}^d \tag{6.20}$$

As in the Binomial Model of Cox, Ross and Rubinstein it is assumed that the value of the portfolio is known for the duration of one period, given that in this period no disaster occurs. Consequently, the portfolio value after a local up move V_{t+1}^u must be equal to the portfolio value after a local down move V_{t+1}^d. A subtraction of equation (6.20) from equation (6.19) allows for an elimination of B:

$$\Delta \cdot S_t = -\frac{C_{t+1}^u - C_{t+1}^d}{u-d} \tag{6.21}$$

$$\Rightarrow B = \frac{1}{1+r} \cdot \left[(C_{t+1}^u - C_{t+1}^d) \cdot \frac{1+u+D}{u-d} - C_{t+1}^u \right] \tag{6.22}$$

These expressions for Δ and B may then be inserted in equation (6.18), yielding:

$$V_t = -\frac{1}{1+r} \cdot \left[C_{t+1}^u \cdot \underbrace{\frac{(r-D)-d}{u-d}}_{=p} + C_{t+1}^d \cdot \underbrace{\frac{u-(r-D)}{u-d}}_{=(1-p)} \right] + C_t = 0 \tag{6.23}$$

A closer look at equation (6.23) clarifies that it has been deviated from the Binomial Model in Section 6.2 only in so far as the risk free interest rate r within the risk neutral probabilities p has been reduced by the dividend payment D. In the model of Cox, Ross and Rubinstein such a risk free portfolio, that has zero value in t, must also be valueless in $t+1$. In the considered case on the contrary, the possibility of natural disaster occurrence must be accounted for in the modeling of the portfolio value development. In this respect it may be shown, that it is impossible to perfectly replicate options on underlyings whose price development follows a stochastic jump process. A dynamic portfolio adjustment cannot be self financed in periods where jumps occur [179].

This follows immediately from the fact that the compound stochastic process permits more than two possibilities in the value development in each period. Therefore, it is impossible for the investor to make her portfolio safe against value jumps on basis of two linear independent assets only. In order to be safeguarded, there must be as many financial instruments with linear independent cash flows as there are possible outcomes. Consequently, as the construction of a save portfolio is not possible in the considered case, the option price cannot be derived on basis of arbitrage free pricing only. An additional assumption for the case of disaster occurrence is required.

Up to now, no assumption has been made with respect to the risk attitude of the public decision maker. In the realm of public project decision making under uncertainty it is often referred to the Arrow Lind Theorem [15] in literature. The latter postulates that the public authority should decide on risky projects under risk neutrality, as it is capable of bearing the risk itself through risk pooling and risk spreading, see for instance [164]. This implies that a decision should be made by taking only the expected value of the cash flows into consideration, while completely neglecting the uncertainty inherent in their estimation. While some authors generally take the validity of the Arrow Lind Theorem for granted in particular for governments of industrialized western countries [115], there are other voices that are convinced of the opposite [142]. Brent [35] for instance states that for public investments the theorem is unlikely to hold and therefore a risk adjustment must necessarily be conducted in the cost and benefit estimation.

Relying on the Arrow Lind Theorem is not very useful in the present context as a neglect of uncertainty would make a postponement of the investment only necessary, if the project was not profitable on basis of expected values, i.e. if the expected cost was higher than the expected benefits. This in turn would make an evaluation by means of a real option approach superfluous. As a matter of fact, even if the expected net project value is positive, a postponement of the investment might still be advantageous if it is likely that there will be better information about the cash flows available in the future, and thus uncertainty about the project development can be reduced.

The value of the investment flexibility should therefore preferably be estimated on basis of arbitrage free pricing only and therefore without restricting assumptions concerning the public risk attitudes. With respect to the disaster imposed information gains in contrast, i.e. the jump component of the stochastic process, risk neutrality is assumed in the following. Accordingly, they are included only by means of their expected values. This approach may be justified in particular if the disaster probability is comparatively low and if rather small postponement intervals to acquire better information about the disaster are considered. Under this assumption, the option value can be calculated in terms of an expected value, relying on the artificial probabilities for the occurrence of local information gains, while taking the true probability for the disaster occurrence into account.

To finally determine the option price, firstly the portfolio development is to be modeled for all possible outcomes in $t+1$. For local information gains this is done by a combination of equations (6.18), (6.19) and (6.21):

$$V_{t+1}^{u/d} = C_{t+1}^{u} \cdot p + C_{t+1}^{d} \cdot (1-p) - C_t \cdot (1+r) \tag{6.24}$$

The portfolio value in case of disaster occurrence is determined as follows:

$$\begin{aligned} \widetilde{V}_{t+1}^{y} &= \Delta \cdot S_t \cdot \Big[(1+\widetilde{Y}) + D\Big] + B \cdot (1+r) + \widetilde{C}_{t+1}^{y} \\ &= -\frac{C_{t+1}^{u} - C_{t+1}^{d}}{u-d} \cdot \Big[\widetilde{Y} - (r-D)\Big] - (1+r) \cdot C_t + \widetilde{C}_{t+1}^{y} \end{aligned} \tag{6.25}$$

This value is a random variable as the size of the multiplicative jump factor $\widetilde{Y}$ is entering the equation stochastically. In the following, it is denoted E_y for the expected value operator with respect to the probability distribution of $\widetilde{Y}$ conditional on the disaster occurrence Ψ. As discussed above it is assumed that $E_y[\widetilde{Y}] = 0$. The value of $E_y[\widetilde{C}_{t+1}^{y}]$ in contrast is unknown due to the asymmetric payoff function of the call option and is to be determined dependent on the value of the investment project in the previous period S_t. By taking the periodical probability of disaster occurrence Ψ into account, the expected portfolio value at $t+1$ is calculated under all possible outcomes:

$$\begin{aligned} E\left[\widetilde{V}_{t+1}\right] &= (1-\Psi)\cdot V_{t+1}^{u/d} + \Psi \cdot E_y\left[\widetilde{V}_{t+1}^{y}\right] \\ &= (1-\Psi)\cdot\left[C_{t+1}^{u}\cdot p + C_{t+1}^{d}\cdot(1-p) - C_t\cdot(1+r)\right] \\ &+ \Psi\cdot\left[(C_{t+1}^{u} - C_{t+1}^{d})\cdot\frac{r-D}{u-d} + E_y\left[\widetilde{C}_{t+1}^{y}\right] - C_t\cdot(1+r)\right] \end{aligned} \tag{6.26}$$

Under the assumption of risk neutrality with respect to disaster imposed information gains, the public decision maker will value the considered portfolio equal to a fictive second portfolio, that leads to the same cash flows with certainty. A portfolio that was self financed at time t must be valueless at any time in the future due to the absence of arbitrage. This implies, that portfolio value (6.26) may be equalized to zero. From this observation the first formula for the option price is obtained:

$$\begin{aligned} C_t = \frac{1}{1+r}&\cdot\Big[(1-\Psi)\cdot\Big(C_{t+1}^{u}\cdot p + C_{t+1}^{d}\cdot(1-p)\Big) \\ &+ \Psi\cdot\Big((C_{t+1}^{u} - C_{t+1}^{d})\cdot\frac{r-D}{u-d} + E_y\left[\widetilde{C}_{t+1}^{y}\right]\Big)\ \Big] \end{aligned} \tag{6.27}$$

By defining new artificial probabilities p', this option pricing formula can substantially be simplified. In this respect, equation (6.27) is rearranged in a way that the option values are ordered according to the different possible outcomes in $t+1$:

$$(1+r)\cdot C_t = \Psi\cdot E_y[\widetilde{C}_{t+1}^{y}] + (1-\Psi)\cdot[C_{t+1}^{u}\cdot p' + C_{t+1}^{d}\cdot(1-p')] \tag{6.28}$$

Consequently, the new artificial probabilities are given through:

$$p' = \frac{\frac{r-D}{1-\Psi} - d}{u-d} \qquad \text{and} \qquad 1-p' = \frac{u - \frac{r-D}{1-\Psi}}{u-d} \tag{6.29}$$

As a result, the option price is again derived from risk free pricing, even if this time the artificial probabilities for up and down moves could not be obtained on basis of arbitrage free pricing only. The value C_t is to be interpreted as the value of the decision to postpone the investment for at least one more period and thus, to keep the option alive. As the considered option constitutes an American call option, it has to be constantly evaluated, if an early execution of the option and thus the decision to immediately invest is optimal. Therefore, expression (6.27) needs to be extended to the following form:

$$C_t = \max\left\{S_t - P,\quad \frac{1}{1+r}\cdot\Big(\Psi\cdot E_y[\widetilde{C}_{t+1}^{y}] + (1-\Psi)\cdot\Big[C_{t+1}^{u}\cdot p' + C_{t+1}^{d}\cdot(1-p')\Big]\Big)\right\} \tag{6.30}$$

In each period a decision has to be made between investing immediately and thus exercising the option or to postpone the investment decision to the next period by keeping the option alive. In the first case the project value S_t less the investment cost $C^T = P$ is obtained. The decision to postpone the investment should be made in contrast, if the value of keeping the option alive is higher than the profit resulting from an instant investment. This weigh out between investing immediately and postponing the investment is influenced by all future optimal decisions as they are already anticipated in the current option value, as indicated by the recursive structure of equation (6.30).

6.3.2 Solution Approaches for Real Options of Finite and Infinite Duration

Is the investment opportunity valid for a finite number of periods, it is possible to determine the option price at any point in time through backwards induction starting from expiration time T. However, sometimes it might be possible to postpone the investment arbitrarily in the future so that the investment opportunity never expires or at least not for a predictable number of periods. Nevertheless, also in case of infinite expiration time $T \to \infty$ it is possible to determine the optimal investment strategy. In this respect, it is made use of the repetitive structure of the decision process that is implicit in equation (6.30). Real options with infinite durations have a value that is actually independent of the decision point t, implying that a recursive form of equation (6.30) may be used to price the option. The value of the investment opportunity surely is still dependent on the estimation of the social value of the project S that is estimated with best available information. If $\widetilde{S}'$ is denoted for the estimated value of the project in the next period, the option can be priced by:

$$C(S) = max\left\{S - P, \frac{E[C(\widetilde{S}')|S]}{1+r}\right\} \tag{6.31}$$

This equation follows the Bellman optimality principle [24], which characterizes the property of an optimal policy. It says that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (as expressed by the Bellman equation) [22]. This optimality principle is related to the concept of optimal substructure and fundamental within dynamic programming. It allows breaking down a dynamic optimization problem into simpler subproblems, by assessing the value of a decision problem at a certain point in time in terms of the payoff from some initial choices and the value of the remaining decision problem that results from these initial choices.

Therefore, the Bellman principle allows to comprise all future decisions in a so called continuation value. In the considered case this corresponds to the value of the investment possibility if the decision to postpone the investment is made and may be calculated in dependency of S by taking equation (6.27) into account. The decision with respect to the optimal investment timing is then characterized as a classical optimal stopping problem, which is concerned with determining the optimal time to take action. Stopping in the considered case means initiating the investment and therefore to terminate the decision process.

Dixit and Pindyck [67] show that for such a problem a threshold value S^* for the estimated value of the social risk reduction project exists, from which onwards an immediate investment becomes optimal. Consequently, for all estimated values of the risk reduction project satisfying $S > S^*$ an immediate investment is optimal, whereas in the case $S < S^*$ it is optimal to postpone the investment for at least another period. If equality $S = S^*$ is given, there is indifference between the two actions. The threshold value S^* may be calculated numerically by means of the recursive pricing formula. In this respect, based on a reasonably defined starting function $C(S)$ not only the threshold value but also the option price function must be determined iteratively until equation (6.31) holds.

Is the value of the risk reduction project S without the flexibility to postpone the investment estimated to be higher than the threshold value S^*, an immediate initiation of the project becomes the optimal decision. Consequently, the new decision rule leads to different results in comparison the conventional NPV criterion in all cases where $P < S < S^*$ and the threshold value S^* is calculated to be higher than the investment cost $C^T = P$. Therefore, the difference $S^* - P$ corresponds to the value of the possibility to postpone the investment.

6.4 Discussion

The methodology developed in this chapter constitutes a first move in applying financial real option theory to the management of natural disaster risk. The cost and benefits of natural disaster risk reduction projects are subject to substantial uncertainties that have to be properly accounted for in the project decision. With the possibility to delay the investment initiation to a future point in time, new information can be acquired that allows a more precise estimation of the project's cash flows. This reduction in epistemic uncertainties reduces the risk of misinvestment and has a value that is to be included in the appraisal, but comes at the price that a natural disaster might occur in the meantime that could have been confined by a project implementation in time.

The theoretical basis for the suggested approach has been presented by introducing the discrete Binomial Option Pricing Model of Cox, Ross and Rubinstein. It has been demonstrated that the possibility to invest in a public risk reduction project permits an interpretation in terms of an option that the public authority possesses to acquire the project, similar to that the owner of a financial call option has on buying the underlying. By modeling the social value of the risk reduction project as the stock quotation, the investment cost as the strike price and the time for which the investment possibility exists as the expiration time, the risk reduction project has been integrated in the option pricing context.

The possibility that a natural disaster occurs during a postponement of the investment has been accounted for by introducing a regular dividend payment that is proportional to the social value of the risk reduction project. In this way, forgone benefits in terms of reduced disaster losses could be integrated in the model. By defining a stochastic process that describes the evolution of the project's social price over time on basis of steady and disaster imposed information gains, eventually a discrete pricing formula for the value to postpone the investment has been derived.

The limitations of the suggested approach have clearly been outlined. An application of financial real option theory to public project appraisal certainly goes in line with the requirement that the prerequisites for applying real option theory are fulfilled. Critical in this respect was the assumption that the project's cash flows could be replicated by the cash flows of a twin security that is traded on free markets or alternatively, the assumption of market completeness. The replicability of the cash flows must be satisfied in order to be able to rely on arbitrage free pricing, which is the fundamental principle of option pricing theory. In reality however, the replicability assumption is a rather strong requirement that will hardly be fulfilled. Therefore, the above presented derivation of the option price is to be seen as an approximation only.

The second encountered difficulty constituted the incorporation of disaster imposed information gains in the model as they caused the social value of the risk reduction project to jump in arbitrary directions and amounts. This further contributed to the complexity to construct a dynamic replicating portfolio. Therefore, it was assumed that the public decision maker was risk neutral with respect to the jump component, so that it could be included in the model by means of its expected value only. In comparison to the often placed hypothesis of public risk neutrality with respect to the whole project however, this constitutes a relaxation.

So far the derivation of the option pricing methodology has been a purely theoretical exercise. In the following chapter an application of the real option methodology to public risk mitigation measures in the flood risk context is presented in order to demonstrate the consistency of the above results.

Chapter 7

Application of the Proposed Methodology

The final chapter of this thesis is assigned to the task to demonstrate how the introduced risk management concepts are to be applied in practice by means of two selected case studies. In particular, the city of San Francisco, California constitutes the subject of study and is investigated with respect to earthquake risk. For this purpose, different earthquake scenarios are simulated by making use of the disaster loss estimation program HAZUS [108], developed for the U.S. Federal Emergency Management Agency (FEMA). Firstly, the results of the simulations are employed to assess the total risk of the city by systematically following the steps of the probabilistic risk management framework. In a next step, two possible public risk reduction interventions are considered that improve the seismic performance of diverse building classes according to the design levels prescribed by the U.S. Standards. By assessing the reduced disaster losses and potential cost of the two strategies, it is decided within a cost benefit analysis (CBA) whether the interventions are socially worthwhile. In a second illustrative example an application of the real option approach to flood risk management is discussed. In particular, the construction of a rain storage reservoir is evaluated by the new investment criterion and different decision outcomes in comparison to the net present value (NPV) rule are presented.

7.1 General Information about HAZUS

HAZUS, which stands for "Hazards U.S.", is an integrated computer-based framework developed by the National Institute of Building Sciences for the FEMA to provide a tool for estimating losses from natural disasters. In particular, the program enables the assessment of different kinds of losses in any study region throughout the U.S. due to earthquakes, storms and floods. The earthquake loss estimation module, which constitutes the focus of the present chapter, provides a decision support tool for assessing the effects of scenario earthquakes with user-defined magnitude and location. This forecasting capability allows anticipating the consequences of future seismic events and developing plans and strategies for risk reduction.

The methodology can be applied to study local, regional and state areas with differently exposed population. It is designed to produce loss estimates for use by federal, state, regional and local governments in planning for earthquake risk mitigation, preparedness, response and recovery. The program provides damage and loss estimates in a set of predefined building classes, infrastructural elements such as bridges, essential facilities such as hospitals, schools and fire stations, and high potential loss facilities such as nuclear power plants, dams and military installations. Thus, the program covers nearly all aspects of the built environment. Extensive national databases are embedded within HAZUS, containing information such as population demography in a study region, square footage for different occupancies of buildings and number and location of essential infrastructures.

HAZUS allows for a profound damage analysis of different system components. The damage of a structure is described in terms of nature and extent of damage exhibited by its components (beams, columns, walls, ceilings, piping, etc.). For example, such component damage descriptions as "shear walls are cracked", "ceiling tiles fell", "diagonal bracing buckled" or "wall panels fell out", used together with such terms as "some" and "most" would be sufficient to describe the nature and extent of overall building damage. Accordingly, damage is described by five damage states: none, slight, moderate, extensive and complete, as visualized in Figure 7.1.

Based on these damage state descriptions, the HAZUS methodology supplies two different damage functions for each building type and for structural and non-structural damage: fragility curves that describe the probability of reaching or exceeding different damage states given peak building response, and building capacity (push-over) curves that are used to determine peak building response. As the focus of this chapter lies on the estimation of losses, it is referred to the HAZUS technical manual for further details about damage evaluations [106].

Figure 7.1: Damage state classification [HAZUS User Manual [107]]

In the loss estimation module, HAZUS basically distinguishes between three types of losses: direct loss, indirect loss and casualties. In the direct economic loss estimation the previously gathered damage state information is evaluated and converted to monetary units (US$ 1994). The loss appraisal covers structural and nonstructural repair and replacement costs, the associated loss of building content and business inventory as well as loss of functionality. The latter includes in particular business interruption and rental income losses. Indirect economic losses in contrast, account for possible dislocations in economic sectors not sustaining direct damage. The extent of such losses depends upon such factors as the availability of alternative sources of supply and markets for products, the length of the production disturbance and deferability of production. The losses are given in terms of employment and income effects for the study region and are available up to 15 years after disaster occurrence.

In the assessment of human losses HAZUS takes into account the strong correlation between building damage (both structural and nonstructural) and the number and severity of casualties. Depending on earthquake intensities and associated damage states, HAZUS estimates the number of people that will potentially be injured and killed by the considered earthquake, based on population distribution data, inventory information (building stock distribution), damage state probabilities and casualty rates revised from ATC-13 [17]. The casualties are classified into four severity levels that describe the extent of the injuries and potential fatalities:

1. Severity Level 1: Injuries that require medical attention but hospitalization is not needed.

2. Severity Level 2: Injuries that require hospitalization but are not considered life-threatening.

3. Severity Level 3: Injuries that require hospitalization and may become life threatening if not adequately treated.

4. Severity Level 4: Victims are killed by the earthquake.

The casualty estimates are provided for three times of day: 2:00 AM, 2:00 PM and 5:00 PM. These times represent the periods of the day where different sectors of the community are at their peak occupancy loads. The 2:00 AM estimate considers that the residential occupancy load is maximum, the 2:00 PM estimate considers that the educational, commercial and industrial sector loads are maximum and 5:00 PM represents peak commute time.

To sum up, in a simplified form, the steps to perform a hazard scenario within HAZUS include:

1. The selection of the study area. The region of interest may be created on basis of census tracts, census blocks, counties or even the whole state.

2. The specification of the magnitude and location of the scenario earthquake. In developing the scenario earthquake, consideration should be given to potential fault locations.

3. Optionally, the provision of additional information describing local soil and geological conditions.

4. The use of the formulas embedded in HAZUS to compute probability distributions for damage to different classes of buildings, facilities and lifeline system components.

5. The use of damage and functionality information to compute estimates of direct economic loss, casualties and shelter needs and indirect economic impacts on the regional economy for the years following the earthquake.

These steps are now applied for selected earthquakes scenarios affecting the city of San Francisco, California.

7.2 Seismic Risk Management for the City of San Francisco

In this section, earthquake scenarios corresponding to five different return periods and magnitudes are simulated for the city of San Francisco, California. These in turn are employed for a total seismic risk estimation and the identification of risk treatment alternatives by consecutively following the steps of the probabilistic risk management framework introduced in Chapter 2.

7.2.1 System Definition: Characteristics of the Study Region

As outlined in Chapter 2, the risk management framework starts with the definition of the system and the identification of risks that might put the functionality of the system in danger. The sole risk to be analyzed in the following is associated with the occurrence of seismic events in the region of San Francisco, California. The geographical size of the region is 47.26 square miles and it is composed of 176 census tracts, as depicted in Figure 7.2. There are over 329,000 households living in the region and the total population amounts to 776,733 people (US Census Bureau data for the year 2000).

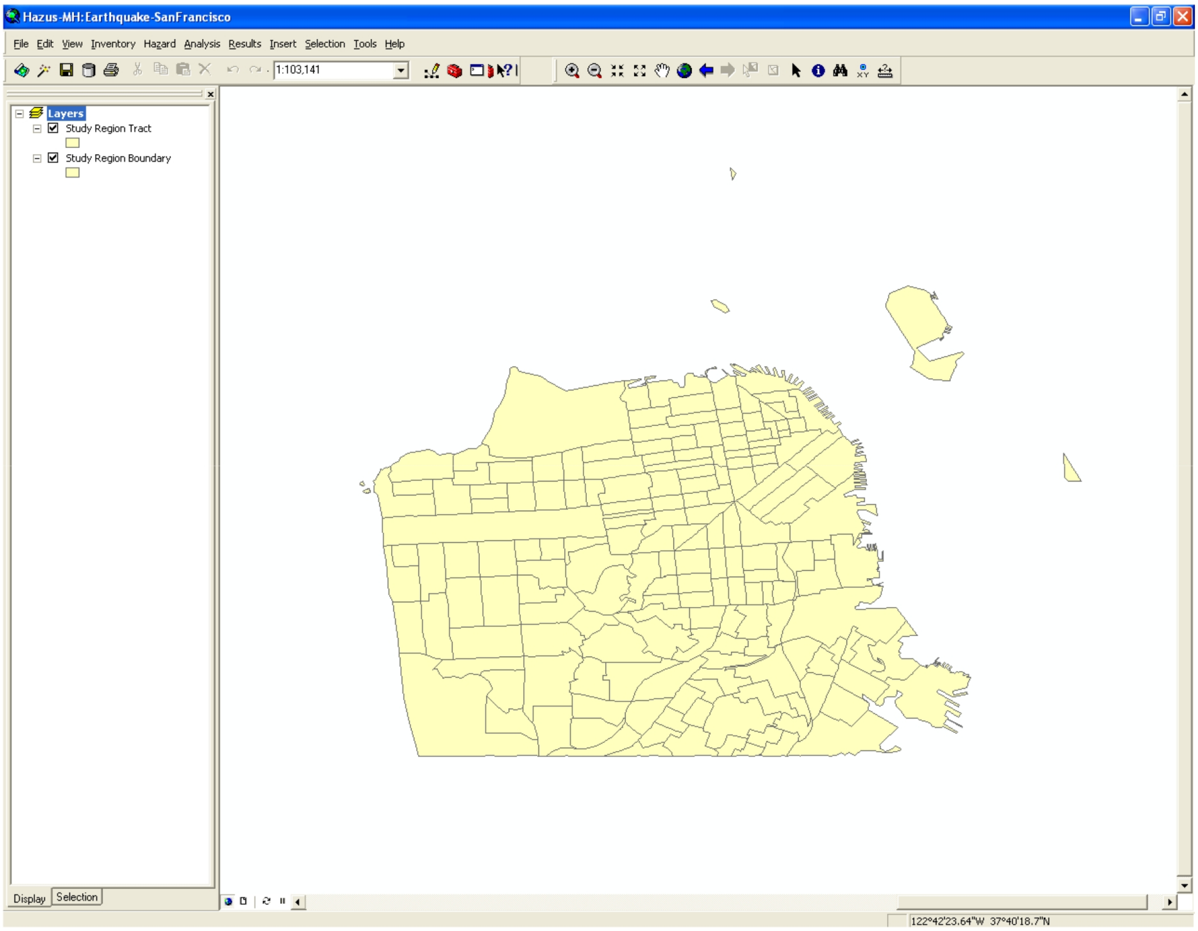


Figure 7.2: Map of the study region with census tract boundaries [HAZUS [108]]

The total building stock of the city is composed of an estimated 158,000 buildings with a total building replacement value of around US$ 84 billion[1], excluding contents. Approximately 94% of the building stock and 73% of the building value are associated with residential housing. The replacement value of the transportation and utility lifeline systems is estimated to be US$ 2,337 and US$ 627 million, respectively. However, for reasons of simplicity, damages and losses corresponding to transportation and utility lifelines are not being accounted for in the subsequent analysis.

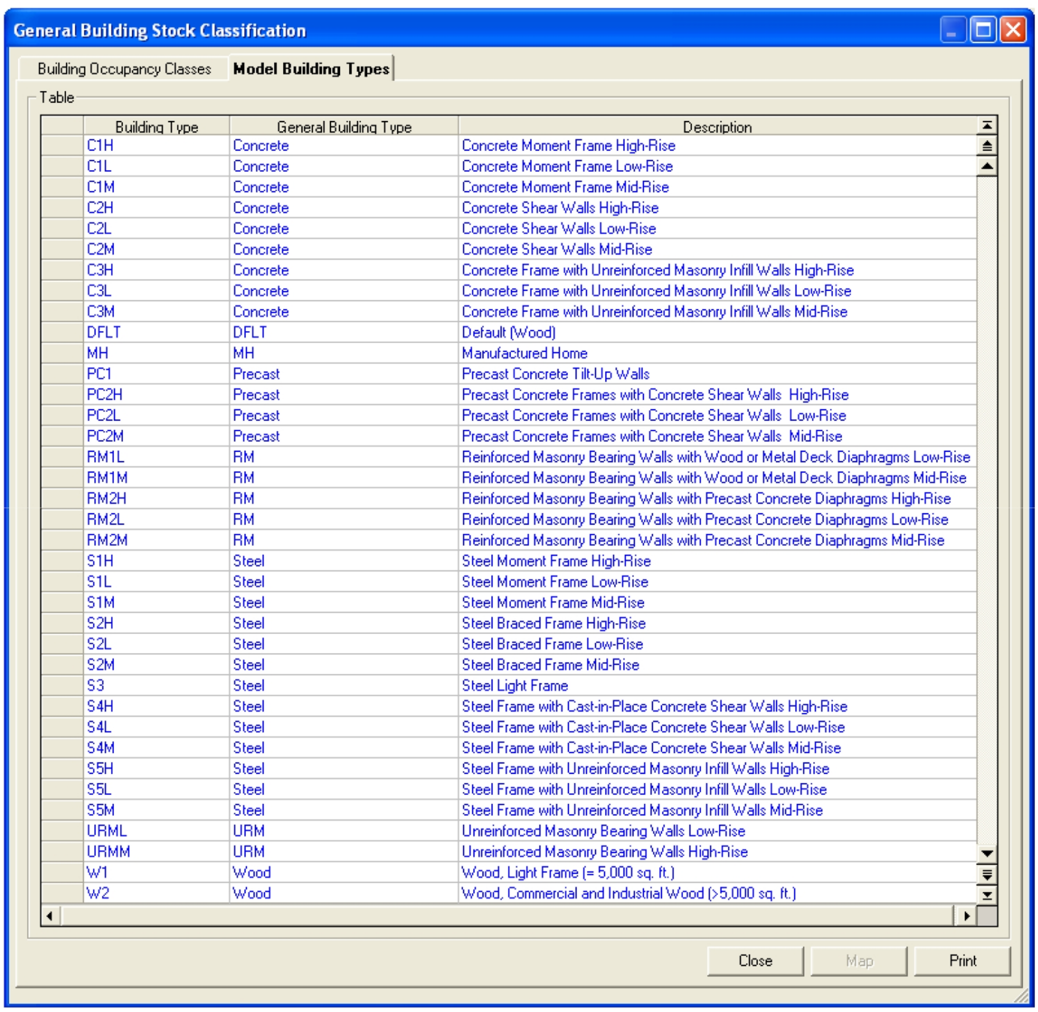
7.2.2 Hazard Assessment

In order to initiate the risk analysis procedure, a hazard assessment for the study region has to be carried out. The first step of hazard assessment constitutes the hazard analysis phase where the intensity and frequency of earthquake events have to be estimated that are relevant for the study region. The two elements frequency and intensity are closely related and their modeling often

[1] Corresponds to US$ 117.5 billion in 2007 dollars.

requires similar sets of data. One possibility is to rely purely on historical records of past events. For this purpose, HAZUS provides the possibility to select from a huge database of past events that are available within the deterministic procedure for simulating earthquake scenarios offered by the program. By this feature, historical events may be reconstructed and parameters modified to study potential risk reduction effects.

The second and more extensive procedure to estimate the relevant intensity and frequency parameters is to rely on probability distributions that have been constructed on basis of historical data, expert opinions and statistical techniques that take into account well-established scientific principles and an understanding of how natural hazards behave. In this way, it is possible to include also rare extreme events that exceed historical records. For this intent, HAZUS enables characterizing the ground shaking demand by means of probabilistic seismic hazard contour maps developed by the United States Geological Survey (USGS). The methodology includes maps for eight probabilistic hazard levels, ranging from 100-year to 2500-year return periods.



General Building Stock Classification

Building Occupancy Classes | Model Building Types

Table

Building Type	General Building Type	Description
C1H	Concrete	Concrete Moment Frame High-Rise
C1L	Concrete	Concrete Moment Frame Low-Rise
C1M	Concrete	Concrete Moment Frame Mid-Rise
C2H	Concrete	Concrete Shear Walls High-Rise
C2L	Concrete	Concrete Shear Walls Low-Rise
C2M	Concrete	Concrete Shear Walls Mid-Rise
C3H	Concrete	Concrete Frame with Unreinforced Masonry Infill Walls High-Rise
C3L	Concrete	Concrete Frame with Unreinforced Masonry Infill Walls Low-Rise
C3M	Concrete	Concrete Frame with Unreinforced Masonry Infill Walls Mid-Rise
DFLT	DFLT	Default (Wood)
MH	MH	Manufactured Home
PC1	Precast	Precast Concrete Tilt-Up Walls
PC2H	Precast	Precast Concrete Frames with Concrete Shear Walls High-Rise
PC2L	Precast	Precast Concrete Frames with Concrete Shear Walls Low-Rise
PC2M	Precast	Precast Concrete Frames with Concrete Shear Walls Mid-Rise
RM1L	RM	Reinforced Masonry Bearing Walls with Wood or Metal Deck Diaphragms Low-Rise
RM1M	RM	Reinforced Masonry Bearing Walls with Wood or Metal Deck Diaphragms Mid-Rise
RM2H	RM	Reinforced Masonry Bearing Walls with Precast Concrete Diaphragms High-Rise
RM2L	RM	Reinforced Masonry Bearing Walls with Precast Concrete Diaphragms Low-Rise
RM2M	RM	Reinforced Masonry Bearing Walls with Precast Concrete Diaphragms Mid-Rise
S1H	Steel	Steel Moment Frame High-Rise
S1L	Steel	Steel Moment Frame Low-Rise
S1M	Steel	Steel Moment Frame Mid-Rise
S2H	Steel	Steel Braced Frame High-Rise
S2L	Steel	Steel Braced Frame Low-Rise
S2M	Steel	Steel Braced Frame Mid-Rise
S3	Steel	Steel Light Frame
S4H	Steel	Steel Frame with Cast-in-Place Concrete Shear Walls High-Rise
S4L	Steel	Steel Frame with Cast-in-Place Concrete Shear Walls Low-Rise
S4M	Steel	Steel Frame with Cast-in-Place Concrete Shear Walls Mid-Rise
S5H	Steel	Steel Frame with Unreinforced Masonry Infill Walls High-Rise
S5L	Steel	Steel Frame with Unreinforced Masonry Infill Walls Low-Rise
S5M	Steel	Steel Frame with Unreinforced Masonry Infill Walls Mid-Rise
URML	URM	Unreinforced Masonry Bearing Walls Low-Rise
URMM	URM	Unreinforced Masonry Bearing Walls High-Rise
W1	Wood	Wood, Light Frame (= 5,000 sq. ft.)
W2	Wood	Wood, Commercial and Industrial Wood (>5,000 sq. ft.)

Close | Map | Print

Figure 7.3: Building classification by model building type [HAZUS [108]]

The secondly introduced probabilistic approach has been adopted in the present study. In particular, five different hazard scenarios have been considered, corresponding to return periods of 100, 250, 500, 1000 and 2500 years and user defined moment magnitudes of 6.9, 7.2, 7.5, 7.7 and 7.9, respectively. The moment magnitude values have been specifically derived for the city of San Francisco as proposed in [239]. In particular, the peak ground accelerations (PGA) corresponding to different exceedance probabilities have been deduced and related to the earthquake magnitudes using the following equation[2]:

$$\log PGA = 0.67 + 0.43 M_w - 1.08 \log(D_e^2 + 7^2)^{\frac{1}{2}} \pm 0.35 \tag{7.1}$$

Formula (7.1) is suitable for moderate to large earthquakes and has been extracted from a huge data set. In this equation PGA represents the peak ground acceleration measured in g, M_w corresponds to the moment magnitude, D_e is the distance to the epicenter in kilometers and the last term expresses the standard deviation of the predicted value. To determine the distance to the epicenter, a standard value of 10 km has been assigned, which approximately corresponds to the shortest distance of San Francisco to the San Andreas fault. The San Andreas fault is a continental transform fault that runs at a length of around 1,300 km through California and forms the tectonic boundary between the North American and the Pacific plate. Throughout recorded history it has been observed that the epicenters of numerous significant earthquakes in California were allocated along the San Andreas fault [270].

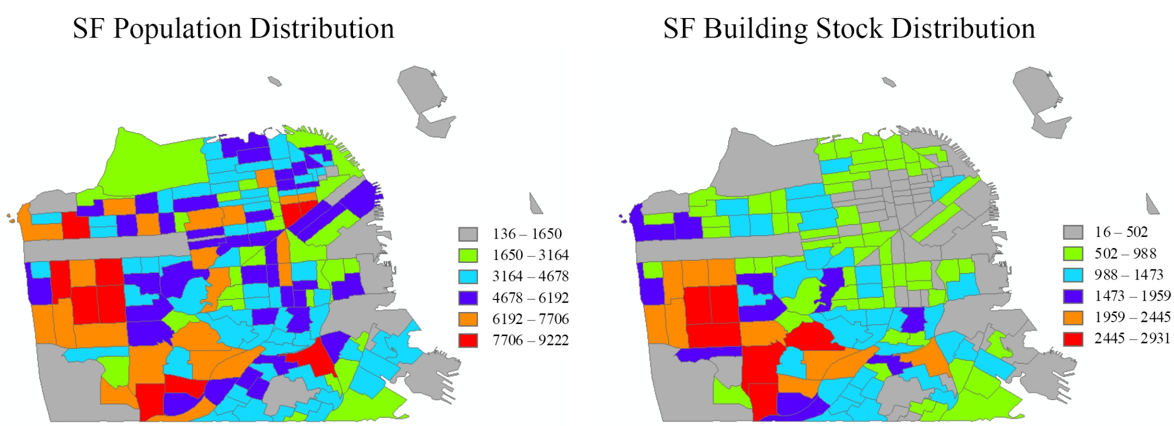


Figure 7.4: Distribution of population and building stock in SF [HAZUS [108]]

As the relevant intensities and frequencies of the earthquake hazards have been estimated, it is proceeded with the decomposition of the system in the elements at risk (EaR). The elements at risk subdivision is carried out within the HAZUS program automatically. In particular, buildings with similar characteristics influencing their structural response are classified into model building types which correspond to the EaR classes, as outlined in Chapter 2. Typical characteristics of each model building type are construction material (wood, steel, concrete, masonry), number of stories and square footage, following the classification system of FEMA 178 [80]. An overview of the building classification is provided in Figure 7.3. Furthermore, for each EaR class additional information about occupancy in terms of functional use of the building (residential, commercial, industrial, educational, etc.), number of occupants, dollar exposure and foundation type is provided. The total population and building stock exposure distributed over the census tracts is depicted in Figure 7.4.

[2]It is worth noting that a number of alternative equations that cover the relationship between PGA and moment magnitude are available in literature [43], highlighting the fact that there is no general scientific consensus on this issue.

The final step in the hazard assessment phase is to convert the hazard intensity occurring at the epicenter to a hazard load that is affecting the EaR classes. This is partly covered by equation (7.1) that transforms the moment magnitude M_w into an approximate PGA for the city of San Francisco. The further evolution of the hazard within the city is carried out by HAZUS automatically, taking into account local soil and geographical characteristics, so that eventually the hazard load for each EaR is obtained.

7.2.3 Damage Assessment

A crucial characteristic that significantly influences the performance of a building under seismic excitation is its seismic design level as dictated by modern codes, such as the 1976 Uniform Building Code, the 1985 NEHRP Provisions or later editions of these model codes. Model building types may be designed to either high-code, moderate-code, or low-code seismic standards, or not seismically designed at all. The program integrated damage functions in terms of capacity and fragility curves are differentiated with respect to the above mentioned design levels.

The study area of San Francisco is appropriately modeled using building damage functions with a seismic design level that corresponds to the seismic zone 4 of the 1976 Uniform Building Code or map area 7 of the 1985 NEHRP Provisions. In areas of high seismicity like San Francisco, buildings of newer construction (i.e. post 1973) are described by high-code damage functions, while buildings of older construction are represented by moderate-code damage functions, if built after about 1940, or mostly by low-code damage functions, if built before about 1940. Accordingly, the whole building stock of San Francisco consists of 2.2% low-code buildings, 64.5% moderate-code buildings and 33.3% high-code buildings, which corresponds to a total number of 3,431, 102,076 and 52,802 buildings, respectively.

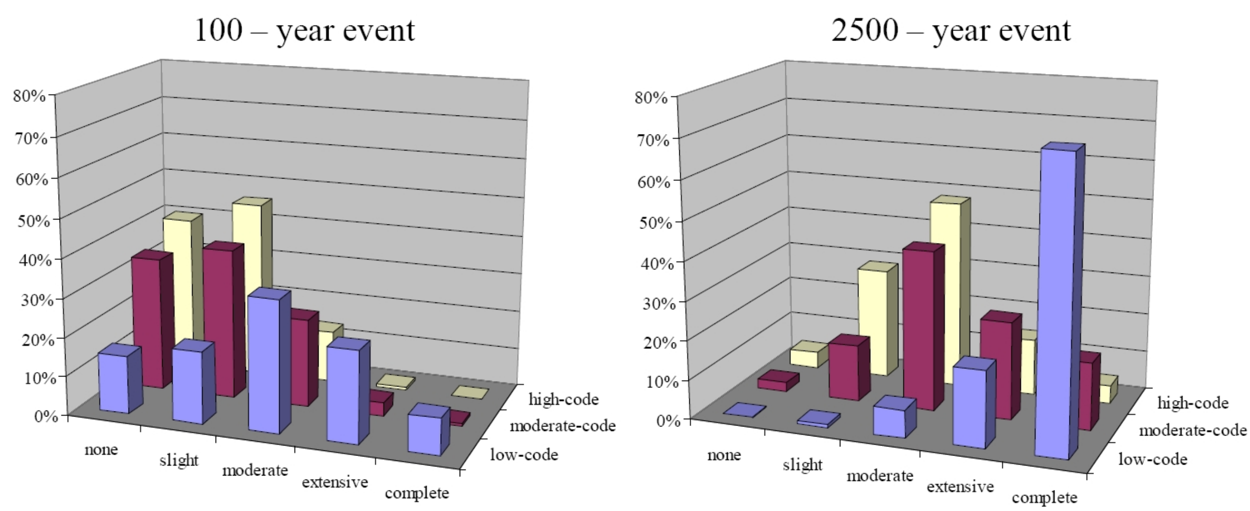


Figure 7.5: Damage states for different design levels

In the five different earthquake scenarios that have been simulated for the city of San Francisco, the total damage of the city's building stock has been assessed with respect to the damage states introduced above and damaged buildings have been grouped according to their design code levels. In Figure 7.5 the simulation results have been processed for the smallest and largest considered event, i.e. a 100-year earthquake with a moment magnitude of 6.9 and a 2500-year earthquake with a moment magnitude of 7.9, respectively. In particular, it is shown for each of the three design levels, how damage distributes over the considered damage states. Each column in the graph represents the fraction of buildings falling into a certain damage state relative to the total building population of each design level in the city.

As expected, the low design buildings exhibit generally much higher probabilities that high damages will occur across all simulated earthquake scenarios. Whereas for the 100-year event the low code buildings are almost normally distributed over the damage states, the moderately and even more the highly designed buildings are heavily skewed to the left, providing a much higher level of safety. The probability mass of the distribution moves consecutively to the right as the earthquake intensity is increased and more rare events are considered. This is especially emphasized by the 2500-year chart, where over 70% of the low designed buildings are expected to collapse, 66% of the moderate code buildings exhibit moderate or extensive damage, wheras 81% of the highly designed buildings feature at most moderate damage and thus, still perform very well. A detailed overview of the damage state results for each considered return period and design level is shown in Appendix F.1.

7.2.4 Loss Assessment

As the damage states across the total building stock of San Francisco have been assessed for the different earthquake scenarios and processed with respect to the different design levels, the next step of the risk analysis phase constitutes the estimation of losses that are expected to go in line with the given damage states. As outlined above, there are in particular three major loss categories that are assessable within HAZUS: casualties, direct economic losses and indirect economic losses. With respect to these loss categories, HAZUS generally does not provide the possibility to directly assign the respective losses to each design level. Instead, losses are supplied by location, occupancy and building type as well as for the city as a whole by means of global summary reports. As it is intended to study the effects of public risk reduction interventions in terms of an update of seismic design codes however, the assignment of losses to design levels is necessary for performing a CBA. How this has been achieved is explained subsequently for each loss category separately.

Casualties

The casualty estimation has been calibrated with respect to the day time of 2:00 PM, which showed the most severe results throughout all performed simulations. The casualty data supplied by the program is available either by occupancy (commercial, residential, etc.) and building type (W1, W2, etc.), and is subdivided by the above sketched severity levels indicating the degree of injury or fatal outcome. However, this data was of limited use for the present analysis since no reference to the design levels is provided. Thus, in order to allocate the casualties to the three building design levels, the calculation of casualties has been carried out separately. For this purpose, firstly the average number of occupants per building has been approximated by the following equation:

$$\text{Average no. of people/building} = \frac{\text{Total population of SF}}{\text{Total no. of buildings}} = 4.9 \tag{7.2}$$

Thus, the simplifying assumption has been made that all inhabitants linger inside buildings at the moment of disaster occurrence, so that only indoor casualties have been accounted for in the simulations. Based on this number it has been calculated subsequently, how many people were staying in buildings of a certain design level damaged according to a certain damage state. In this respect, the previously obtained data of the damage assessment phase have been used, which are summarized in Appendix F.1. Having the average number of occupants per damaged building of each design level on hand, the casualty rates provided in the HAZUS manual [106] have been employed to calculate the corresponding number of casualties. The casualty rates indicate the likelihood of suffering a certain degree of injury when staying in a building of a certain damage state and are inferred from data statistics and combined with expert opinion. An exemplary table of these coefficients is provided in Appendix F.6 for the case of extensive building damage. The so obtained results are summarized in Appendix F.3 for each design level and earthquake scenario.

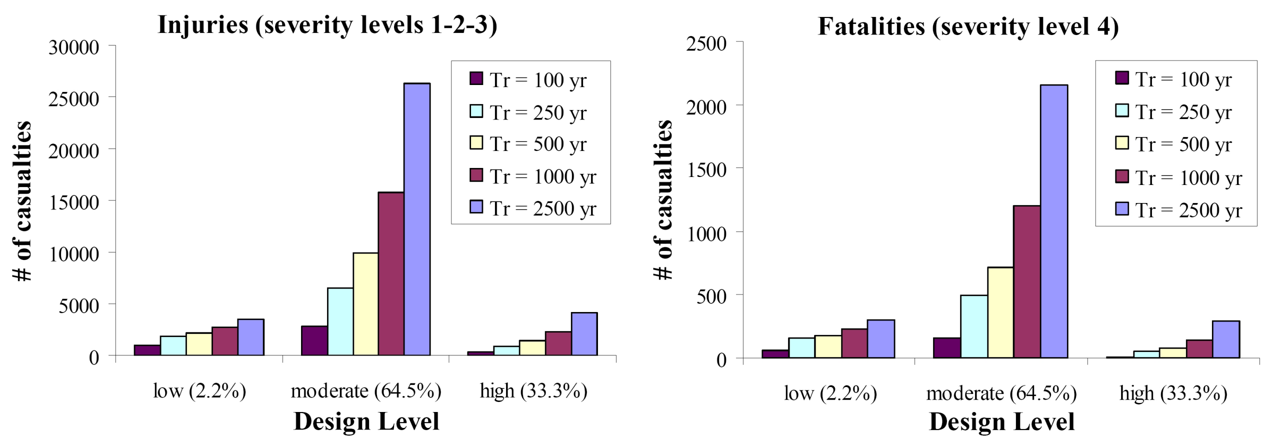


Figure 7.6: Casualties by design level and hazard scenario

In Figure 7.6 the absolute number of casualties for each design code level is displayed for each hazard scenario, subdivided by non fatal and fatal casualties. The percentage numbers in brackets on the axis of abscissae correspond to the fraction of respectively designed buildings relative to San Francisco's total building population. This explains the comparatively higher number of injuries and fatalities within the moderate design class in comparison to the low design class. If the casualties are scaled per 1000 buildings of the respective design classes in contrast, the expected outcome of having the greatest number of casualties in the low design class, followed by the moderate and the high design classes is observed, as shown in Figure 7.7.

In particular, in the move from an average low design class to an average moderate design class building, a decrease of around 85% in the fatality rate is recorded in media throughout all hazard scenarios, while the move from an average moderate design to an average high design building leads to a decrease in the fatality rate of around 80%. Similar observations can be made with respect to the number of injuries. The total number of fatalities for the lowest considered 100-year event amount to 234 at city level, while for the highest 2500-year event 2,756 fatalities are recorded.

In order to obtain a statement about the accuracy of the casualty estimates, the separately calculated results have been summed over the three design levels to assess the casualties at city level, which in turn have been confronted with the direct output of the program. Throughout all hazard scenarios, the compliance with the HAZUS results turned out to be sound as the maximum observed deviation was in the range of 15%.

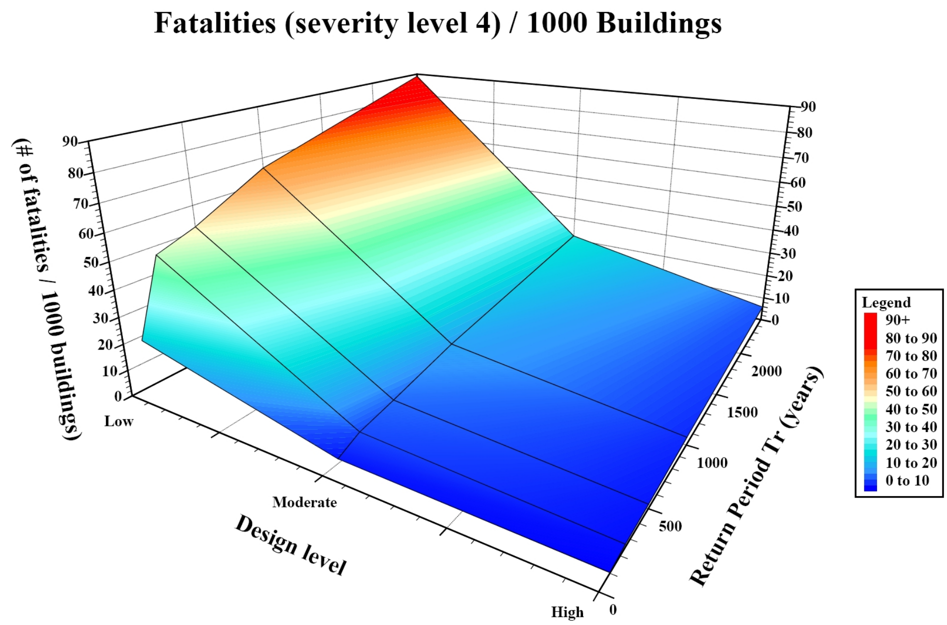


Figure 7.7: Fatalities per 1000 buildings of respective design class

Direct Economic Losses

In the assessment of direct economic losses, HAZUS distinguishes between capital stock and income losses. The capital stock losses subsume in particular the cost of structural and non-structural damage as well as content and inventory loss, while the income loss category mainly comprises the cost due to reduced building functionality, i.e. relocation-, capital related-, wage- and rental losses. For structural and non-structural damage costs, the results are displayable by design code level, whereas this information is not available for losses of the other categories. To overcome this lack of information, the assumption has been made that the losses related to the other categories arise proportionally to those of structural damage.

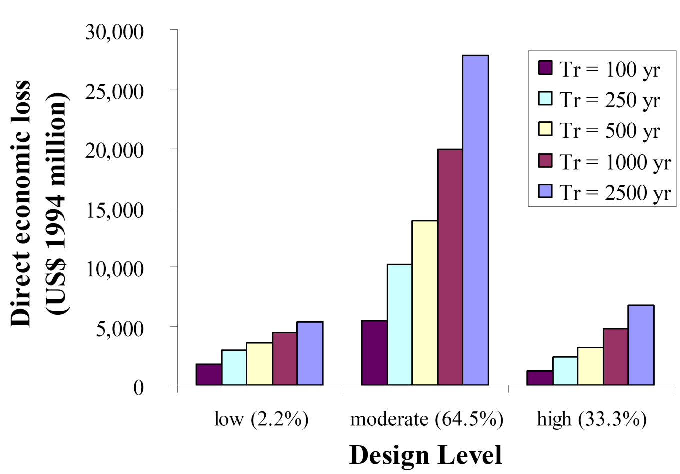


Figure 7.8: Total direct economic loss by design level and hazard scenario

In Figure 7.8 the total economic losses corresponding to the five considered hazard scenarios have been processed with respect to the building design levels where they accrue. For reasons mentioned above, the highest fraction of losses in absolute terms occurs in the moderate design building class throughout the considered scenarios, as 64.5% of San Francisco's total building stock is accordingly designed. The low and high design classes in contrast contribute to the total losses to approximately equal extent. The low design class however corresponds to only 2.2% of the total building stock, while highly designed buildings make up one third of the total building population.

If evaluated in relative terms on a per building level though, the losses are significantly higher in low design buildings and descending until the high design code is reached, confirming the intuition. The total direct economic losses for the 100-year earthquake throughout all design levels on city level sum up to US$ 8,489 million, while the total losses for the strongest considered 2500-year event amount to US$ 39,923 million. In Appendix F.4 detailed summary tables of the total direct economic losses are listed by design level, damage state and earthquake scenario.

Also for the direct economic losses the above sketched approach to assign direct economic losses to design code levels has been verified by confronting the results to the HAZUS outcome at city level. Also in this case the approximation matched the HAZUS results quite well and the deviations throughout the considered scenarios have been below 10%.

Indirect Economic Losses

In the assessment of indirect economic losses, HAZUS distinguishes between income and employment losses over the first 15 years following disaster occurrence. The discounting procedure to the current date is automatically done by the program. Whereas the income losses are given in monetary terms, the employment losses cover the change in the city's employment situation in number of workers. As it was not clear if the employments losses are already processed and evaluated within the income loss category, only the first indirect loss category has been included in the analysis.

Again, the program does not offer the possibility to display these losses by design code level, due to the fact that indirect losses generally occur throughout single building classes and potentially affect the whole system. Nevertheless, indirect losses have been set in proportion to total direct losses and accordingly distributed over the three design levels to enable their inclusion in a cost benefit analysis for design code updates.

Figure 7.9 displays the indirect losses for the simulated earthquake events. As the indirect losses have been related to the direct losses, the loss distribution over design levels resembles that of Figure 7.8, with different scales however. The indirect losses amount to roughly 30% of the direct losses throughout the considered scenarios. Accordingly, the 100-year event leads to indirect economic losses of US$ 2,664 million, whereas the 2500-year event results in US$ 12,820 million of indirect losses. The indirect economic loss results are summarized in Appendix F.5 by design level, occurrence in time and earthquake scenario.

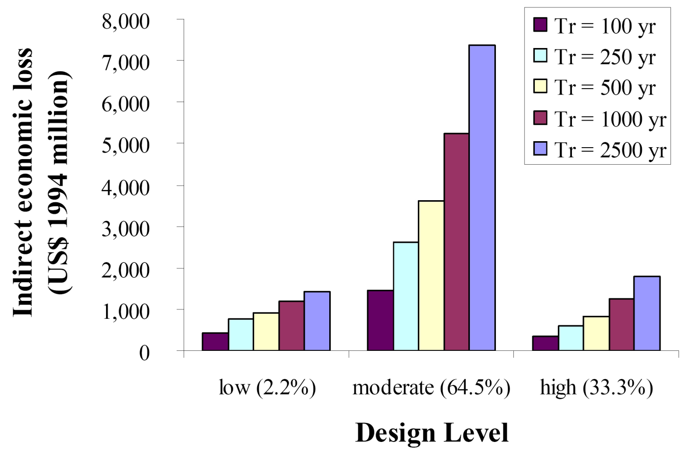


Figure 7.9: Total indirect economic loss by design level and hazard scenario

As performed for the other two loss categories, the separately calculated indirect economic losses have been compared to the HAZUS results showing differences of around 10% for each hazard scenario, which is considered to be in an acceptable range.

7.2.5 Total Risk Assessment

After the damage states for each design level and considered hazard scenario and the corresponding losses have been assessed, the final step in the risk analysis phase constitutes the calculation of risk. For the purpose of this case study, it is focused on losses and thus on the assessment of total risk. The total risk comprises all previously collected information about losses and links them to their probability of occurrence, to finally obtain an expected earthquake loss estimation for the city of San Francisco on an annual basis.

For this purpose, firstly the above obtained loss values for all three loss categories have been comprised in Table F.10 of the appendix, separated by hazard scenario. The next step to be performed in the risk calculation is to convert these values in annualized losses, by means of their annual occurrence probabilities. In this respect, it is necessary to firstly transform the return period information in annual exceedance probabilities by means of formula (2.1) and then to convert the latter to annual occurrence probabilities. The relation between exceedance probabilities and annual occurrence probabilities is provided by formula (2.4). Table 7.1 summarizes the correspondence between return periods, exceedance probabilities and occurrence probabilities for the five considered hazard scenarios.

Tr (years)	Exceedence probability	Probability of occurrence
2500	0.0004	0.0004
1000	0.0010	0.0006
500	0.0020	0.0010
250	0.0040	0.0020
100	0.0100	0.0060
0	1.0000	0.9900

Table 7.1: Calculating annual occurrence probabilities from return periods

Having the annual occurrence probabilities for each hazard scenario on hand, the annualized losses for each hazard scenario are easily calculated by taking the product of the total loss values provided in Table F.10 and the annual occurrence probabilities, listed in Table 7.1 above. The so obtained results for the annualized losses are summarized in Table 7.2 by design level and hazard scenario.

Annualized losses							
Return period (years)	*Design level*	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
				severity 1	severity 2	severity 3	severity 4
100	*low*	10.7	3.1	4.10	1.26	0.19	0.38
	moderate	32.9	10.5	12.87	3.37	0.48	0.95
	high	7.3	2.4	1.74	0.30	0.04	0.07
250	*low*	5.9	1.8	2.69	0.81	0.16	0.32
	moderate	20.4	6.3	9.83	2.66	0.50	1.00
	high	4.7	1.4	1.43	0.30	0.05	0.10
500	*low*	3.6	1.1	1.55	0.57	0.09	0.18
	moderate	13.9	4.4	7.18	2.38	0.36	0.72
	high	3.2	1.0	1.05	0.28	0.04	0.08
1000	*low*	2.7	0.9	1.17	0.38	0.07	0.14
	moderate	11.9	3.8	6.98	2.11	0.35	0.72
	high	2.8	0.9	1.05	0.27	0.04	0.09
2500	*low*	2.1	0.7	0.99	0.34	0.06	0.12
	moderate	11.1	3.6	7.61	2.48	0.41	0.86
	high	2.7	0.9	1.23	0.36	0.06	0.12

Table 7.2: Total annualized losses by design level and hazard scenario

From Table 7.2 the relative contributions of the considered earthquake scenarios to the total annualized losses become obvious. While comparatively frequent events with lower return periods lead to lower absolute losses, their relative share in the total annualized losses is rather significant due to their higher annual occurrence probabilities. In terms of direct and indirect economic losses the contribution of the 100-year event to total annualized losses is approximately three to five times higher than that of the 2500-year earthquake across all design levels. For human losses in contrast, the differences across hazard scenarios are less emphasized. This is due to the fact that stronger intensity events lead to a greater fraction of severely damaged buildings within the total damaged building stock which in turn disproportionately strongly increases the probability of getting injured or even killed by the disaster. This in turn compensates the low associated occurrence probabilities.

Total Risk						
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	25.0	7.6	10.5	3.4	0.6	1.1
moderate	90.2	28.6	44.5	13.0	2.1	4.2
high	20.7	6.6	6.5	1.5	0.2	0.5
total	135.9	42.8	61.5	17.9	2.9	5.8

Table 7.3: Total earthquake risk by design levels

To finally obtain the total earthquake risk for the city of San Francisco, the annualized losses need to be summed up over all hazard intensities, in line with formula (2.19). As outlined in Chapter 2, this accumulation may either be performed for each loss category separately or alternatively over all categories simultaneously, if a reasonable way has been found to convert the intangible disaster losses to monetary value. In Table 7.3 the total earthquake risk is listed for each considered loss category apart, still subdivided into the three different design levels, which will turn out to be useful in the subsequent CBA.

Casualty severity level	Fraction of VSL	Value (US$ 2007)
1	0.002	8,312
2	0.037	151,697
3	0.475	1,974,141
4	1.000	4,156,087

Table 7.4: Value of statistical life and injury valuation

The last step to be performed in order to comprise the accumulated loss data into a single value, which then represents the total risk covering all loss categories, is to place a monetary value on the separately listed injuries and fatalities. How to value statistical lives has been one major topic of this thesis and extensively studied throughout Chapters 4 and 5. For the purpose of total risk calculation, the valuation is based on the value of statistical life (VSL) obtained from the Life Quality Index (LQI). The estimations of LQI based VSLs for different countries have been carried out previously and are summarized in Table 5.3. The value for the U.S. amounted to US$ 4.156 million which is adopted for the present calculation.

As holds for the VSL, also the valuation of injuries is a highly debated issue within safety literature and so far no general consensus has been achieved. An interesting approach to include non fatal human consequences in form of injuries or diseases in LQI based safety evaluations is presented in [151]. For the present study however, it is necessary to distinguish between the above outlined severity levels 1, 2 and 3 in appraising the injuries. For this purpose, the U.S. Department of Transportation [71] suggests to value injuries as a proportion of the VSL depending on severity. The proposed fractions have been transformed to the injuries classification of HAZUS and are summarized in Table 7.4.

For the final total risk calculation it has to be noted that the HAZUS dollar values of direct and indirect economic losses listed in Table 7.3 are provided in terms of US$ 1994, whereas the VSL obtained from the LQI has been estimated in US$ 2007 . Therefore, for reasons of consistency, the HAZUS results have been converted to US$ 2007 by using the consumer price index to account for inflation. Accordingly, the value of US$ 1 in 1994 corresponds to US$ 1.4 in 2007 [269]. Accounting for these adjustments, the following estimate for the total earthquake risk of San Francisco is obtained

$$\text{Total SF Earthquake Risk} \quad = \quad 283 \quad \left[\frac{\text{US\$ 2007 million}}{\text{year}}\right] \tag{7.3}$$

which includes all loss categories and in particular the monetized human losses. The fraction of direct economic losses in the total risk is 67.2%, that of indirect economic losses 21.1%, while the contributions of fatalities and total non fatal injuries amount to 8.6% and 3.1%, respectively. The comparatively low share of human losses in the total risk highlights the fact that the seismic safety standards in San Francisco are already on a very advanced level.

7.2.6 Risk Reduction Interventions

After the total earthquake risk of San Francisco has been assessed, risk reduction interventions may be implemented within the risk treatment phase. In particular, two public risk reduction interventions are considered in the following. Both strategies examine a hypothetical update of the buildings' design code level in order to meet the modern code requirements at portfolio, i.e. city level. The first strategy consists in rehabilitating all low design code buildings to reach the moderate design code level. The second strategy in contrast, analyzes the hypothetical retrofitting of all low and moderate design code buildings until they attain the high design code. In the subsequent section, a detailed CBA shall reveal if these potential mitigation strategies are socially worthwhile.

7.3 Cost Benefit Analysis

As the status quo seismic risk analysis of San Francisco has been completed, it is subject of this section to study the effects of the above outlined public risk mitigation interventions and to judge whether they are cost effective. In this respect it has to be investigated, how the interventions impact the total expected disaster losses over time and an estimation of the rehabilitation cost has to be carried out. As risk reduction interventions on a portfolio level are not directly implementable within HAZUS, the cost and benefit estimations of the mitigation measures have been carried out aside from the program, as illustrated in the following.

7.3.1 Cost Estimation

The cost component of the two considered public risk reduction interventions is given by the rehabilitation cost that is necessary to elevate the respective parts of San Francisco's building stock to an advanced seismic performance level. The rehabilitation costs have been calculated according to FEMA 156 [81], that includes a comprehensive set of costs for the seismic rehabilitation of existing buildings based on a 2,088 buildings database. According to this regulation, the total rehabilitation cost C^T can be expressed as the product of different cost factors. These include in particular a typical cost (so called Building Group Mean Cost C_1) which represents the mean structural cost for the seismic rehabilitation of a building, excluding the cost of replacing architectural finishes. The typical cost is then building (-group) specifically modified by some adjustment factors C_i, i = 2,...,5. The adjustment factors account for building surface, seismicity of the area and performance objective, location and time to adjust for inflation.

The total cost C^T to seismically rehabilitate a building is thus estimated using the following equation

$$C^T = C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot C_5 \tag{7.4}$$

where C_1 is the Building Group Mean Cost (provided in US$ 1993/sq.ft.), C_2 corresponds to the Area Adjustment Factor, C_3 is the Seismicity and Performance Objective Adjustment Factor, C_4 is the Location Adjustment Factor and C_5 represents the Time Adjustment Factor. The values of each coefficient are given in [81], differentiating between eight building groups into which all building types listed in Figure 7.3 are clustered, based on cost distribution similarities. The specific values employed in the considered case study are summarized in Table 7.5. In the first column of the table the classification of the HAZUS building types according to [81] is provided, while the second column contains the number of buildings in San Francisco that fall into the respective category, including their medium building surface in 1000 sq.ft., which is included in the third column. The last two columns provide an overview over the factors C_1 and C_2, while the cost factors C_3, C_4 and C_5 have not been listed as they are equal for all building groups in San Francisco for each considered rehabilitation option.

FEMA 156 group no. (*)	# buildings	Average surface/building (1000 sq.ft.)	Group mean cost C_1 (US$ 1993/sq.ft.)	Area adjustment factor C_2
1 (URML)	1,458	9.0	15.3	1.01
2 (W1, W2)	141,280	2.8	12.3	1.02
3 (RM1L, PC1)	6,530	10.6	14.0	1.07
4 (C1L, C3L)	573	13.0	20.0	1.06
5 (S1L)	1,081	11.4	18.9	1.14
6 (S2L, S3)	1,357	9.7	7.2	1.12
8 (C2L, S4L, PC2L, RM2L)	6,047	10.1	17.3	1.08

(*) In brackets, the corresponding building types comprised in each group

Table 7.5: Rehabilitation cost factors by building group [FEMA 156 [81]]

The value for C_3 however, is particularly interesting in performing CBA for seismic design code updates. It contains information about the cost adjustment that is necessary to reach a certain performance objective. Therefore, it has been of special relevance to estimate the cost of the two considered portfolio mitigation measures. In this respect, it has to be noted that FEMA distinguishes between three rehabilitation levels, i.e. Life Safety (LS), Damage Control (DC) and Immediate Occupancy (IO), that do not coincide with the design levels defined in HAZUS, i.e. low, moderate and high design code. Therefore, the HAZUS code levels have been assigned to the FEMA performance levels with reasonable assumptions. In particular, rehabilitation to DC level is hypothesized to coincide with the attainment of moderate design code and rehabilitation to IO level is assumed to equal the retrofit to high design code. Accordingly, the coefficient C_3 for the retrofit from low to moderate design has been extracted to be 1.43, while the rehabilitation coefficient from low to high design is 2.08.

Eventually, the C_4 coefficient for location adjustment has been set to 1.12, which corresponds to the city of San Francisco, and the C_5 coefficient for the time adjustment has been determined by converting the US$ 1993 into US$ 2007 on basis of the consumer price index to account for inflation, resulting in a value of 1.43 [269].

Table 7.6 provides an overview about the three possible risk treatment options and their corresponding cost. The cost for the first RR intervention has been estimated as the rehabilitation cost necessary to rise the 2.2% low designed buildings of the total city's building stock to moderate code. In this respect, the simplifying assumption has been made, that the low code building distribution over building types coincides with that of the whole city to reduce complexity in calculation[3]. The final cost has been obtained by multiplying the mean rehabilitation cost in US$/sq.ft. per building group, given in formula (7.4), and the average surface of low design code buildings within the building group. For the second risk reduction intervention, the rehabilitation cost consists of bringing

[3]In reality however, in particular concrete, reinforced masonry and unreinforced masonry buildings take a disproportionately high share in the low design building fraction. The precise building distribution over building types is displayable for each design class within the program's inventory module.

the 2.2% of low design buildings and the 64.5% of moderate design buildings to the high code level. Also here it has been assumed for reasons of simplicity, that the moderate design buildings are distributed over building types representatively for the whole city's building stock[4]. The costs have been assessed analogously to the first risk reduction intervention.

Risk treatment strategies - Cost			
	Status quo	RR intervention 1	RR intervention 2
Strategy	no action	rise low design to moderate	rise low and moderate design to high
Cost (US$ 2007 million)	0	400	5,909

Table 7.6: Risk reduction interventions and their cost estimates

To validate the estimated cost data for the two considered risk reduction interventions, they have been compared to the results of the FEMA provided Online Seismic Rehabilitation Cost Calculator [82], which constitutes an interactive tool to calculate the seismic rehabilitation cost for distinct building types to attain certain performance objectives in different locations throughout the U.S.. As the calculator is based on the same regulation FEMA 156, the online results matched the manually calculated results very well.

7.3.2 Benefit Estimation

After the cost of the two considered interventions have been assessed, it is proceeded with the estimation of the measures' benefits to figure out if they are socially worthwhile. As outlined above, the benefits of a public risk reduction intervention consist of the reduced disaster losses after project implementation in comparison to status quo state. In this respect, the latter have to be assessed throughout the different loss categories and over time. The time period over which the benefits occur has been assumed to equal the design life, which is generally 50 years for buildings and common structures. Table 7.7 summarizes the benefits from each considered risk reduction strategy, subdivided in prevented (direct and indirect) economic losses and number of prevented injuries and fatalities. These benefits occur statistically every year, if the respective risk treatment option is implemented.

In order to estimate the benefits of the first considered risk reduction strategy to retrofit all low designed buildings in the city until they reach moderate design, the total annualized disaster losses provided in Table 7.3 have been recalculated by assuming that San Francisco's building stock was composed of 66.7% (= 64.5% + 2.2%) of moderately and 33.3% of highly designed buildings. In this respect, the losses accruing in the low design class have firstly been subtracted from the status quo annualized losses and in a second step the moderate losses have been augmented by the 2.2% according to their increased fraction in the city's total building population. The so obtained updated annualized losses after the implementation of the first intervention have finally been subtracted from the status quo annualized losses to net out the benefits of the intervention.

[4]Here, the assumption is well supported by the program's inventory information.

Risk treatment strategies - Benefit					
Strategy	Total economic benefit (US$ 2007 million)	# reduced casualties			
		severity 1	severity 2	severity 3	severity 4
Status quo	0	0	0	0	0
RR intervention 1	40.0	15.5	4.4	0.7	1.0
RR intervention 2	95.2	26.5	8.9	1.5	3.5

Table 7.7: Risk reduction interventions and their annual benefits

The assessment of benefits for the second risk reduction intervention to rehabilitate all non highly designed buildings to the high code standards has been performed by recalculating the total annualized status quo disaster losses under the assumption that San Francisco's total building stock was homogeneously consisting of high design code buildings only. With this ambition, firstly the losses accruing in the low and moderate design classes have been subtracted from status quo annualized losses and in a consecutive step, the losses emerging in the high design class have been proportionally augmented by 66.7%, which represents the increase of the high design building fraction in the total building population. Eventually, the so obtained updated disaster losses after the implementation of the second mitigation measure have been subtracted from annualized status quo losses to extract the benefits of the intervention.

7.3.3 Project Decision

In order to finally decide about the risk reduction measures under investigation, the projects' cost need to be confronted with the benefits that are expected to go in line with it. As the cost and benefits of the interventions have already been estimated and are on hand, what remains to be done is to convert the projects' benefits to monetary units and to make the cost and benefit streams time consistent by discounting future values back to the decision point. With this ambition, Table 7.4 above is firstly taken into account to place a monetary value on saved lives and prevented injuries based on the LQI's VSL estimations for the U.S.. Then, by means of Table 7.7 the cost and benefits of the two risk mitigation options relative to status quo state may be formulated in terms of cash flows occurring at different points in time, as shown in Table 7.8.

Cashflows (US$ 2007 million)			
Time (years)	Status quo	RR intervention 1	RR intervention 2
$t = 0$	0	-400	-5,909
$t = 1$	0	46.4	114.2
$t = 2$	0	46.4	114.2
...	...	...	...
$t = 50$	0	46.4	114.2

Table 7.8: Cash flows of the risk treatment strategies

As mentioned above, in the calculation of the projects' benefit stream it has been assumed that the structural retrofit of the two considered risk reduction strategies remains effective over the whole design period of the structures, which has been set equal to 50 years. Furthermore, it is hypothesized that the cost for the risk reduction interventions are instantly due in period $t = 0$, while the structural retrofitting is completed at the beginning of period $t = 1$ and remains equally effective over the whole design period until the end of the service life in $t = 50$ is reached. This in turn implies that the benefits of the risk reduction measures occur statistically every year and to equal amounts.

	Project Decision		
	Status quo	RR intervention 1	RR intervention 2
Cost (US$ 2007 million)	0	400	5,909
Benefit (US$ 2007 million)	0	932	2,296
NPV (US$ 2007 million)	0	532	-3,613
IRR (%)	0	11.53	-0.13
B/C ratio	0	2.33	0.39

Table 7.9: Project outcomes by decision rules

In order to make the benefits comparable to the project cost however, they need to be discounted back to the decision point $t = 0$ by means of an appropriate discount rate. According to the discussion in Section 5.7.4, the discount rate applied in this study has been determined according to formula (5.84) of the Ramsey Neoclassical Growth Model, which resulted in a discount rate of $r = 4.4\%$ for the U.S..[5] Taking into account this discount factor, the NPVs of the project alternatives are easily calculated by means of formula (3.8).

For project alternative one to retrofit all low design buildings until they are in compliance with the moderate code standards, the initial project cost have been calculated to be US$ 400 million, while the expected discounted benefits sum up to US$ 932 million over the 50 year design life, so that a NPV of US$ 532 million is obtained. Accordingly, this results in a benefit to cost ratio B/C of 2.33 and an internal rate of return (IRR) of 11.53%. Consequently, the expected benefits clearly outweigh the cost of the project by over 200%, so that a reliable recommendation to implement the project may be given.

For the second risk mitigation project under investigation to rehabilitate all non highly designed buildings of San Francisco to the high building code standards in contrast, a completely different outcome is obtained. In this case, the initial cost of the intervention sums up to an impressive US$ 5,909 million, as roughly 67% of the total city's building stock needs be retrofitted to attain the high design, while the expected discounted benefits of the measure are estimated to be less than half the cost, namely US$ 2,296 million. This yields to a benefit to cost ratio B/C of 0.39 and an IRR of -0.13%. As a result, the discounted benefits that are expected to go in line with the risk reduction strategy are significantly lower than the cost, so that clear advice to reject the project can be given. The results of the CBA are schematically summarized in Table 7.9.

[5] See Table 5.3 for details.

Sensitivity Analysis

In order to gain additional insight about the sensitivity of the project decision with respect to the dependent variables, a sensitivity analysis has been performed. As most of the variables impacting the final value of the risk reduction projects are implemented within HAZUS or predefined by the FEMA regulations and are thus modifiable only with prudent reasoning, the sensitivity analysis has been restricted to variations in the VSL and the employed discount rate r. In particular, the VSL estimates have been varied between US$ 1 and 10 million, representing the range where the great majority of values obtained in Chapters 4 and 5 fell into, while the discount rate has been changed in the range of 1 to 10%.

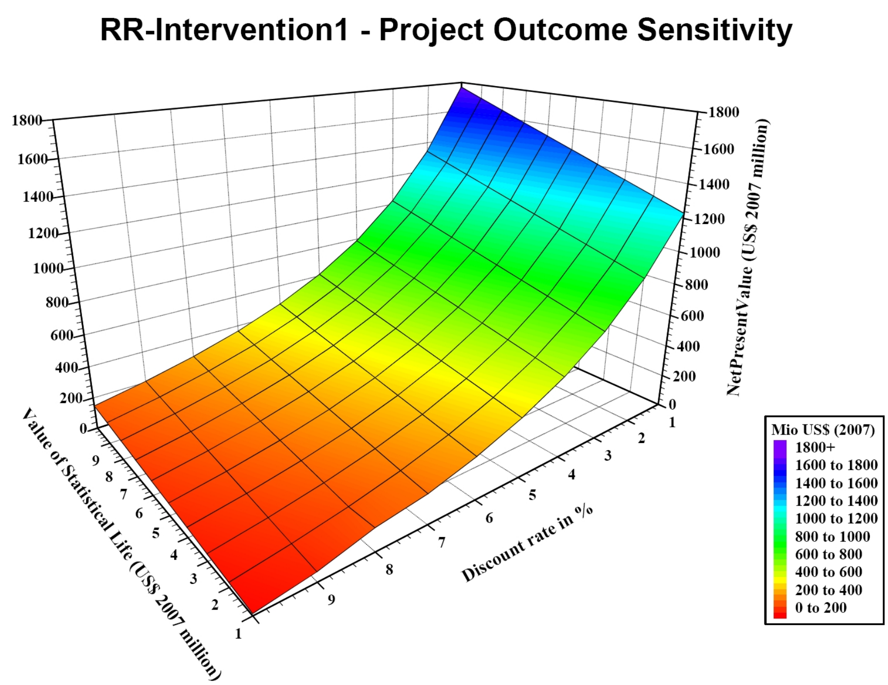


Figure 7.10: Sensitivity of project decision for RR intervention 1

The results of the sensitivity analysis for the first risk mitigation project are displayed in Figure 7.10 for all possible combinations of VSL and discount rate ranges. It is easily seen that project one is cost effective, i.e. yields a positive NPV, for all employed value combinations, so that the implementation of the project should strongly be supported by the public authority. For project two in contrast, the reverse situation turned out to hold: even if the lowest discount factor of 1% in combination with the highest VSL of US$ 10 million were assigned, the project still disclosed a negative NPV. Therefore, the decision to reject the project is well confirmed by the sensitivity analysis.

Discussion

The above performed CBA for the two considered risk reduction interventions delivered unambiguous results that could directly be employed to give sound project recommendations. In both cases, the cost and benefits of the respective risk reduction measures differed in order of magnitudes. Both initial outcomes have further been confirmed by a sensitivity analysis, varying the VSL and discount rate in reasonable ranges. As several parameters influencing the project outcome could not be reasonably changed for above discussed reasons and the difference between project cost and benefits has been that significant, the real option approach to account for uncertainties in the project decision turned out to be unsuitable for the present case study and has not been applied. To demonstrate the applicability of the latter, a separate illustrative case study is performed in the next section.

To draw a conclusion, the results of the performed risk analysis and CBA confirmed the very advanced earthquake safety standards in San Francisco. Only roughly 10% of the expected annual disaster losses are attributed to human losses, while the majority of losses are direct economic losses with a share of approximately 70% and indirect economic losses accounting for roughly 20%. Furthermore, it could be shown that a great fraction of total annualized losses accrue due to rather frequent events with lower return periods, especially when economic losses are concerned. With respect to fatalities and injuries in contrast, a more proportional distribution of losses throughout the considered earthquake scenarios is observed.

Finally, it has been demonstrated that the retrofit of the low design code buildings to moderate code could bring along large benefits on portfolio level. The rehabilitation to high code in contrast, is to be generally evaluated with care when confronting the benefits with the project cost. In certain cases of essential and high potential loss facilities[6] however, the attainment of high code standards generally constitutes a reasonable objective as additional benefits (such as cultural social historical (CSH) and environmental), that have not been accounted for in the present analysis, are expected to occur. If the project recommendations are transferable on an individual building level, the city's current distribution over code classes seems sound and is well supported by the present analysis.

7.4 Application of the Real Option Approach

In the concluding section of this thesis it shall be illustrated, how the innovative real option pricing model may be applied to decide on an investment possibility under uncertainty with flexibility about the investment timing. In particular, a hypothetical preventative risk reduction measure in flood control management is considered, which deals with the construction of a rain storage reservoir that reduces the expected losses of a model region in case of river flooding. It is assumed that the cost to implement the measure can be estimated with some accuracy, while the benefits in terms of reduced disaster losses are highly uncertain and hardly predictable reliably.

If a flood event with a 50-year return period is considered, both the expected losses in status quo condition as well as the estimation of the reduced losses after the implementation of the measure may only be approximated due to the epistemic uncertainties involved and the lack of reliable information on past events and expert opinions. It is further assumed that the flood probabilities have been assessed on basis of incomplete information about historical water levels that were only available for locations with a certain distance from the study region and are thus questionable in their reliability. Consequently, the estimation of the social value of the risk reduction project S_0 at the decision point is seen as a rough estimation only and the investment decision has to be made under substantial uncertainties. Furthermore, it is clear that the cost to construct the rain storage

[6] Such as nuclear power plants, military installations, hospitals and so on.

reservoir are not easily recaptured in case of adverse project development, so that the investment is irreversible.

In order to obtain better information about the effectiveness of the project, there is now the possibility to perform a study in the model region to collect further data about water levels at different measure points directly within the affected area. As the measurement requires some time to deliver accurate data about the variability of the water levels, a time period of two years is granted by the local authority to perform the study. The results of the study after the first year however might be processed to come to a precocious decision, if the data uncertainty could have already been sufficiently reduced. Accordingly, the maximum time to delay the investment decision is two years in order to obtain better information. For reasons of simplicity it is further hypothesized that the study is free of cost as it has already been scheduled prior to the evaluation of the intervention.

The influence of the information gains obtained by the study on the social value of the risk reduction project S_0 is modeled by the two parameters u and d, which describe the value of steady information gains in the evolution of S_0 over time. In particular, the probability of an upturn in the value is set equal to $u = 0.1$, while the probability of a downturn is fixed at $d = -0.09$, as it is assumed that the previously available information on historical water levels in status quo state systematically underestimates the true flooding probabilities, and thus $|u| > |d|$ holds. The annual probability that a 50-year or even a greater flood event occurs in the model region is calculated by taking the reciprocal of the return period according to equation (2.1) and is thus equal to 0.02.

Furthermore, it is hypothesized that smaller flood events than the 50-year flood do not lead to significant changes in the social value of the risk reduction project S_0 and are therefore not considered as disaster events that cause the value of the project to jump due to disaster imposed information gains. The jump component that causes an abrupt change in the value S_0 is to be applied only if a 50-year or stronger event occurs and can adopt the values -0.3 and 0.3, respectively. Accordingly, in case of disaster occurrence, the social value of the risk reduction project is to be either reduced or increased by the respective proportion and it is not known beforehand in which direction the disaster imposed information gain will make an adjustment in the estimated value necessary.

The risk free interest rate is set equal to 5% for the purpose of the considered application and the investment cost $C^T = P$ to construct the rain storage reservoir is arbitrarily set to US$ 1 million. The value of the risk reduction measure is assumed to be US$ 1.1 million under the information level at time $t = 0$[7]. Furthermore, the dividend rate D is assumed to be equal to 7%, indicating that the annualized benefits of the risk reduction project in terms of reduced disaster losses amount to US$ 77,000 per year on average. Accordingly, if a 50-year flood occurs in the model region, a reduction in total losses after the construction of the rain storage reservoir in the size of US$ 3.85 million (US$ 77,000/0.02) in comparison to the status quo situation is expected to be realized.

Under these assumptions, the net value of the possibility to invest in the construction of a rain storage reservoir including the flexibility to delay the investment can be assessed by calculating the call option price C_0 on the project. This is done through backwards induction from the second period $t = 2$: firstly, the option value is calculated in period $t = 2$ for all possible values of S_2, by making

[7] The assigned values for the construction of the rain storage reservoir as well as the project value have been arbitrarily chosen for illustrative purposes and do not claim to be in an authentic range.

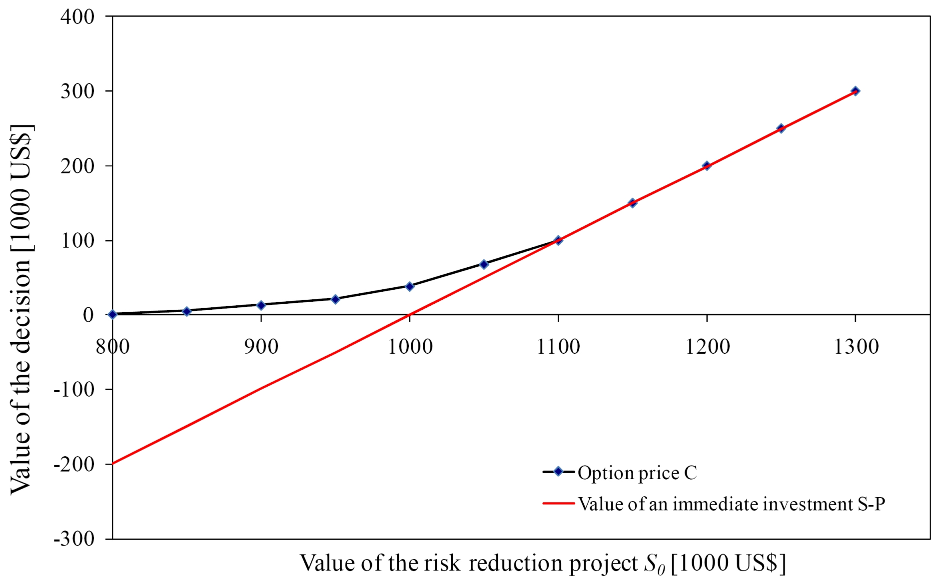


Figure 7.11: Variation of the project value

use of equation (6.1). These in turn are needed in order to determine the state contingent option values in $t = 1$, that are calculated by means of equation (6.30) under anticipation of the optimal investment strategy. The same procedure is then repeated to obtain the option value in $t = 0$ in dependence of the respective $t = 1$ values. Having calculated the option prices in the relevant time periods, the optimal investment strategy under the flexibility to postpone the investment can be determined under the new real option investment criterion.

For this purpose it is firstly analyzed, how a variation in the estimation of the initial project value S_0, that has been carried out under substantial uncertainty, affects the optimal investment decision. In Figure 7.11 the option value C is plotted against the payoff that would be realized if the project was immediately implemented. By means of the classical investment criterion based on the NPV, it would be advantageous to initiate the project, as soon as the value of the risk reduction project outweighed the investment cost, i.e. if $S > P$ was fulfilled and a payoff of $S - P$ would be realized. In the graph this corresponds to the area from which on the red straight line $S - P$ takes on positive values, which holds for $S >$ US$ 1 million.

With the new decision criterion including the possibility to delay the investment in contrast, an immediate project initiation is optimal only if the project value S is higher than the investment cost augmented by the value to postpone the investment and thus higher than the threshold value S^*, that has been characterized in Section 6.3.2. In the graph this is firstly fulfilled at the point where the option value curve C opens out into the straight line $S - P$ and for all values of S that are beyond this point. Accordingly, S^* takes on a value of around US$ 1.1 million and the NPV and the real option criterion give different advices if S falls in the range between US$ 1 and 1.1 million. Has the value of the risk reduction project been estimated to be higher than S^* however, the expected advantage in terms of a high immediate payoff outweighs the expected additional value of information derived from the study to prevent a misinvestment.

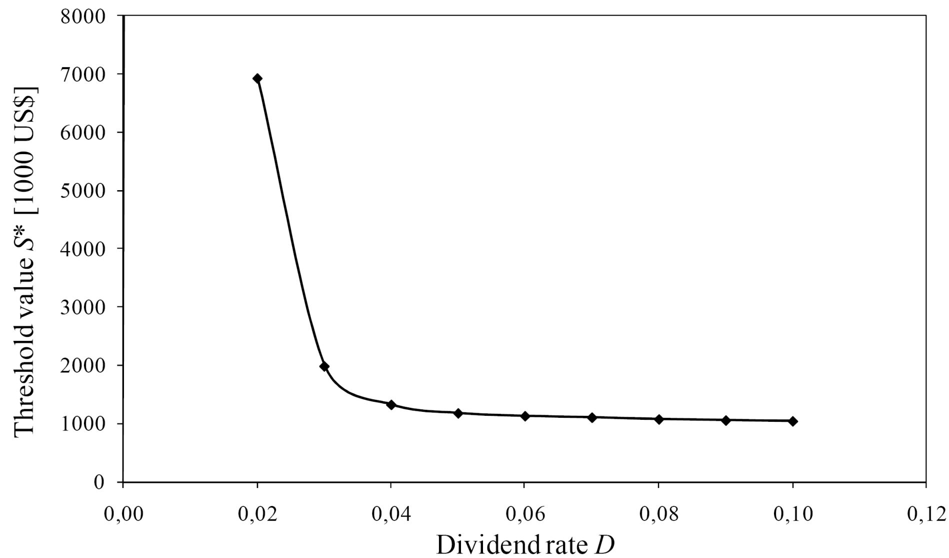


Figure 7.12: Variation of the dividend rate

Analogous quantitative observations are made when the dividend rate D is varied, holding all other variables at default settings. The dividend rate provides the incentives for an immediate project initiation, as it models the fraction of losses that accrue in case of flood occurrence in the time of postponing the investment, which could have been prevented by a project implementation in time. Therefore, the dividend payment is an indicator for the effectiveness of the risk reduction measure, with higher values corresponding to greater efficiency. This leads to the fact that the threshold value S^*, from which on an immediate investment becomes optimal, falls as the dividend rate is increased.

This logical interrelation is illustrated in Figure 7.12. If the dividend rate D takes on comparatively low values in the range of 0-5%, even a slight increase in the dividend rate leads to large decreases in the threshold value S^*, making an immediate investment more attractive. In this respect, it is to a lesser extent the absolute value of the dividend rate that is crucial for the investment decision, but rather the level of the dividend rate relative to the risk free interest rate r. The ratio of these two variables decisively influences the payoff of the invested means during the postponement of the investment and thus constitutes a major factor in determining the opportunity cost.

Accordingly, an increase in the dividend rate D leads to the fact that a delay of the investment becomes comparatively more expensive. If the dividend rate takes on values that are significantly above the risk free interest rate, an asymptotic convergence of the threshold value S^* to the investment cost P can be observed. This is seen in the figure from dividend rates of approximately 10% onwards. In this case, the risk of not having taken appropriate precautious action in the case of disaster occurrence outweighs the uncertainty with respect to project efficiency and the corresponding risk of misinvestment.

Chapter 8

Synopsis

This thesis is devoted to present a general approach for managing disaster risk in public interest. The main focus constitutes the investigation of public risk reduction interventions that affect human safety. In the following, a summary of the thesis' results is provided and new achievements are reviewed. Eventually, directions for future research are presented.

8.1 Summary

At the outset of this thesis it has been demonstrated, how widely the definitions and understandings of the term risk can range. Applied across various disciplines and often used in multidisciplinary collaborations, so far no consistency in the understanding of disaster risk has been achieved. The heterogeneity of risk definitions is mainly attributed to different understandings of the basal terms hazard, vulnerability, exposure, consequences, damage and loss. These terms occur repeatedly throughout different risk definitions and are often used interchangeably.

This lack of a harmonized concept has been addressed by introducing a clear and flexible risk management framework, that provides assistance in analyzing, comparing and treating disaster risk. A mathematical description of the basal terms has been provided and each risk management step has been clearly outlined, leaving some range for problem specific modifications. Frequently used risk definitions originating from literature have been reviewed and integrated in the concept, so that their interrelations were disclosed. In a next step, the necessity for the government to engage in risk reduction activities has been discussed and risk reduction projects have been characterized economically. By demonstrating how the cost and benefits of a risk reduction project may systematically be assessed, the foundation for project appraisal by means of cost benefit analysis (CBA) was laid.

The theoretical basis for CBA is found in welfare economics, that provides guidelines for analyzing both individual and social welfare. The two fundamental principles that are propagated in welfare economics are that individual preferences are to be taken as the basis for social welfare measurement and that social welfare can only be improved by a change, if each member of society is made better off without making anyone worse off. A public risk reduction project constitutes such a change in resource allocation that impacts personal and social wellbeing. The two central objectives of welfare economics are the efficient allocation of resources and their fair distribution among the members of society.

While efficiency aspects can be tackled by means of the Pareto and Kaldor-Hicks criteria, the analysis of distribution entails the formulation of a social welfare function, implicitly including normative value judgments about what is considered to be fair. The measurement of individual wellbeing is based on ordinal utility functions that represent preferences and are neither comparable across individuals nor do they possess a significance in absolute terms. Therefore, changes in welfare have to be converted to monetary value to allow for an application of the Kaldor-Hicks test within CBA. This is done by means of the willingness to pay (WTP) concept.

The WTP concept is a widely applied approach in economics particularly designed to derive prices for goods that are not traded on markets. Human safety constitutes such a non market good that may be valued by the WTP and standardized in the value of statistical life (VSL). It has clearly been emphasized that the VSL does not place a value on a particular identifiable human life. Instead, it is based on the statistical view that prevented fatalities due to an enhancement of safety standards convert to reductions in mortality risk that are in turn enjoyed by any member of society. These reductions in mortality risk permit the interpretation in terms of an economic good that is appraisable by economic theory. Three commonly applied approaches to assess the VSL have been reviewed and sample values from distinct studies have been collected from literature. While labor market studies yielded VSL estimates in the range of US$ 7-8 million, consumer market studies and stated preference approaches delivered values around US$ 5 million. Throughout the single studies a large variance in the VSLs estimates could be observed.

As the single studies are designed to extract a VSL solely by focusing on one particular market or a particular subgroup of the total population, it is not clear which of the values is best employed in a public policy analysis potentially affecting the entire population. Addressing this question, a theoretical model has been developed that quantifies the influence of safety on a market economy in keeping with the individualistic welfare economic postulate. As the safety level that is enjoyed by each individual is the result of both private and public precaution, good reason has been presented to model the fraction of personal safety that is impacted by public risk reduction projects as a public good.

This public safety level uniformly affects all consumers and producers interacting in the economy and determines the circumstances under which economic decisions are made. Under this assumption it has been shown, that an increase in public safety leads to the formation of new single market equilibria and thus of a new general equilibrium. The social price for safety could then be determined by monetizing the individual welfare effects in the move from status quo to post project equilibrium and in a second step, their aggregation on social level. This has been shown to be admissible as an increase in public safety permits the interpretation of an increase in public resource endowments and accordingly, in economic efficiency.

Because of the stringent information requirements about peoples' preferences, the valuation of non market goods on population level often requires the departure from individualistic approaches and the deployment of representative agents. These rely on few characteristics only and allow an assessment in general equilibrium based on average values. This generally comes at the cost of a loss in welfare economic soundness, as it is theoretically admissible under restrictive conditions only. Based on a representative agent model, safety pricing rules in general equilibrium have been derived. It turned out that the total social value of increases in the publicly provided safety level is decomposable in the direct benefits that accrue to the representative consumer and an increase in total economic output. Safety imposed price changes in contrast, could be neglected in the valuation. The representative agent model provided the microfoundations for the Life Quality Index (LQI) concept and is to be seen as the transition model from individualistic bottom up safety pricing strategies to aggregate top down safety pricing models, that directly obtain a price on social level and are macroeconomic in nature.

In presenting top down safety pricing strategies, firstly the traditional human capital approach has been introduced. It is straightforward to apply and has frequently been used in the past, but it has a number of serious drawbacks when its implications are examined in detail: the human capital approach views life just as a factor of production and estimates the loss in production that accrues due to a premature death. The quality of life that the individual derives from the pleasure of living is not valued as individual preferences are completely neglected. The LQI has been developed to approach this weakness. Based on the assumption that the average individual of society or representative consumer derives satisfaction from income, longevity and leisure time, it combines three macroeconomic indicators in one measure of utility or life quality.

It has been outlined however, that traditional LQI derivation approaches suffer from conceptual weaknesses and a new economic consistent derivation approach in general equilibrium has been presented that clearly reveals its relative position within economic theory. Based on the so obtained results, the traditional net benefit criterion has been extended to account also for increases in economic output that accrue as a result of an increased life expectancy. These new safety pricing rules have been obtained for the single period model and extended for intertemporal aspects.

The innovative safety pricing rule yielded VSL estimates in the range of US$ 3-4 million for most western economies. Furthermore, the new derivation approach revealed that it is advisable to employ the full GDP in the safety pricing formula (instead of only 60%) in order to obtain a consistent value of time. These advancements jointly led to a total increase well above 50% in the VSL estimates in comparison to conventional LQI applications. Considering the fact that the traditional LQI based VSLs are allocated at the lower end of the VSL range observed in safety literature, this is seen as a valuable improvement.

Accounting for the fact that all estimations of cost and benefits of public risk reduction projects are highly uncertain, a real option approach has been developed to include these uncertainties in project appraisal. With the possibility to delay the investment initiation to a future point in time, new information may be acquired that allows for a more precise estimation of the project's cash flows. The value of the possibility to postpone the investment has been assessed by modeling the evolution of the total social value of the risk reduction project over time by means of a discrete stochastic process. The latter accounted for expected value movements due to newly acquired information as well as the possibility that a disaster occurs in the time of investment delay. By demonstrating the analogy between financial options on stocks and the possibility to invest in a public risk reduction project, eventually a price for the possibility to delay the investment has been derived on basis of a modified version of the Cox, Ross and Rubinstein Binomial Option Pricing Model.

In order to validate the presented models, firstly a real world application based on the loss estimation model HAZUS has been carried out. Subject of study constituted the city of San Francisco, California that has been investigated with respect to earthquake risk. In particular, five different hazard scenarios have been simulated with return periods ranging from 100 to 2500 years. By systematically analyzing damages, direct and indirect economic and human losses that are expected to go in line with the events, the total risk of the city was estimated to equal US$ 283 million per year. In a consecutive step a CBA has been carried out to evaluate two potential risk reduction strategies of seismically retrofitting buildings of different design levels. It turned out that a rehabilitation of the city's low designed buildings to moderate code on portfolio level is to be considered as a sound investment, while the second option to retrofit all non highly designed buildings to high code is generally not cost efficient and thus recommendable only for essential and high potential loss facilities that bring along further benefits not included in the analysis.

Eventually, an illustrative example for the application of the real option approach has been presented, by considering a risk reduction investment to protect a model region from flood risk by the construction of a rain storage reservoir. Here, the general approach to practically implement the procedure has been shown and the influence of the parameters employed in the option model on the optimal project decision has been outlined. Lastly, reasonable calibrations have been presented where the innovative real option criterion differs from the classical net present value (NPV) investment criterion to prevent the risk of misinvestment.

8.2 Conclusions and Outlook

To conclude, the present work developed new insights and concepts in the following fields:

1. The development of a clear structured framework for managing disaster risk. The important contribution of this work in comparison to the author's previous joint achievements [201] and [208] is to be seen in the mathematization of the risk management process and the risk defining terms. This is expected to contribute to more transparency in application.
2. The application of public good theory to model the impact of safety changes on a market economy in a general equilibrium setting.
3. The economic analysis of the LQI and the presentation of a new time consistent derivation approach in a general equilibrium setting.
4. The development of innovative LQI based safety pricing rules in general equilibrium.
5. The application of real option theory to public disaster risk reduction projects.

In the discussion of disaster risk management it has been outlined, that the common practice to purely focus on expected values in the risk definition, evaluation and treatment is acceptable as disaster risk constitutes a pure downside risk that is spread among a large number of people and constitutes one among many other competing risks. In a sound risk treatment however, it is advisable to include additional information about the statistical nature of risk in the management process. As has been demonstrated, financial risk management offers a wide range of measures and advanced methodologies to process risk information in decision making processes and might further be explored in order to embed established financial risk management strategies in the context of disaster risk. In this way, further advancements are expected to be achieved.

One of the main findings of this thesis in the context of WTP assessment has been the observation that there exists a certain discrepancy between theoretical soundness and practicability of the prevailing approaches. Individualistic bottom up approaches are conceptually most satisfactory but a number of problems may arise in an implementation on large scales. Top down approaches in contrast, are easy to apply and deliver stable values, but come at a violation of the individualistic principle of welfare economics in general. The LQI has been demonstrated to be a first step to merge aggregate individual preference information by means of the labor leisure trade-off and macroeconomic indicators in one concept. An interesting idea for future research would be to "individualize" the LQI or related representative agent models on a group level to improve the characteristics of the agent by including individual preferences more precisely. In this way, the theoretical foundations would be improved and at the same time practical tractability maintained.

Although the goal of identifying means to place a value on reductions in mortality risk that are stable and employable across different fields has been formulated, it may not be possible (or desirable) to have a unique value that can uniformly be used in all applications. Instead, it may turn out that preferences are such that there is one value for a change in the probability of dying in a job accident, another value for a change in the probability of dying in a traffic accident, and yet a third value for a change in the probability of dying in a natural disaster - even for similar persons with identical baseline risks and similar safety improvements. Given the great diversity of values now implicit in public decision making, such a result would not be unexpected, but entails further empirical and conceptual research.

It has been demonstrated, that the most crucial step in the LQI derivation and calibration lies in the determination of the parameter q, which constitutes the elasticity of life quality utility. It has been determined by relying on the labor leisure trade-off which may be interpreted as a decision that individuals make short term. The common LQI practice to transfer this parameter to intertemporal models however, is to be seen as rather critical, as the utility exponent in life cycle models refers to the concept of intertemporal substitution between present and future consumption and thus, serves a distinct purpose. In intertemporal models with endogenous labor supply these two effects are usually being accounted for by defining utility functions with two distinct exponents. Further research on this issue is needed, which entails the exploration of the LQI methodology in an intertemporal macroeconomic growth model.

Lastly, the developed real options approach to price the flexibility to postpone the investment in an uncertain world is to be considered as a first step in applying real options theory to disaster management. Although several quite stringent simplifications and assumptions had to be placed to guarantee the fulfillment of real option theory requirements, the model has provided sound insights in the influence of parameter uncertainty on the project decision. One goal for future research would be to improve the stochastic process that describes possible developments of the social risk reduction project value over time. In particular, it would be desirable to render more precisely the project value evolution in response to steady and disaster imposed information gains and to consistently integrate the model in a continuous time framework. Accordingly, the model could be extended for an application to more complex real world risk reduction investment decisions.

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Appendix A

Risk Management Glossary

System:
The object of investigation for which all Hazard sources are identified and Risk Analysis is being performed. The System may be composed of a single building or infrastructure element, a suburb of a city, a whole urban region or even an entire country.

Hazard: A potentially adverse physical event, phenomenon or human activity that may cause harm to the predefined System. Harm can include injury or Loss of life, property Damage, CSH and economic disruption or environmental degradation.

Hazard Analysis: Consists of three steps: Hazard identification, the determination of relevant intensity levels and the estimation of the corresponding occurrence probabilities in a predefined time period. Depending on the size of the System, the results may differ for each Element at Risk.

Element-at-Risk (EaR) and Elements-at-Non-Risk (EaNR): A single or a group of persons or objects within the predefined System that are susceptible and exposed to the impact of a Hazard are referred to as Elements-at-Risk (EaR). System elements that are not endangered by the impact of the hazard are termed Elements-at-Non-Risk (EaNR). In order to guarantee a complete coverage, all EaR and EaNR collectively should compose the entire System that is being investigated. This is referred to as the "principle of completeness".

Exposure: System elements whose hazard load is expected to exceed a certain threshold level for a given hazard intensity, so that damage is expected to occur. All exposed system elements constitute Elements at Risk.

Structural Vulnerability: Is a specific characteristic of an Element at Risk that indicates the physical susceptibility towards the impact of a Hazard. Thus, Structural Vulnerability links the Hazard load to the Damage of an Element at Risk.

Damage: Describes the physical, biological or chemical effect on an Element at Risk caused by the impact of a Hazard of a given intensity. Damage captures the material harm and is not expressed in monetary terms.

System Vulnerability: Is a specific characteristic of an Element at Risk, that indicates the total potential of a Hazard of a given intensity. Thus, System Vulnerability assigns a Loss value to each given Damage state of an Element at Risk. It is best described by a function that evaluates the Consequences of a certain Damage state by taking into account the value of the Element at Risk itself as well as its designated functionality within the System.

Consequences: This term captures and quantifies the various adverse effects a natural disaster event of a certain intensity may have on the different Elements at Risk. Consequences can be subdivided into Direct and Indirect Consequences.

Direct Consequences: Are harms that occur simultaneously to the time the disaster takes place or by immediate follow-up physical destruction such as fires. Therefore, they can directly be related to the disaster itself.

Indirect Consequences: Usually occur with a time shift as a result of the Direct Consequences. They can be interpreted as follow up costs that result from the Element at Risk being unable to carry out its designated functionality within the System after the disaster has occurred.

Moreover, Direct as well as Indirect Consequences may further be subdivided and classified into economic, human, ecological and CSH Consequences due to the measure that is in use for their quantification. As it is possible to assign a monetary value to economic Consequences in a direct way, they are referred to as tangible. All other Consequences classes are termed intangible. In the following several Direct and Indirect Consequences are outlined divided by consequence classes:

1. Direct Consequences:
   - economic: Adverse effects on capital stock resulting from physical Damage of economic value carrying objects.
   - human: Injuries and fatalities due to the Damage of objects.
   - ecological: Ground, air and water pollution, contamination of the environment or other devastating effects on ecosystems caused for instance by releases of toxic substances.
   - CSH: Adverse effects on capital stock resulting from physical Damage of CSH value carrying objects, such as schools, hospitals, historical buildings and so on.
2. Indirect Consequences:
   - economic: Business interruption, wage losses, production downtime and other harms on the economy in the long term.
   - human: The spread of diseases resulting from the absence of satisfactory hygiene within the affected area, psychological post-disaster effects.
   - ecological: Penalties due to the violation of environmental regulation laws.
   - CSH: Adverse effects on the wellbeing of society resulting from the absence of the CSH value carrying object.

Loss: Subdivided by consequence classes, this term accumulates all Direct and Indirect Consequences a natural disaster of a certain intensity may have at the time the disaster occurs. To quantify the Loss, the sum of all Direct and Indirect Consequences belonging to the considered consequence class for each Element at Risk being part of the System has to be calculated. In this connection, the Indirect Consequences need to be discounted dependent on the time they occur by taking a consequence class specific discount factor into account. The Loss can either be subdivided into human, economic, ecological and CHS Loss or a total Loss measure may be implemented, if a reasonable way has been found to convert the intangible losses to monetary value.

Risk: Risk is expressible in two distinctive ways. One possibility is to express Risk with respect to the structural Damage (here called "Structural Risk"). The second way is to relate Risk to the resulting Loss (here called "Total Risk").

Structural Risk: The Structural Risk is calculated by taking the products of the annual occurrence probabilities of the Damages and the Damages themselves and summing up these products over all Damage states.

$$\text{Structural Risk} = \text{Probability x Damage [Damage measure / year]}$$

Alternatively, the Structural Risk is directly relatable to the Hazard source, by taking the annual Hazard occurrence probabilities times the Damages that are expected to go in line with them and summing up these products over all Hazard intensities.

Total Risk: For each consequence class the Total Risk is calculated by taking the products of the annual probabilities of occurrence of the Losses and the Losses themselves and summing up these products over all Loss levels.

$$\text{Total Risk} = \text{Probability x Loss [Loss unit / year]}$$

Alternatively, the Total Risk is directly relatable to the Hazard source, by taking the annual Hazard occurrence probabilities times the Losses that are expected to go in line with them and summing up these products over all Hazard intensities.

In analogy to the Loss, the Total Risk may be split into the human, the economic, the ecological and the CSH risk or alternatively, an overall Total Risk may be calculated, if a reasonable way has been found to convert the intangible consequences to monetary units.

Risk Review: Due to the ever changing environment of the Risk influencing variables, the primary purpose of Risk Review is to constantly include all new information, knowledge and experience about the Risk and to perform a Risk update on a regular basis. It should be emphasized that the Risk Review step is only being performed for already identified Risks which have run through the Risk Assessment and Risk Treatment phase at least once. Consequently, in each Risk Review iteration the effectiveness of possibly performed Risk reduction interventions is assessable.

Risk Monitoring: Accompanying all the steps of the Risk Management Framework, the Risk Monitoring procedure captures the exchange of information among all stakeholders actively or passively involved or participating in the Risk Management process. It includes the awareness of the System to constantly survey already identified Risks and the record of potentially newly occurring Hazards. As a result of the monitoring procedure the Risk evolution over time is registered.

Risk Management: Risk Management is defined as the systematic application of management policies, procedures and practices to the tasks of identifying, assessing, treating, communicating, reviewing and monitoring Risk.

Appendix B

Probability Theory

B.1 The Total Probability Theorem

For two continuous random variables X and Y that have the joint probability density function $f_{X,Y}$ the marginal probability density functions of X and Y are calculated to be:

$$\begin{aligned} f_X(x) &= \int f_{X,Y}(x,y)dy \\ f_Y(y) &= \int f_{X,Y}(x,y)dx \end{aligned} \tag{B.1}$$

The conditional probability density functions are then defined by:

$$\begin{aligned} f_{X|Y}(x,y) &= \frac{f_{X,Y}(x,y)}{f_Y(y)} \\ f_{Y|X}(x,y) &= \frac{f_{X,Y}(x,y)}{f_X(x)} \end{aligned} \tag{B.2}$$

Then, from equations B.1 and B.2 the marginal probability density functions of X and Y may be reformulated to the following expression:

$$\begin{aligned} f_X(x) &= \int f_{X,Y}(x,y)dy = \int f_{X|Y}(x,y)f_Y(y)dy \\ f_Y(y) &= \int f_{X,Y}(x,y)dx = \int f_{Y|X}(x,y)f_X(x)dx \end{aligned} \tag{B.3}$$

This is called the law of total probability. Consequently, the marginal densities of X and Y can be determined by integrating the product of conditional densities and the marginal density of the respective other random variable over the respective other random variable, without referring to the joint probability.

B.2 First Order Stochastic Dominance

In first order stochastic dominance, a random variable X is said to stochastically dominate a random variable Y, if for any parameter a the following relation holds

$$P(X \leq a) \leq P(Y \leq a) \tag{B.4}$$

indicating that the probability that the random variable X takes a value that is lower than a is smaller than the probability that the random variable Y takes a value smaller than a. In other words, higher values are more likely to be realized by X than by Y.

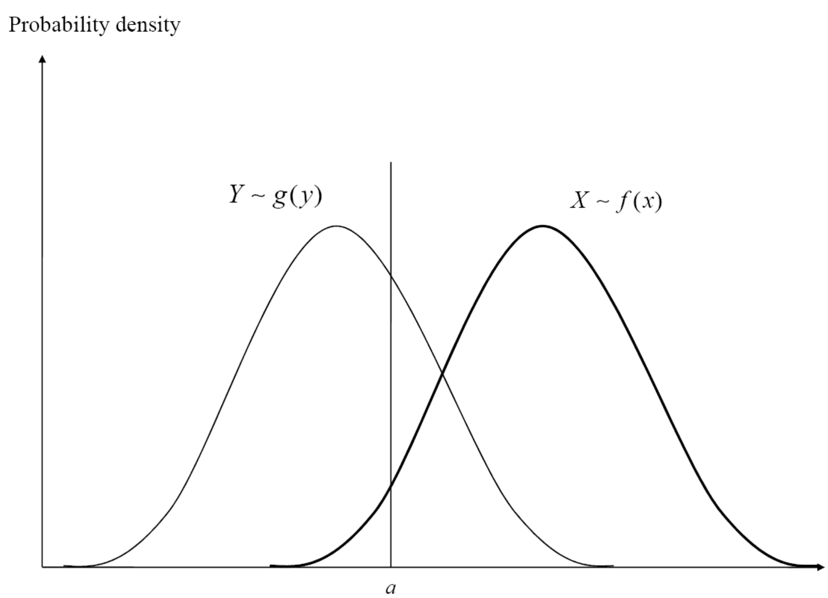

Figure B.1: First order stochastic dominance

If F and f denote the cumulative probability function and the PDF of the random variable X, respectively and G and g represent the two respective functions of the random variable Y, first order stochastic dominance may also be expressed as follows:

$$F(a) \leq G(a) \qquad \text{for all } a \tag{B.5}$$

and

$$\int_{-\infty}^{a} f(x)dx \leq \int_{-\infty}^{a} g(y)dy \qquad \text{for all } a \tag{B.6}$$

As a consequence of first order stochastic dominance the following relation holds for the expected values:

$$E[X] \geq E[Y] \tag{B.7}$$

Graphically, this implies that a dominating cumulative distribution F may never lie above the dominated distribution G and that a dominating PDF f is located with more probability mass "to the right" than the dominated PDF g, as illustrated in Figure B.1.

Appendix C

The Envelope Theorem in (Constrained) Optimization Problems

A maximum (or minimum) value function is an objective function where the choice variables have been assigned their optimal values. These optimal values of the choice variables are in turn functions of the exogenous variables and parameters of the problem. Once the optimal values of the choice variables have been substituted into the original objective function, the function indirectly becomes a function of the parameters (through the parameters' influence on the optimal values of the choice variables). Thus the maximum value function is also referred to as the indirect objective function.

What is the significance of the indirect objective function? Consider that in any optimization problem the direct objective function is maximized (or minimized) for a given set of parameters. The indirect objective function gives all the maximum values of the objective function as these parameters vary. Hence, the indirect objective function is an "envelope" of the set of optimized objective functions generated by varying the parameters of the model.

C.1 The Unconstrained Case

To illustrate, consider the following maximization problem with two choice variables x and y, and one exogenous parameter α:

$$\max_{x,y} U(x, y, \alpha) \tag{C.1}$$

The first order necessary conditions are:

$$\begin{aligned} \frac{\partial U(x, y, \alpha)}{\partial x} &= 0 \\ \frac{\partial U(x, y, \alpha)}{\partial y} &= 0 \end{aligned} \tag{C.2}$$

If the second order conditions are met, these two equations implicitly define the solutions:

$$
\begin{aligned}
x &= x^*(\alpha) \\
y &= y^*(\alpha)
\end{aligned}
\tag{C.3}
$$

If these solutions are substituted into the objective function, a new function is obtained:

$$
V(\alpha) = U(x^*(\alpha), y^*(\alpha), \alpha) \tag{C.4}
$$

This function is the value of U when the values of x and y are those that maximize $U(x, y, \alpha)$. Therefore, $V(\alpha)$ constitutes the maximum value function (or indirect objective function). If V is differentiated with respect to α, the following expression is obtained:

$$
\frac{\partial V(\alpha)}{\partial \alpha} = \frac{\partial U}{\partial x} \cdot \frac{\partial x^*}{\partial \alpha} + \frac{\partial U}{\partial y} \cdot \frac{\partial y^*}{\partial \alpha} + \frac{\partial U}{\partial \alpha} \tag{C.5}
$$

However, from the first order conditions it is known that $\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} = 0$. Therefore, the first two terms disappear and the result becomes:

$$
\frac{\partial V}{\partial \alpha} = \frac{\partial U}{\partial \alpha} \tag{C.6}
$$

This result states that, when α varies at the optimum, with x and y allowed to adjust optimally, leads to the same result as if x and y were held constant. Note that α enters the maximum value function provided in (C.4) in three places: one direct and two indirect (through x and y). Equations (C.5) and (C.6) show that, at the optimum, only the direct effect of α on the objective function matters. This is the essence of the envelope theorem. The envelope theorem states that only the direct effects of a change in an exogenous variable need to be considered, even though the exogenous variable may enter the maximum value function indirectly as part of the solution to the endogenous choice variables.

C.2 The Constrained Case

Now, the attention is switched to the case of constrained optimization problems. Again, an objective function U, two choice variables x and y and the exogenous parameter α are given. Additionally, the following constraint is introduced

$$
g(x, y, \alpha) = 0 \tag{C.7}
$$

so that the constrained optimization problem becomes:

$$
\begin{aligned}
&\max_{x,y} U(x, y, \alpha) \\
&\text{s.t. } g(x, y, \alpha) = 0
\end{aligned}
\tag{C.8}
$$

The Lagrangian for this problem is:

$$
\Lambda = U(x, y, \alpha) + \lambda g(x, y, \alpha) \tag{C.9}
$$

The first order conditions are:

$$\frac{\partial \Lambda}{\partial x} = \frac{\partial U}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0 \tag{C.10}$$
$$\frac{\partial \Lambda}{\partial y} = \frac{\partial U}{\partial y} + \lambda \frac{\partial g}{\partial y} = 0 \tag{C.11}$$
$$\frac{\partial \Lambda}{\partial \lambda} = g(x, y, \alpha) = 0 \tag{C.12}$$

Solving this system of equations yields:

$$x = x^*(\alpha); \qquad y = y^*(\alpha); \qquad \lambda = \lambda^*(\alpha) \tag{C.13}$$

Inserting these solutions into the objective function U gives

$$V(\alpha) = U(x^*(\alpha), y^*(\alpha), \alpha) \tag{C.14}$$

where $V(\alpha)$ constitutes the indirect objective function, or maximum value function. It is the maximum value of U for any x and y that satisfy the constraint for a given level of α.

To address the question of how $V(\alpha)$ varies as α changes, V is differentiated with respect to α:

$$\frac{\partial V(\alpha)}{\partial \alpha} = \frac{\partial U}{\partial x} \cdot \frac{\partial x^*}{\partial \alpha} + \frac{\partial U}{\partial y} \cdot \frac{\partial y^*}{\partial \alpha} + \frac{\partial U}{\partial \alpha} \tag{C.15}$$

In the constrained case however, equation (C.15) will not simplify to $\frac{\partial V}{\partial \alpha} = \frac{\partial U}{\partial \alpha}$, since $\frac{\partial U(\alpha)}{\partial x} \neq 0$ and $\frac{\partial U(\alpha)}{\partial y} \neq 0$ holds, as may be verified from the first order conditions (C.10) and (C.11). To proceed further, the optimal values $x^*(\alpha)$ and $y^*(\alpha)$ are substituted in the constraint g to produce the identity:

$$g(x^*(\alpha), y^*(\alpha), \alpha) = 0 \tag{C.16}$$

Differentiating equation (C.16) with respect to α yields:

$$\frac{\partial g}{\partial x}\frac{\partial x^*}{\partial \alpha} + \frac{\partial g}{\partial y}\frac{\partial y^*}{\partial \alpha} + \frac{\partial g}{\partial \alpha} \tag{C.17}$$

Multiplying equation (C.17) by λ, combining the result with equation (C.15) and rearranging terms, the following expression is obtained:

$$\frac{\partial V(\alpha)}{\partial \alpha} = (\frac{\partial U}{\partial x} + \lambda \frac{\partial g}{\partial x})\frac{\partial x^*}{\partial \alpha} + (\frac{\partial U}{\partial y} + \lambda \frac{\partial g}{\partial y})\frac{\partial y^*}{\partial \alpha} + \frac{\partial U}{\partial \alpha} + \lambda \frac{\partial g}{\partial \alpha} \tag{C.18}$$

From the first order conditions (C.10) and (C.11) it is seen that the first two terms are equal to zero so that the following final expression is derived:

$$\frac{\partial V(\alpha)}{\partial \alpha} = \frac{\partial U}{\partial \alpha} + \lambda \frac{\partial g}{\partial \alpha} \tag{C.19}$$

By sharply considering equation (C.19) it follows that the partial derivative of the value function V with respect to any given indirect parameter α is equal to the partial derivative of the Lagrange function Λ with respect to this parameter:

$$\frac{\partial V}{\partial \alpha} = \frac{\partial \Lambda}{\partial \alpha} \tag{C.20}$$

This result holds also if more than one exogenous parameters enter the value function, as considered in the main text. Consequently, the Langrange functions serves as the objective function in deriving the indirect objective function.

It is important to note that some of the comparative static results depend critically on whether the exogenous parameters enter only the objective function or whether they enter only the constraints, or enter both. If an exogenous parameter enters only in the objective function, the comparative static results are the same as for the unconstrained case. In any case, equation (C.20) is the proper tool to analyze how the value function responds due to exogenous parameter changes.

Appendix D

The Consumer's Utility Maximization and Expenditure Minimization Problem

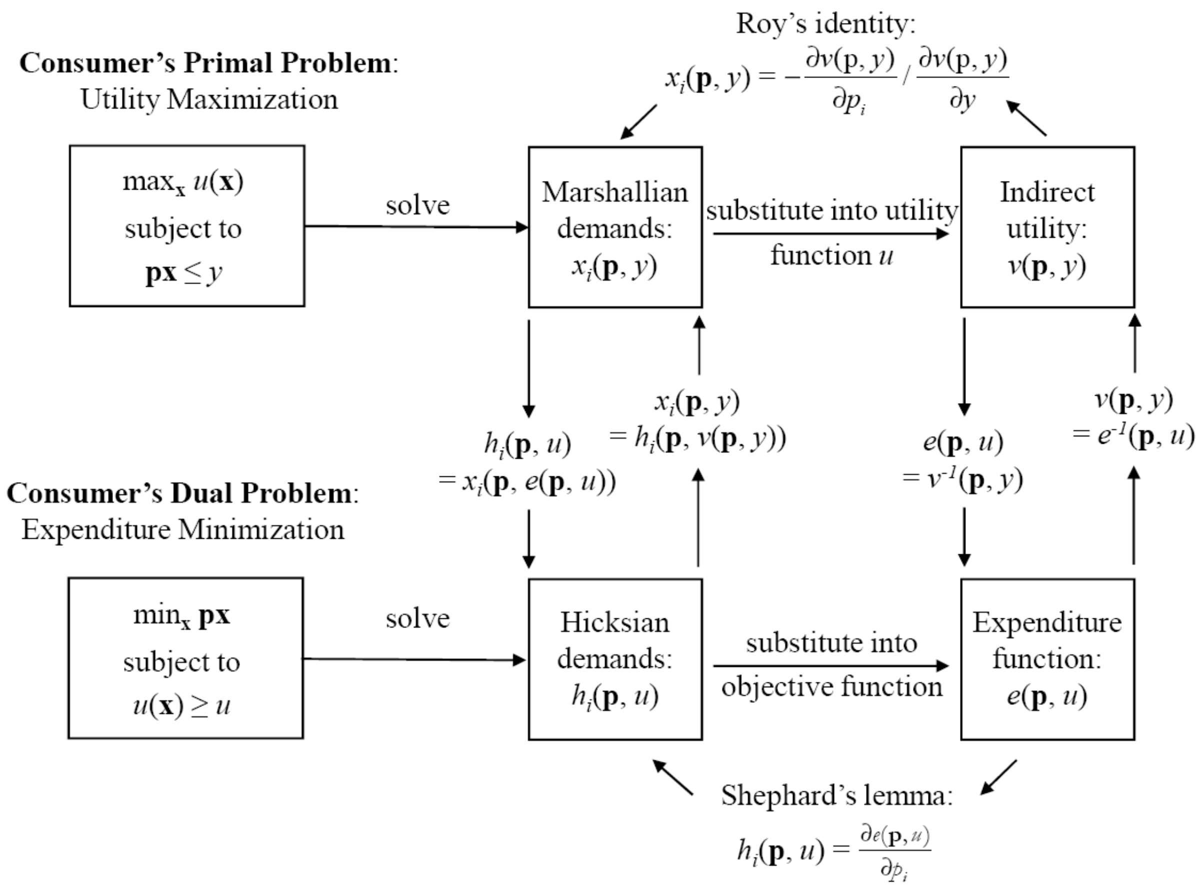

Figure D.1: Utility maximization and expenditure minimization

Appendix E

Life Quality Index Derivations

E.1 The Original Derivation

The original life quality index derivation provided in [181] was carried out on basis of a differential equation approach. As the whole life quality index discussion emerged from this first derivation, it is briefly reviewed in the following.

It is firstly assumed that life quality has two dimensions and is basically derived from two factors: intensity and duration of life. In this respect, the intensity of life is deduced from wealth production and is measured by an increasing continuous monotonic differentiable function f of the GDP per capita y. Then, another increasing continuous monotonic differentiable function h of time $\widehat{l}$ is taken into account to incorporate a measure for the duration of life to enjoy that wealth into the concept. The time $\widehat{l}$ represents the leisure time available to a person outside occupational activities that produce y. If e denotes the total life expectancy at birth and a represents the total fraction of lifetime that is devoted to income producing activities, the leisure time available to a person is $\widehat{l} = (1-a)e$. Thus, any product of the form

$$L = f(y) \cdot h(\widehat{l}) \tag{E.1}$$

represents a compound indicator of the duration and intensity of life. Now, in order to analyze how marginal changes in income dy and leisure time $d\widehat{l}$ influence the general compound indicator (E.1), the total differential is taken into account:

$$\begin{aligned} dL &= \frac{\partial L}{\partial y} \cdot dy + \frac{\partial L}{\partial \widehat{l}} \cdot d\widehat{l} \\ &= \frac{df}{dy} \cdot h(\widehat{l}) \cdot dy + \frac{dh}{d\widehat{l}} \cdot f(g) \cdot d\widehat{l} \end{aligned} \tag{E.2}$$

Dividing equation (E.2) by L in order to make relative impacts on the compound indicator visible and by a rearrangement of terms the following expression is obtained:

$$\begin{aligned}\frac{dL}{L} &= \underbrace{\frac{y}{f(y)} \cdot \frac{df(y)}{dy}}_{:=k_1} \cdot \frac{dy}{y} + \underbrace{\frac{\widehat{l}}{h(\widehat{l})} \cdot \frac{dh(\widehat{l})}{d\widehat{l}}}_{:=k_2} \cdot \frac{d\widehat{l}}{\widehat{l}} \\ &= k_1 \cdot \frac{dy}{y} + k_2 \cdot \frac{d\widehat{l}}{\widehat{l}}\end{aligned} \tag{E.3}$$

The two factors k_1 and k_2 constitute the elasticities of $df(y)$ and $dh(\widehat{l})$, respectively. The ratio of these two factors $\frac{k_1}{k_2}$ describes the relative impact of proportionate changes in GDP per person and leisure time, which is dependent on life expectancy. It is further hypothesized, that the relative impact of these two factors is independent of their absolute values, e.g. $\frac{k_1}{k_2} = \text{const.}$ Under this assumption the factors k_1 and k_2 must also be constant:

$$k_1 = \frac{y}{f(y)} \cdot \frac{df(y)}{dy} = \text{const.} \quad \text{and} \quad k_2 = \frac{\widehat{l}}{h(\widehat{l})} \cdot \frac{dh(\widehat{l})}{d\widehat{l}} = \text{const.} \tag{E.4}$$

Equation E.4 constitutes a set of two first order differential equations which can be solved to yield $f(y) = y^r$ and $h(\widehat{l}) = \widehat{l}^s$, leading to:

$$L = y^r \cdot \widehat{l}^s = y^r \cdot ((1-a) \cdot e)^s \tag{E.5}$$

It is then assumed that individuals on average choose a to maximize their total life quality, so that the function L is maximized with respect to a, i.e. the labor leisure trade-off is applied. The first order condition $\frac{dL}{da}$ along with equation (E.5) leads to:

$$r = s \cdot \frac{a}{1-a} \tag{E.6}$$

Without loss of generality it is further assumed that $r + s = 1$, so that $r = a$ and $s = (1-a)$. Thus, from (E.5) the following equation is obtained:

$$L = y^a \cdot ((1-a) \cdot e)^{1-a} \approx y^a \cdot e^{1-a} \cdot (1-a)^{1-a} \tag{E.7}$$

It is then proceeded by dropping the term $(1-a)^{1-a}$ as it is practically constant, especially for industrialized and developed countries [196], to obtain the LQI in its following final expression:

$$L = y^a \cdot e^{1-a} \tag{E.8}$$

Although the derivation of the LQI is a straightforward mathematical excercise, it provides little intuitive explanation and conceptual discussion about its mathematical form and implications in decision making, as admitted by the authors themselves [196].

E.2 The Lifetime Utility Approach

The basic idea behind the lifetime utility approach is that a person's enjoyment of life stems from a continuous stream of consumption over the remaining lifetime. If $c(t)$ denotes the consumption rate at time t and if it is further assumed that a time separable instantaneous utility function $u(c(t))$

exists that is designed to quantify the level of satisfaction from consumption at each moment in time, the lifetime utility U of a person of age x is represented by the following integral

$$U(x) = \int_x^{t_max} u(c(t))dt \tag{E.9}$$

where t_{max} denotes the maximum attainable age, taken from life tables. Due to several psychological and economic reasons, people tend to undervalue a prospect of future consumption in relation to that of present consumption. This phenomenon is integrated in the lifetime utility concept by introducing a suitable discount rate for utility r, which accounts for the individual's time preferences and economic growth:

$$U_d(x) = \int_x^{t_max} u(c(t)) \exp\left[-\int_x^t r(\tau - x)d\tau\right] dt \tag{E.10}$$

The realization of utility $U(c(t))$ at age $t \geq x$ is conditional on the individual's survival up to age t. If the probability of surviving until (at least) age t is given by the survival function $S(t)$, the expected value of remaining lifetime utility at age x is obtained:

$$L(x) = E\left[U(x)\right] = \frac{1}{S(x)} \int_x^{t_max} u(c(t))S(t) \exp\left[-\int_x^t r(\tau - x)d\tau\right] dt \tag{E.11}$$

The survival function $S(t)$ may easily be expressed by means of the age specific mortality rates $\mu(t)$:

$$S(t) := \exp\left[-\int_0^t \mu(\tau)d\tau\right] \tag{E.12}$$

Furthermore, it is possible to include the discounting effect in the survival term so that

$$S_d(t) := \exp\left[-\int_0^t \mu(\tau) + r(\tau - x)d\tau\right] \tag{E.13}$$

is obtained, which is equal to the discounted probability of survival to age t. As a consequence, equation (E.11) can further be simplified to:

$$\begin{aligned} L(x) &= \int_x^{t_max} u(c(t)) \exp\left[-\int_x^t \mu(\tau) + r(\tau - x)d\tau\right] dt \\ &= \int_x^{t_max} u(c(t))S_d(t|x)dt \end{aligned} \tag{E.14}$$

As discussed in the main text, under perfect markets with fair insurance available and time preferences equal to the interest rate, it can be shown that a constant consumption pattern is optimal, i.e. $c(t) = c$, so that equation (E.14) is rearranged to:

$$L(x) = u(c) \int_x^{t_max} S_d(t|x)dt \tag{E.15}$$

By remembering that total remaining life expectancy at age x is given through

$$e(x) = \int_x^{t_max} \exp\left[-\int_x^t \mu(\tau)d\tau\right] dt = \int_x^{t_max} S(t|x)dt \tag{E.16}$$

and total discounted remaining life expectancy at age x is defined as:

$$e_d(x) = \int_x^{t_max} \exp\left[-\int_x^t \mu(\tau) + r(\tau - x)d\tau\right] dt = \int_x^{t_max} S_d(t|x)dt \tag{E.17}$$

equation (E.15) may eventually be expressed as

$$L(x) = u(c)e_d(x) \tag{E.18}$$

which is the optimal remaining lifetime utility and a surrogate measure of quality of life of a person of age x.

Actual societies consist of people of all ages, so that equation (E.18) has to be averaged over the age distribution of society in order to arrive at a life quality measure for the average individual. This is done by taking the age distribution h into account

$$h(x, n) := \frac{\exp[-nx]S(x)}{\int_0^{t_{max}} \exp[-nx]S(x)dx} \tag{E.19}$$

where n denotes annual population growth. In real world applications population growth rates are changing over time, so that (E.19) is to be seen as an approximation. Averaging $e_d(x)$ over the whole population consisting of all age classes yields society's age averaged discounted life expectancy $\widetilde{e}_d$:

$$\widetilde{e}_d = E_h[e_d(x)] = \int_0^{t_{max}} e_d(x)h(x, n)dx \tag{E.20}$$

By means of the age averaged discounted life expectancy, social discounted lifetime utility may be expressed as

$$L = u(c)\widetilde{e}_d \tag{E.21}$$

which is to be seen as a social indicator, measuring the average individual's life quality over the remaining lifetime. Eventually, by assuming that $u(c)$ represents a power utility function, the parameter q, that has been derived as illustrated in the main text, is substituted for the exponent of the general utility function $u(c)$ and consumption c is replaced by income y, yielding the social life quality index with the following functional form:

$$SLQI = y^q\widetilde{e}_d \tag{E.22}$$

In order to judge on the effectiveness of safety improvements, equation (E.22) is then taken into account for an application in the net benefit criterion (5.18) formulated in the main text.

Appendix F

HAZUS Simulation Results

F.1 Damage States by Design Level and Earthquake Event

Low Seismic Design Level	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	33	70	97	22	6	229
Steel	21	27	74	57	26	205
Concrete	63	86	192	144	47	531
Precast	14	18	51	61	22	166
Reinforced Masonry	127	95	253	250	64	789
Unreinforced Masonry	251	333	466	257	152	1,459
Manufactured Home	3	9	21	15	3	51
Total	**513**	**637**	**1,155**	**806**	**320**	**3,431**

Moderate Seismic Design Level	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	32,804	37,222	19,897	1,877	534	92,334
Steel	549	603	1,154	667	118	3,090
Concrete	389	552	725	387	75	2,128
Precast	213	229	415	220	38	1,116
Reinforced Masonry	1,104	718	971	408	33	3,234
Manufactured Home	11	32	71	51	9	174
Total	**35,070**	**39,356**	**23,233**	**3,610**	**808**	**102,077**

High Seismic Design Level	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	19,316	22,737	6,185	284	46	48,569
Steel	157	155	147	25	1	484
Concrete	610	621	281	41	2	1,555
Precast	105	111	101	17	2	336
Reinforced Masonry	878	496	309	39	4	1,725
Manufactured Home	9	24	54	39	7	133
Total	**21,075**	**24,144**	**7,077**	**443**	**62**	**52,802**

Table F.1: Damage states by design level for 100-year event

Low Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	10	37	106	53	23	229
Steel	4	9	46	80	65	205
Concrete	18	39	142	194	138	531
Precast	4	8	35	62	56	166
Reinforced Masonry	46	49	196	304	194	789
Unreinforced Masonry	80	202	467	380	331	1,459
Manufactured Home	1	5	17	20	8	51
Total	**163**	**349**	**1,009**	**1,095**	**815**	**3,431**

Moderate Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	17,320	35,946	32,243	5,179	1,646	92,334
Steel	249	407	1,131	994	309	3,090
Concrete	148	374	769	607	230	2,128
Precast	78	142	414	362	119	1,116
Reinforced Masonry	390	522	1,197	920	206	3,234
Manufactured Home	3	16	59	69	28	174
Total	**18,187**	**37,407**	**35,813**	**8,132**	**2,538**	**102,077**

High Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	9,845	24,380	13,029	1,108	206	48,569
Steel	80	134	203	63	4	484
Concrete	349	633	464	100	8	1,555
Precast	54	103	137	36	6	336
Reinforced Masonry	573	546	501	93	12	1,725
Manufactured Home	2	12	45	53	21	133
Total	**10,904**	**25,809**	**14,379**	**1,453**	**257**	**52,802**

Table F.2: Damage states by design level for 250-year event

Low Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	5	25	93	67	39	229
Steel	2	5	31	77	90	205
Concrete	9	23	104	187	207	531
Precast	2	5	27	55	78	166
Reinforced Masonry	26	32	152	284	295	789
Unreinforced Masonry	36	126	393	420	484	1,459
Manufactured Home	0	3	14	21	14	51
Total	**81**	**218**	**814**	**1,110**	**1,207**	**3,431**

Moderate Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	10,411	30,469	38,907	9,195	3,352	92,334
Steel	151	298	1,009	1,123	509	3,090
Concrete	95	293	720	667	353	2,128
Precast	43	97	362	415	199	1,116
Reinforced Masonry	209	376	1,117	1,129	403	3,234
Manufactured Home	1	10	46	71	46	174
Total	**10,911**	**31,543**	**42,160**	**12,601**	**4,862**	**102,077**

High Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	6,603	22,605	16,919	2,021	422	48,569
Steel	51	111	217	95	10	484
Concrete	226	568	575	166	20	1,555
Precast	36	90	149	51	10	336
Reinforced Masonry	426	524	607	146	22	1,725
Manufactured Home	1	7	35	54	35	133
Total	**7,343**	**23,906**	**18,501**	**2,533**	**519**	**52,802**

Table F.3: Damage states by design level for 500-year event

Low Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	2	10	63	81	74	229
Steel	0	1	12	60	132	205
Concrete	3	9	54	146	319	531
Precast	1	2	14	36	113	166
Reinforced Masonry	9	14	88	214	464	789
Unreinforced Masonry	11	57	266	410	716	1,459
Manufactured Home	0	1	8	18	24	51
Total	**25**	**92**	**505**	**966**	**1,842**	**3,431**

Moderate Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	5,000	21,946	42,626	15,714	7,047	92,334
Steel	57	154	737	1,194	949	3,090
Concrete	39	174	590	718	607	2,128
Precast	16	52	271	436	341	1,116
Reinforced Masonry	99	234	916	1,234	752	3,234
Manufactured Home	0	4	27	62	81	174
Total	**5,211**	**22,563**	**45,167**	**19,358**	**9,778**	**102,077**

High Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	3,687	18,833	21,276	3,830	943	48,569
Steel	24	75	212	148	26	484
Concrete	109	429	676	286	55	1,555
Precast	16	65	154	80	22	336
Reinforced Masonry	253	443	724	253	52	1,725
Manufactured Home	0	3	21	48	61	133
Total	**4,089**	**19,848**	**23,061**	**4,645**	**1,159**	**52,802**

Table F.4: Damage states by design level for 1000-year event

Low Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	0	2	29	74	124	229
Steel	0	0	3	32	170	205
Concrete	1	2	20	83	425	531
Precast	0	0	6	17	143	166
Reinforced Masonry	2	4	38	117	629	789
Unreinforced Masonry	3	20	147	332	958	1,459
Manufactured Home	0	0	4	12	35	51
Total	**6**	**30**	**244**	**667**	**2,484**	**3,431**

Moderate Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	2,453	14,608	40,156	22,204	12,911	92,334
Steel	13	52	395	1,024	1,606	3,090
Concrete	10	70	384	659	1,006	2,128
Precast	4	19	156	381	556	1,116
Reinforced Masonry	28	100	594	1,153	1,359	3,234
Manufactured Home	0	1	12	41	120	174
Total	**2,508**	**14,850**	**41,697**	**25,462**	**17,559**	**102,077**

High Seismic Design Level

	# of Buildings					
	None	Slight	Moderate	Extensive	Complete	Total
California						
San Francisco						
Wood	2,047	14,403	23,701	6,441	1,977	48,569
Steel	9	42	176	199	59	484
Concrete	52	296	676	404	126	1,555
Precast	7	42	140	105	41	336
Reinforced Masonry	146	338	749	379	113	1,725
Manufactured Home	0	1	9	31	91	133
Total	**2,262**	**15,122**	**25,452**	**7,559**	**2,407**	**52,802**

Table F.5: Damage states by design level for 2500-year event

F.2 Casualty Rate Coefficients

#	Building Type	Casualty Severity Level Severity 1 (%)	Severity 2 (%)	Severity 3 (%)	Severity 4 (%)
1	W1	1	0.1	0.001	0.001
2	W2	1	0.1	0.001	0.001
3	S1L	1	0.1	0.001	0.001
4	S1M	1	0.1	0.001	0.001
5	S1H	1	0.1	0.001	0.001
6	S2L	1	0.1	0.001	0.001
7	S2M	1	0.1	0.001	0.001
8	S2H	1	0.1	0.001	0.001
9	S3	1	0.1	0.001	0.001
10	S4L	1	0.1	0.001	0.001
11	S4M	1	0.1	0.001	0.001
12	S4H	1	0.1	0.001	0.001
13	S5L	1	0.1	0.001	0.001
14	S5M	1	0.1	0.001	0.001
15	S5H	1	0.1	0.001	0.001
16	C1L	1	0.1	0.001	0.001
17	C1M	1	0.1	0.001	0.001
18	C1H	1	0.1	0.001	0.001
19	C2L	1	0.1	0.001	0.001
20	C2M	1	0.1	0.001	0.001
21	C2H	1	0.1	0.001	0.001
22	C3L	1	0.1	0.001	0.001
23	C3M	1	0.1	0.001	0.001
24	C3H	1	0.1	0.001	0.001
25	PC1	1	0.1	0.001	0.001
26	PC2L	1	0.1	0.001	0.001
27	PC2M	1	0.1	0.001	0.001
28	PC2H	1	0.1	0.001	0.001
29	RM1L	1	0.1	0.001	0.001
30	RM1M	1	0.1	0.001	0.001
31	RM2L	1	0.1	0.001	0.001
32	RM2M	1	0.1	0.001	0.001
33	RM2H	1	0.1	0.001	0.001
34	URML	2	0.2	0.002	0.002
35	URMM	2	0.2	0.002	0.002
36	MH	1	0.1	0.001	0.001
B1	Major Bridge	N/A	N/A	N/A	N/A
B2	Continuous Bridge	N/A	N/A	N/A	N/A
B3	S.S. Bridge	N/A	N/A	N/A	N/A

Table F.6: Indoor casualty rates for extensive structural damage by building type [HAZUS technical manual [106]]

F.3 Number of Casualties by Design Level and Earthquake Event

T_r = 100 yr			*# Casualties*		
Design level	severity 1	severity 2	severity 3	severity 4	total
low	683	209	31	63	987
moderate	2,144	562	80	159	2,944
high	289	50	6	12	358

T_r = 250 yr			*# Casualties*		
Design level	severity 1	severity 2	severity 3	severity 4	total
low	1,347	406	80	160	1,993
moderate	4,915	1,329	250	498	6,992
high	714	151	25	51	941

T_r = 500 yr			*# Casualties*		
Design level	severity 1	severity 2	severity 3	severity 4	total
low	1,545	569	89	178	2,381
moderate	7,177	2,380	359	716	10,632
high	1,046	277	38	77	1,438

T_r = 1000 yr			*# Casualties*		
Design level	severity 1	severity 2	severity 3	severity 4	total
low	1,952	638	108	226	2,924
moderate	11,634	3,509	577	1,200	16,920
high	1,754	449	68	142	2,414

T_r = 2500 yr			*# Casualties*		
Design level	severity 1	severity 2	severity 3	severity 4	total
low	2,473	857	146	305	3,781
moderate	19,028	6,207	1,035	2,155	28,424
high	3,082	895	142	296	4,415

Table F.7: Casualties by design level and earthquake scenario

F.4 Direct Economic Loss by Design Level and Earthquake Event

T_r = 100 yr	*Direct Economic Losses (US$ 1994 million)*				
Design level	slight	moderate	extensive	complete	total
low	21.6	225	866	672	1,784
moderate	560	1,871	2,231	820	5,481
high	360	576	238	50.5	1,224

T_r = 250 yr	*Direct Economic Losses (US$ 1994 million)*				
Design level	slight	moderate	extensive	complete	total
low	10.9	184	1,105	1,664	2,963
moderate	514	2,633	4,440	2,595	10,182
high	382	1,079	682	194	2,337

T_r = 500 yr	*Direct Economic Losses (US$ 1994 million)*				
Design level	slight	moderate	extensive	complete	total
low	7.1	148	1,082	2,317	3,554
moderate	430	2,932	5,992	4,514	13,868
high	353	1,344	1,116	380	3,193

T_r = 1000 yr	*Direct Economic Losses (US$ 1994 million)*				
Design level	slight	moderate	extensive	complete	total
low	2.8	86.5	896	3,502	4,487
moderate	301	2,969	8,082	8,520	19,872
high	289	1,622	1,943	855	4,710

T_r = 2500 yr	*Direct Economic Losses (US$ 1994 million)*				
Design level	slight	moderate	extensive	complete	total
low	0.7	36.7	574	4,752	5,363
moderate	194	2,629	9,691	15,294	27,808
high	221	1,761	3,006	1,765	6,752

Table F.8: Total direct economic loss results by design level and earthquake scenario

F.5 Indirect Economic Loss by Design Level and Earthquake Event

	Indirect Economic Losses (US$ 1994 million)						
T_r = 100 yr							
Design level	1^{st} year	2^{nd} year	3^{th} year	4^{th} year	5^{th} year	years 6 to 15	total
low	12.4	73.8	104	107	107	107	510
moderate	42.8	254	357	366	366	366	1,752
high	9.8	58.2	82.1	84.0	84.0	84.0	402
T_r = 250 yr							
Design level	1^{st} year	2^{nd} year	3^{th} year	4^{th} year	5^{th} year	years 6 to 15	total
low	24.2	133	187	191	192	192	919
moderate	83.1	458	642	658	658	658	3,157
high	19.1	105	147	151	151	151	725
T_r = 500 yr							
Design level	1^{st} year	2^{nd} year	3^{th} year	4^{th} year	5^{th} year	years 6 to 15	total
low	29.9	162	227	232	232.6	233	1,117
moderate	117	632	886	906	908	908	4,357
high	26.9	145	204	209	209	209	1,003
T_r = 1000 yr							
Design level	1^{st} year	2^{nd} year	3^{th} year	4^{th} year	5^{th} year	years 6 to 15	total
low	39.0	208	291	298	297.8	298	1,431
moderate	173	920	1,288	1,318	1,319	1,319	6,337
high	41.0	218	305	312	313	313	1,502
T_r = 2500 yr							
Design level	1^{st} year	2^{nd} year	3^{th} year	4^{th} year	5^{th} year	years 6 to 15	total
low	47.5	250	350	358	358	358	1,722
moderate	246	1,296	1,815	1,856	1,858	1,858	8,930
high	59.8	315	441	451	451	451	2,168

Table F.9: Total indirect economic loss results by design level and earthquake scenario

F.6 Total Loss Overview by Design Level and Earthquake Event

$T_r = 100$ yr	Total Losses					
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	1,784	510	683	209	31	63
moderate	5,481	1,752	2144	562	80	159
high	1,224	402	289	50	6	12

$T_r = 250$ yr	Total Losses					
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	2,963	919	1347	406	80	160
moderate	10,182	3,157	4915	1329	250	498
high	2,337	725	714	151	25	51

$T_r = 500$ yr	Total Losses					
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	3,554	1,117	1545	569	89	178
moderate	13,868	4,357	7177	2380	359	716
high	3,193	1,003	1046	277	38	77

$T_r = 1000$ yr	Total Losses					
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	4,487	1,431	1952	638	108	226
moderate	19,872	6,337	11634	3509	577	1200
high	4,710	1,502	1754	449	68	142

$T_r = 2500$ yr	Total Losses					
Design level	Direct Economic (US$ 1994 million)	Indirect Economic (US$ 1994 million)	# Casualties			
			severity 1	severity 2	severity 3	severity 4
low	5,363	1,722	2473	857	146	305
moderate	27,808	8,930	19028	6207	1035	2155
high	6,752	2,168	3082	895	142	296

Table F.10: Total loss by design level and earthquake scenario